

**GRAPHING CALCULATORS
AND THE ZONE OF PROXIMAL DEVELOPMENT:
A STUDY OF FOURTEEN YEAR OLD MALAYSIAN STUDENTS'
DEVELOPMENT OF GRAPHICAL CONCEPTS WITH TECHNOLOGY**

Baiduriah Yaakub

**A thesis submitted for the degree of
Doctor of Philosophy**

**Institute of Education
University of London**

2002



ABSTRACT

This study investigates 14 year-old students' development of graphical concepts using graphing calculators. Two learning models based on two broad interpretations of Vygotsky's "zone of proximal development" were implemented to gauge the role of graphing calculators in technology-based learning. Epistemological case studies were used to ascertain the extent to which the graphing calculator facilitated the learning of key graphical concepts. To this end, students of different levels of mathematical attainment were observed to determine the different kinds of understanding they derived from using the technology.

The 24 students participating in the study were pre and post tested, and formed into two groups. One group was taught according to a structured, teacher-led learning model, and the other group was taught according to an open-ended, activity-led learning model.

What emerges from the study is the complexity of the teaching and learning situation when technology is incorporated. A student's learning of graphical concepts with the graphing calculator was the result of an interplay between his/her knowledge of the functionality of the graphing calculator, existing mathematical knowledge and the nature of teacher intervention.

The use of the graphing calculator raises the issues of the ordering or sequencing of learning of graphs from simple linear equations to those perceived as more complex polynomials. With the graphing calculator, students were able to learn much more than was thought possible. Changes in students' mathematical learning were accompanied by a change in the role of the calculator from a static display tool to a mediational tool. The study also highlights the issues of teachers' roles when technology is incorporated including teacher's content knowledge, and the ways in which teachers intervene with students, in particular how teachers deal with students' semantic and syntactic errors in using the calculator.

ACKNOWLEDGMENTS

My thanks go primarily to my supervisor, Dr. Ian Stevenson for having had the patience to guide and assist me through the completion of the thesis. His insights gave me invaluable formative experiences, and his constant encouragement to persevere was a source of inspiration.

My heartfelt thanks also go to my husband, Yusoff, for providing me the moral and financial support, and patiently tolerated my preoccupation with the thesis. To him I dedicate this thesis.

This study would not have been possible without the cooperation of the principal, the teachers and the students at the school where the research was conducted. My deepest thanks to the two teachers who had the willingness, and the courage to try the teaching modules and use the graphing calculator, a tool that they encountered and used for the first time in their teaching career.

I am also particularly grateful to Prof. Celia Hoyles for her critical comments and suggestions.

These acknowledgements would not be complete without thanks to all the staff in the Mathematical Sciences Group whose friendship over the years and helpfulness in so many ways, contributed in creating a supportive working environment.

Finally, my sincere thanks goes to Dr. Philipp Kent for his reading and constructive comments of the thesis.

TABLE OF CONTENTS

	Page
Abstract	ii
Acknowledgements	iii
Table of contents	iv
List of appendices	viii
List of figures	ix
List of tables	xi
List of excerpts	xii
CHAPTER 1 BACKGROUND	
Introduction	1
1.1 Why the graphing calculator?	3
1.2 Graphing in Malaysian schools	5
1.3 Technology and teachers	9
1.4 Aims of this study	10
1.5 Contents of the thesis	10
CHAPTER 2 A REVIEW OF THE LITERATURE	
2.1 Graphs and graphing	13
2.2 Misconceptions and difficulties in graphing	16
2.2.1 Linearity	17
2.2.2 Continuity	18
2.2.3 Representations	20
2.2.4 Scaling	22
2.2.5 Variable	23
2.3 The use of a graphing calculator in students' learning	24
2.3.1 Graphical concepts	25
2.3.2 Errors related to graphing calculator use	29
2.3.2.1 Visual illusions	30
2.3.2.2 View-windows	32
2.3.2.3 Tool artefacts	34
2.3.3 Issues of instructional approach	36

2.4	The role of technology in students' learning	39
2.5	The role of the teacher with technology	49
2.6	Conclusion	54

CHAPTER 3 THE THEORETICAL FRAMEWORK

3.1	The Vygotskian perspective on knowledge construction	56
3.2	Different interpretations of the zone of proximal development	63
3.3	The learning models for this study	67
3.4	The study	72
3.5	The research questions	73

CHAPTER 4 METHODOLOGY

4.1	The design of the study	74
4.2	The design of the lessons	76
4.3	The design of the assessment	82
4.4	Pilot study	84
4.5	Outcomes of the pilot study and implications for the main study	84
4.6	Main study implementation	87
4.6.1	Teacher selection	88
4.6.2	Teacher training	88
4.6.3	Selection of students	89
4.6.4	Videotaping in the classroom	90
4.6.5	Data collection method	90
4.7	The analytic framework	94
4.7.1	Preliminary data analysis	95
4.7.2	Selection of case study students	96
4.7.3	The case studies	96

CHAPTER 5 CASE STUDY 1: THE CONCEPT OF CONTINUITY

5.1	Background	101
5.2	Expected learning outcomes with the graphing calculator on continuity	103
5.3	Analysis of case study students in the Structured Learning Model	105
5.4	Conclusion on the concept of continuity in the Structured Learning Model	123
5.5	Analysis of case study students in the Interpretative Learning Model	124

5.6	Conclusion on the concept of continuity in the Interpretative Learning Model	139
-----	--	-----

CHAPTER 6 CASE STUDY 2: THE CONCEPT OF GRADIENT

6.1	Background	140
6.2	Expected learning outcomes with the graphing calculator on gradient	145
6.3	Analysis of case study students in the Structured Learning Model	146
6.4	Conclusion on the concept of gradient in the Structured Learning Model	166
6.5	Analysis of case study students in the Interpretative Learning Model	167
6.6	Conclusion on the concept of gradient in the Interpretative Learning Model	182

CHAPTER 7 CASE STUDY 3: THE CONCEPT OF LINEAR GRAPHS

7.1	Background	183
7.2	Expected learning outcomes with the graphing calculator on linear graphs	187
7.3	Analysis of case study students in the Structured Learning Model	189
7.4	Conclusion on the concept of linear graphs in the Structured Learning Model	206
7.5	Analysis of case study students in the Interpretative Learning Model	207
7.6	Conclusion on the concept of linear graphs in the Interpretative Learning Model	224

CHAPTER 8 CASE STUDY 4: THE CONCEPT OF NON-LINEAR GRAPHS

8.1	Background	226
8.2	Expected learning outcomes with the graphing calculator on non-linear graphs	229
8.3	Analysis of case study students in the Structured Learning Model	231
8.4	Conclusion on the concept of non-linear graphs in the Structured Learning Model	242
8.5	Analysis of case study students in the Interpretative Learning Model	243
8.6	Conclusion on the concept of non-linear graphs in the Interpretative Learning Model	259

CHAPTER 9 FINDINGS AND DISCUSSION

9.1	Mathematical knowledge	262
9.2	Functionality of the calculator	266
9.3	Teacher intervention	268
9.4	The learning models	270
9.5	Models of students' learning	272

CHAPTER 10 CONCLUSION

10.1	The impact of technology on students' learning	276
10.2	The value of using Vygotsky's idea of the zone of proximal development	278
10.3	Implications for the integration of graphing calculators in students' learning	279
10.4	Limitations of the study and suggestions for future research	281
10.5	Concluding remarks	283

BIBLIOGRAPHY	284
---------------------	-----

LIST OF APPENDICES

	Page
Appendix 1 Excerpt of a student's disapproval of the open learning approach	304
Appendix 2 Teaching Module for Structured Learning Model	305
Appendix 3 Teaching Module for Interpretative Learning Model	343
Appendix 4 Students' Worksheet for Structured Learning Model	379
Appendix 5 Students' Worksheet for Interpretative Learning Model	411
Appendix 6 Graphing Calculator Manual	437
Appendix 7 Assessment 1	459
Appendix 8 Assessment 2	460
Appendix 9 Assessment 3	461
Appendix 10 Assessment 4	463
Appendix 11 Assessment 5	464
Appendix 12 Pre-test	466
Appendix 13 Post-test	475

LIST OF FIGURES

		Page
Figure 2.01	Graphs of a family of parabolas	15
Figure 2.02	Item on scaling in the CSMS study	22
Figure 2.03	Graph of $y=-0.5x-2$	30
Figure 2.04	Graph of $y=-0.5x+2$	30
Figure 2.05	Graph of $y=-2x-3$	30
Figure 2.06	Graph of $y=-2x+3$	30
Figure 2.07	Graph of $y=-0.5x+2$	31
Figure 2.08	Graph of $y=-x-2$ in the INIT window	31
Figure 2.09	Graph of $y=-x-2$ when “zoomed out”	31
Figure 2.10	Graph of $y=x^2-x-1$ in the INIT window	31
Figure 2.11	Graph of $y=x^2-x-1$ when “zoomed out”	31
Figure 2.12	Graphs of $y=5x^2-2x+1$ and $y=5x^2-2x-2$	32
Figure 2.13	Graphs of $y=2x+3$ and $y=-0.5x-2.5$ in the “standard window”	32
Figure 2.14	Graph of $y=x$ drawn in different view-windows	32
Figure 2.15	Graph of $y=3x$ in the standard window	33
Figure 2.16	Graph of $y=3x+400$	34
Figure 2.17	The view-window setting	34
Figure 2.18	The graph of $y=2x^3-16x^2+12x+6$	34
Figure 4.01	Flowchart of instrument implementation	91
Figure 4.02	The layout of the classroom and students	92
Figure 5.01	A view-window with large values	106
Figure 5.02	A previous view-window	108
Figure 5.03	Movement of the tracer from one point to the next point	122
Figure 5.04	Graph of $y=x$ generated in the G-CON and G-PLT with range values of x from 1 to 6	124
Figure 5.05	Effects of the change in pitch value	125
Figure 5.06	An unusual view-window setting by Grace	125
Figure 6.01	Graphs of equations with different values of “ m ”	141
Figure 6.02	Gradient as the ratio of (y_2-y_1) to (x_2-x_1)	141
Figure 6.03	Graphs of $y=x+1$, $y=x$ and $y=x-1$ in two different view-windows	147
Figure 6.04	Wan’s sketch of graphs with different gradients	149
Figure 6.05	Graphing calculator graphs	155

Figure 6.06	Ng's sketch	155
Figure 6.07	Ng's hand draw graphs of $y=mx$	158
Figure 6.08	Graphs of $y=Mx$ generated in the DYNA Mode for range of values of "M" between -3 and 3	164
Figure 6.09	Emy's imperfect STARBURST picture in the STD window	180
Figure 7.01	Replication of the graphic display	197
Figure 7.02	Amin's work on determining the gradient	204
Figure 7.03	"Graph Func" syntax on both screens	215
Figure 8.01	The parabola generated in the G-PLT and G-CON command with range from $x=-3$ to $x=3$	230

LIST OF TABLES

	Page
Table 4.01 Differences between the Structured and Interpretative Learning Model	75
Table 4.02 The learning activities in the learning models	77
Table 4.03 Example of tasks and teacher interventions in the two learning models	81
Table 4.04 General intervention questions suggested in the pilot study	86
Table 4.05 A summary of the types of case studies	99
Table 5.01 Students' performance on continuity in the StuctLM	102
Table 5.02 Students' performance on continuity in the InterLM	103
Table 6.01 Students' performance on gradient in the StuctLM	143
Table 6.02 Students' performance on gradient in the InterLM	144
Table 7.01 Students' performance on linear graphs in the StuctLM	185
Table 7.02 Students' performance on linear graphs in the InterLM	186
Table 8.01 Students' performance on non-linear graphs in StructLM	228
Table 8.02 Students' performance on non-linear graphs in the InterLM	228

LIST OF EXCERPTS

		Page
Excerpt 5.01	Wan's understanding of the trace facility and number sense	105
Excerpt 5.02	Wan's understanding of the view-window in relation to the movement of the pointer	107
Excerpt 5.03	Ng's understanding of the view-window and tracer	111
Excerpt 5.04	Ng's understanding of the view-window in relation to the appearance of the graph	114
Excerpt 5.05	Ng's difficulty with the tracer	115
Excerpt 5.06	Amin's difficulty with choosing a view-window	119
Excerpt 5.07	Teacher intervention with Grace	126
Excerpt 5.08	Presha's difficulty with the calculator operations	128
Excerpt 5.09	Presha's difficulty with the TABLE Mode	130
Excerpt 5.10	Emy's difficulty with the mathematical terms	133
Excerpt 5.11	Emy's difficulty with the functionality of the TABLE Mode	135
Excerpt 6.01	Wan's understanding of the functionality of the DYNA Mode	148
Excerpt 6.02	The influence of Wan's mathematical language	150
Excerpt 6.03	The influence of Wan's algebraic competency	152
Excerpt 6.04	The graphing calculator as a display tool	154
Excerpt 6.05	Ng's difficulty with the DYNA Mode	156
Excerpt 6.06	Teacher's ineffective intervention	161
Excerpt 6.07	Knowledge compartmentalisation arising from difficulty in making sense of graphic display	165
Excerpt 6.08	Grace's understanding of the DYNA Mode	168
Excerpt 6.09	The influence of Grace's mathematical knowledge	169
Excerpt 6.10	The graphing calculator as a display tool	172
Excerpt 6.11	The role of the GRAPH Mode	176

Excerpt 6.12	Emy's misconceptions	178
Excerpt 7.01	Using the graphing calculator to support conjectures	189
Excerpt 7.02	Wan's ability to follow the curriculum script	190
Excerpt 7.03	Ng's inability to follow the curriculum script	194
Excerpt 7.04	Amin's lack of mathematical knowledge	200
Excerpt 7.05	Amin's lack of consciousness on what is keyed in	204
Excerpt 7.06	Grace's knowledge in decimals and fractions	208
Excerpt 7.07	Teacher's corrective feedback on syntactic error	210
Excerpt 7.08	Presha's inability to make sense of the graphic display	212
Excerpt 7.09	Intervention that did not resolve Presha's difficulty and syntactic error	214
Excerpt 7.10	Intervention that focuses on obtaining the correct answer	217
Excerpt 7.11	Emy's difficulty in hand graphing	220
Excerpt 7.12	Teacher's scaffolding in Emy's exploration of graphs of $y=mx+c$	222
Excerpt 8.01	Transformation in Wan's activity	232
Excerpt 8.02	Inappropriate use of calculator facility	235
Excerpt 8.03	Using the graphing calculator as a display tool	238
Excerpt 8.04	Grace's specific questions	243
Excerpt 8.05	Interpretation of the graphic display	245
Excerpt 8.06	The quality of teacher's knowledge	246
Excerpt 8.07	Knowledge source from classroom teaching	247
Excerpt 8.08	Intervention on semantic error and Presha's difficulty with the task	250
Excerpt 8.09	The impact of the quality of the teacher's knowledge	251
Excerpt 8.10	The influence of lack of mathematical knowledge and teacher intervention	254

CHAPTER 1

BACKGROUND

Introduction

Education occurs within a sociocultural context of which technology is an integral part, that continues to evolve and therefore cannot be ignored. In mathematics education, it has been documented that technology can perform a wide range of mathematical work including algebraic manipulation, solving equations, finding integrals and derivatives, and plotting graphs, thereby having an impact on the mathematics learnt in school. Such technology stimulates an ongoing debate about the appropriate content and organization of the mathematics curriculum when technology is used. More importantly the capability to provide different ways of calculating, and therefore new opportunities for analyzing and solving problems, makes technology a vehicle to transform the teaching and learning process itself. However, technology in and of itself will not change the way teachers teach and the way students learn, what matters is how technology is used (Sandholtz et al., 1997). If it is used in a “meaningful” way, it can transform learning (Sandholtz et al., 1997; Doerr and Zangor, 2000), offering different ways and opening new doors in the teaching and learning of mathematics. Meaningful here means involving the learner, encouraging autonomy, and engagement in questioning, and reflection (Burton and Jaworski, 1995). The challenge therefore is to examine how and to what extent a particular technology can offer support in students’ learning. Correspondingly, how can technology be integrated in the classroom: what is its role and what is the role of the teacher in relation to it?

This brings us to the focus of this thesis. The research presented here was initially motivated by the concern to reform the traditional form of instruction in Malaysian schools, which can be summed up as teacher-centered and based on the transmission model of learning (where the teacher imparts knowledge to the students). The recommendation in the curriculum review in 1989 for the development of a more learner-centered approach, making the learner primarily responsible for his/her own learning in the active construction of knowledge, had met with little practical success up

till 1995 (Pusat Perkembangan Kurikulum, 1995). The *SMART School*¹ curriculum (Ministry of Education Malaysia, 1997), to be implemented in stages from 1999 further stresses “moving away from memory-based learning” (p. 9) and catering for individual abilities in the form of technology-supported learning.

The use of scientific and graphing calculators is non-existent in Malaysian secondary schools. Four function calculators are used in secondary schools as a tool to aid in computations, but in general, the use of calculators has no place in mathematics learning. The almost negligible impact of calculators in mathematics classrooms in Malaysia may be attributed to two major reasons. First, simple four function calculators are allowed in the public examination only at the *Sijil Pelajaran Malaysia* level, taken at the end of the fifth year in secondary school by seventeen year-olds. In Mathematics, calculator use is restricted to one of the two papers taken in which all questions are of “problem-solving” type. Second, the Mathematics Syllabus for secondary school implemented in 1989 preceded the policy on allowing the use of simple calculators in the *Sijil Pelajaran Malaysia* examination in 1995. The syllabus had not explicitly stated nor suggested that calculators can be used as a teaching tool in the teaching and learning of mathematics (Pusat Perkembangan Kurikulum, 1989a, b, c; 1990a, b). Since the purpose of calculators in the examination is to aid in carrying out the four basic operations and their use restricted to straightforward computations, calculators in classrooms are also used only to expedite or to check computations. There is very little acknowledgment of their potential in supporting knowledge construction and understanding. It is thus challenging in the Malaysian context to carry out research using graphing calculators, an evolving personal technology considered by many to have the potential to revolutionize mathematics education (Dick, 1992; Ruthven, 1992, 1996; Oldknow, 1995; Waits and Demana, 1996; Slavitt, 1996; Barling, 1997; Waits, 2000).

¹ The implementation of the *SMART School* concept beginning in 1999 with eventual nationwide implementation in 2010 (Ministry of Education Malaysia, 1997) is very relevant to this thesis. The *SMART School* concept rests upon the need to transform the Malaysian educational system to support the nation’s drive to become a fully industrialized nation by the year 2020 (Dato Seri Dr. Mahathir Bin Mohamad, 1991). This calls for a technologically literate, thinking work force. Technology-supported schools are seen as the catalyst for this transformation. The *SMART School* curriculum has also highlighted the need to offer equal access to learning opportunities and to accommodate differing learning abilities, styles and paces.

Although graphing calculators are suggested in the *SMART School* curriculum, the lack of empirical evidence of their potential, and lack of support and know-how for integration in the mathematics classroom renders them almost useless to the teachers. Having the technology resources in the school does not necessarily imply that teachers will use them in their teaching (Norton et al., 2001). Indeed, teachers require the opportunity to look at new developments and examples of technology as used in educational practice so that they become aware of the opportunities they offer (Reys and Smith, 1994; Dawes, 2001).

1.1 Why the graphing calculator?

Graphing calculators have been shown to provide new learning contexts in the classroom involving “experimental” strategies, new kinds of learning activity among students and the representation of mathematical information in new ways (Shumway, 1990; Ruthven, 1990; 1994b; Hector, 1992; Dunham and Dick, 1994; Drijvers and Doorman, 1996; Graham and Thomas 1997; Gomez and Fernandez, 1997). Depending on the model, the current power of graphing calculators includes generating tables and graphs of linear and polar functions, solving equations, simple programming and computer algebra (see Oldknow, 1995; Kissane, 1997; Oldknow and Taylor, 2000). Hennessy et al. (2001) has outlined how the graphing calculator facilitates students’ graphing. For example, it facilitates the visual representation of functions through dynamic graphing, and speeds up the process of graphing, thereby freeing students to analyze and reflect on mathematical activities. They also found that the graphing calculator encouraged translation back and forth between numeric, graphical and algebraic representations. The ability to manipulate and generate multiple graphs easily provided opportunities for exploring the properties of and relationships between graphs. Ruthven (1990) found that the graphing calculator encouraged both students and teachers to make more use of graphical approaches in solving problems and to develop new mathematical ideas. Rich’s (1991) study compared two teachers, reporting that the teacher who used graphing calculators used more exploration and encouraged conjecturing in learning activities, compared with the teacher who did not use the graphing calculator. Also, the graphing calculator teacher asked more higher-level questions, used examples differently, stressed the importance of graphs and approximation in problem solving, used more graphs and explored the connection between algebra and geometry.

It is argued that for technology to have any impact on the curriculum and the way teachers teach, the contact time between the students and the technology must be considerable (Doerr and Zangor, 2000; Selinger, 2001). This is a vital question of accessibility when integrating technology in the curriculum – how well is it distributed among students and is it readily available or restricted to a laboratory setting? Studies have indicated that an important factor in the general acceptance of technology by teachers is the availability and portability of the technology (Blease and Cohen, 1990; Martin, 1991; Schmidt and Callahan, 1992). The Chicago Mathematics Project (Usiskin, 1993) reported that calculators were easier to integrate into secondary school mathematics compared to computers, citing problems of access to computers at the required times. The same situation can be argued for many schools nowadays in Malaysia, in particular schools in rural areas that lack the necessary infrastructure for computers. The use of graphing calculators seems more viable. They are not only portable, they are relatively cheaper and robust (Shumway, 1990; Oldknow, 1995; Ruthven, 1994a; Goldstein, 1994; Waits and Demana, 1996; Graham and Thomas, 1997; Berger, 1998) and flexible in terms of time and use (Usiskin, 1993, Hennessy et al., 2001). While this renders them more accessible and thus attractive for frequent use, as Ruthven (1995, 1996) recommends, like any other technological tool their value lies in how they can be used to facilitate teaching and learning. This is precisely what this study sets out to do: to characterize the transformation in students' understanding of mathematical concepts when using the graphing calculator in their learning.

The potential of the graphing calculator for widespread use as a personal tool (both inside and outside the classroom) is made possible by portability and relatively low price (Ruthven, 1990, 1996; Goldstein, 1994; Oldknow, 1995). It is possible that this evokes a notion of ownership and acceptance different from desktop computers, giving students the incentive to make use of the calculator and learn how to use it effectively (Goldstein, 1994; Ruthven, 1995). There is some evidence that the small screen of graphing calculators allows students to make mistakes in private, away from the ridicule or scrutiny of their peers, creating opportunities for conjecturing and developing confidence in the process (Smart, 1995). Indeed Ruthven (1990) suggests that the use of a graphing calculator can reduce uncertainty, and therefore anxiety during learning. Several studies found evidence the graphing calculator makes doing mathematics more enjoyable (eg.,

Dick and Shaughnessy, 1988; Graham and Thomas; 1997; Hennessy et al., 2001). This has great implications for dealing with the student attitudes that are increasingly seen as crucial factors affecting their school performance (Schoenfeld, 1989; Galloway et al., 1996).

1.2 Graphing in Malaysian schools

Leinhardt et al. (1990) note that the way equations (functions) and graphs are introduced to students in schools varies from country to country with no proven optimal approach. Since the present study has been developed in the Malaysian school context, the way graphing is approached there will be briefly described. It is useful to point out that, in general, the mathematics curriculum is structured according to content-by-process matrices, in which a list of mathematical topics is outlined as content corresponding to some variant of the Bloom taxonomy² as process (Pusat Perkembangan Kurikulum, 1989a, b, c; 1990, a, b).

Graphing starts in the elementary school and is approached as the display of information in the form of bar graphs, pictographs, circle graphs and line graphs. It is only in Secondary 2, students are taught to plot points on the Cartesian axes (14 year olds), moving on to linear equations in Secondary 3. The concept of the straight line and its equation $y=mx + c$ is covered in Secondary 4. Graphing of other equations restricted to certain polynomials, namely quadratic, cubic and reciprocal is covered in Secondary 5. This is extended to solving equation(s) by graphical methods and determining the region of an inequality in two variables. It is also expected that over time, students should become able to relate certain types of equations with certain standard shapes of graphs, and vice versa, for example, straight line, parabola, cubic and exponential.

Students also do graphing work in their science and other classes. In science, the graphing work usually starts with the data obtained from observation, which is set up as axes construction, points plotted and finally a graph. Graphs are sometimes plotted to identify the existence of a pattern and hence deduce a relationship, or to predict on the outcome of a variable under consideration. Emphasis is on the qualitative interpretation

² Knowledge and comprehension underpin “higher level” thinking processes like application, analysis and evaluation (Bloom, 1956).

of graphs, that is, looking at the graph to a relationship between the two variables. This is different from what is learned in the mathematics classroom where graphing work usually starts from a given algebraic function to generate ordered pairs for a graph. In geography, students are acquainted with pie charts, bar graphs, line graphs or other kinds of frequency graphs. These graphs often only consider one continuous variable (frequency), with the other axis used to show entirely independent items. Students also encounter the ideas of dependence and variables in separate mathematical topics like ratio and proportions. However, such diverse ways of thinking about graphs do not necessarily facilitate students' understanding (see e.g., Leinhardt et al., 1990; Demana et al., 1993).

The structure of the Malaysian curriculum for graphing can in part be identified with the typical mathematics textbooks topics in Grades 1-8 as analysed by Demana et al. (1993). Some of their criticisms apply to the Malaysian school context. One is the late entry of engaging students in constructing the graph of an equation (not until Grade 7) and this is restricted to linear graphs, and students still do not encounter graphing of non-linear equations in Grade 8. The graphing activities tend to have a local, rather than global focus. Demana et al. argue that this results in students having difficulties in making global interpretations of graphs and making the connection between an equation (a function) and its graph. They suggest that graphing activities should start very early, and should be a major part of the curriculum as students need extensive practice in making and reading from graphs. They further assert the need to be mindful of what Herscovics (1989) pointed out: "It is easy to underestimate the level of sophistication involved in the construction of the Cartesian plane schema. Since most students manage to provide the coordinates of points in the plane and, conversely, locate points when given coordinates we conclude that they generally understand the structure involved." (p. 68). This, they argued was not the case. Malaysian students lag two years behind their American counterparts: they start to construct graphs of equations only in their third year of secondary school. This is a disturbing situation given the vast literature on students' difficulty in graphing. A study conducted by Ismail and Salleh (1997) revealed that 14 year old Malaysian students had difficulty in choosing an appropriately scaled set of axes for a given data set. They too concluded that students needed to have substantial guidance and practice in the construction and interpretation of graphs.

My concern with the Malaysian curriculum is that the way it is structured is based on the notion that mathematical learning is hierarchical from “low level” skills to “high level” skills, illustrated for example in its approach to graphing from “simple” linear equations to the supposed more “difficult” higher order polynomials. Graphing is also treated in a fragmented way, presented in bits and pieces, for example, in Secondary 2 students begin to plot points and stop at the determination of the length and the mid-point of a line between two points. It is not until the following year that they begin to draw graphs of some linear equations, and only after that are they formally introduced to the general form of equation for the straight line ($y=mx+c$). Graphs of higher order polynomials are not introduced until another year later. As graphing technology is not used in the learning of graphs, it is common that many graphs of equations of higher order are not explored as this is tedious and time consuming to do. Likewise, the investigation of the properties of graphs of different families is also restricted.

Many have argued that the limited opportunity for students to explore graphs of different polynomials, and to manipulate and compare graphs of the same family, is the root cause of students not recognising the form of an equation and its associated graph (Swan, 1982; Demana et al., 1993; Philipp et al., 1993; Kieran, 1993), and students having a local rather than the global view of the graph (Demana et al., 1993; Kieran, 1993). The limited opportunity to explore and manipulate graphs deprives students from making connections across the three “multiple linked representations” in graphing (algebraic, tabular or numerical, and graphical) (Goldenberg, 1988; Demana et al., 1993; Moschkovich et al., 1993; Kieran, 1993; Wilson and Krapfl, 1994). Students are also obscured from seeing the equation and the graph as objects or entities that can be manipulated as a whole to form new objects as described by Sfard (1991). Sfard’s object or structural conceptualisation³ is useful because it emphasises the importance of detaching the new entity from the process that produced it. In this way, the process of generating a graph has been criticised as one of the factors that obscures students’ understanding of graphing. The time spent graphing by manually plotting individual

³ Sfard posited that abstract concepts could be conceived in two different ways, structurally - as objects, and operationally – as processes. The structural conception refers to seeing a mathematical concept as an abstract object that exists somewhere in space and time that can be manipulated as a whole. The operational conception refers to seeing a mathematical concept as a detailed sequence of operations and algorithms. The structural conception is static, instantaneous and integrated whereas operational conception is dynamic, sequential and detailed.

points obscures the connection between the equation and the graph as “object” (Dick, 1992). As Swan (1982) suggests, these disconnections result in students losing sight of the meaning of graphing tasks.

The advent of graphing technology has been seen as providing a new opportunity for students to explore graphs and to facilitate the forming of connections across representations. Yerushalmy and Schwartz (1993) claim that the use of graphing technology “frees users from the physical action of plotting. This, in turn, may permit them to shift the focus of their attention away from the detail of the process of making the graph and allow them to benefit from the contemplation and manipulation of the graph itself” (p. 46). They also argue that graphing technology which allows the learner to manipulate the graphical representation itself (e.g., to “stretch” or “squeeze”) and to view the impacts of this on the numerical and symbolic representations, or those that allow the user to manipulate the numerical representations and view the result in the graph and the symbolic expression, make it more possible for the learner to view the function and any of its representations as a single entity. It must be pointed out that the current graphing calculator technology is restricted in terms of this kind of interactivity. It cannot provide feedback to the user and the graphs it produces cannot be physically and directly manipulated, for example, rotated, pushed or pulled. Therefore “direct graphing action” described by Smith and Confrey (1992) with the computer software Function Probe, in which the user can “create” an action as a visual experience, taking transformations “back” to their geometric origin, or those mentioned above are not possible with the graphing calculator. However, the graphing calculator in the dynamic mode does enable the user to see what happens to a graph when parameters are changed. This dynamic capability has been argued to provide an environment that enables the user to see graphs as objects that can be transformed, thus drawing students’ attention to the graph globally (Goldenberg, 1988).

We must be mindful of the potential problems that arise when technology is used in students’ learning. Moschkovich et al. (1993) cautioned that it is not the case that “once things are shown clearly on the computer screen, then students will understand” (p. 76). The limitations of the tool (and the user) create the phenomena of “blindness” and “breakdown” as described by Winograd and Flores (1986). These refer to the inevitable fact that no tool will be infinitely flexible. Therefore it will be blind to certain functions

and can inhibit certain kinds of understanding. It can engender certain misconceptions and cause interruption in learning. Some researchers have warned of the potential pitfalls of the graphing tools (e.g., Dion, 1990; Kaput, 1992; Demana et al., 1993), and others have provided evidence of a number of misconceptions that students develop when using these graphing tools (Goldenberg, 1988; Dugdale, 1993; Moschkovich et al., 1993; Ward, 2000). Against the backdrop of the limitations and potentiality of the graphing calculator, it is interesting to see how the graphing calculator facilitates students to understand the relationship between the equation and the graph it represents. To what extent does using the graphing calculator afford the viewing of graphs globally? Does this global view facilitate students' understanding of graph transformations when the parameters in an equation are varied?

1.3 Technology and teachers

There are broadly two main choices for integrating technology into learning. One way is to use it as add-on to facilitate the teaching and learning of some concepts predefined by the curriculum. Alternatively, one can take advantage of the tool's ability to effect deeper changes in the way students learn and the concepts that they learn. Whichever way technology is adopted, the introduction of technology arguably will affect the role of the teacher. For example, the concern towards getting students to be more autonomous and to self-regulate their learning has resulted in some studies that use technology to provide learning environments for student-directed exploration. Hoyles and Sutherland (1989) found that there is a critical balance between teacher-initiated activities and student-directed exploration in student's learning. Student's progress or lack of progress in some areas was found to be closely associated to the presence or absence of appropriate teacher intervention. Noss and Hoyles' (1996) idea of the phenomenon of "didactical paradox" in which "didactical situations 'force' teachers to tell the students what to do; yet the same process empties the learning situation of any cognitive content..." (p. 70), cautions us to be mindful of the role of intervention in student's learning.

The graphing calculator is a powerful classroom resource. The teacher's role is crucial, and the presence of the calculator changes this role. What is the nature and consequences of teacher interventions in the learning process? Specifically, what kinds of

interventions promote and complement graphing calculator use that enhance students' learning?

1.4 Aims of this study

Against the backdrop of the Malaysian secondary school year (14 year-old students), this study therefore sets out to investigate:

- the graphical concepts that students can come to understand with the graphing calculator;
- the role of the graphing calculator in facilitating understanding of graphical concepts;
- the extent to which different learning environments in which the graphing calculator is being used structure student's learning;
- the student's approach to using the graphing calculator in his/her learning (including the strategies used when encountered with difficulties and errors operating the calculator); and
- the nature and consequences of teacher intervention in the learning process.

What is needed is a model of teaching and learning with technology which will enable exploration of these issues. A major aspect of this study will be to develop an appropriate framework.

1.5 Contents of the thesis

Chapter 2 presents a review of literature for this study. It starts with defining the meaning of graphs and what constitute as "graphing". Difficulties and misconceptions that are encountered in the learning of graphing concepts are discussed. Included are issues related to graphing calculator use. The role of technology in student's learning and the role of the teacher when technology is used are reflected upon.

Chapter 3 follows with the theoretical framework of the study. The Vygotskian view on knowledge construction and the concept of *zone of proximal development* (ZPD) is

explained. The rationale for using the Vygotskian view on knowledge construction is discussed and, drawing from two broad interpretations of Vygotsky's idea of the ZPD, two learning models are proposed for use in this study. The two learning models are defined. This chapter concludes by defining the scope of the study and the research questions.

Chapter 4 describes the methodology employed in the study. It begins by presenting the design of the study, and follows with the design of the lessons used in the study. This chapter also illuminates the test instruments used, their design and the purpose of each instrument. Problems encountered in the pilot study are discussed in relation to their implications for the main study. The student sample of the study and its method of selection are also described. The criteria for choosing the teachers to participate is also discussed. A description of how the learning models were implemented in the classroom and how the classroom was organized to enable the recording of data for the study, is included. How the analytic framework, which follows an iterative design was developed is discussed. This chapter concludes with a description of the use of epistemological case studies to analyze the ways in which the graphing calculator facilitated the learning of particular graphical concepts.

Chapters 5, 6, 7 and 8 are each analysis of case study of the graphical concepts of continuity, gradient, linear graphs and nonlinear graphs respectively. Each case study analyzes how students of different mathematical attainment in each of the two learning models learned with a graphing calculator.

Chapter 9 follows with a discussion of findings of the study framed within its theoretical considerations. It also attempts to model the different kinds of understanding that students derived from using the technology, and concludes with an overview of the complexity of teaching and learning situations when technology is integrated into students' learning.

Chapter 10 presents the conclusions of the study. The complexity of the teaching and learning situations observed are considered relative to the framework of the study. Conclusions about the impact of the graphing calculator on learning are put forward.

Finally, the contribution and the limitations of the present study are considered and topics for further research are suggested.

CHAPTER 2

A REVIEW OF THE LITERATURE

This review is organised into six sections. The first section discusses the meaning of graphs and graphing, and issues surrounding its learning. Section 2.2 discusses the difficulties as well as misconceptions that are encountered in the learning of equations and their graphs. Section 2.3 considers factors related to the use of graphing calculators, including student learning of graphical concepts, errors in graphing calculator use, and issues of instructional approaches when integrating graphing calculators into learning. This is followed with a discussion on the role of technology in student's learning with a focus on the role of graphing calculator in section 2.4. Section 2.5 discusses the role of teachers with technology to highlight issues surrounding integrating technology in classrooms. The conclusion of the review is presented in section 2.6

2.1 Graphs and graphing

The learning of graphs is problematic and this is well-documented (e.g., Kerslake, 1981; Bell et al., 1987; Goldenberg, 1988; Friedler and McFarlane, 1997; Swatton and Taylor, 1994; Romberg et al., 1993; Knuth, 2000). Dictionary definitions describe a graph as a diagram or picture that shows the relationship between two or more quantities (Flyfield and Blane, 1994), or between two variables (Nelson, 1998), where the relationship is shown by means of points plotted on a coordinate system (Tapson, 1996) whose coordinates satisfy the relation (Collins Dictionary, 1995). Fry (1984) proposed a generic definition: "A graph is information transmitted by position of point, line or area on a two-dimensional surface" (p. 5), including all spatial designs and excluding displays that incorporate the use of symbols such as words and numbers (e.g., tables).

It seems that graphs are used to record and convey information in a visual and immediate way. A graph can represent either a "situation" or a functional relationship. This representation is used to reason about the information displayed as well as to describe or predict the situation or a relationship it represents. On the one hand graphing involves construction, on the other hand, interpretation. Both processes require a fluency with the

vocabulary of graphical language. Therefore, graphs can also be viewed as a form of symbolism with its own syntax and mode of representation (Barclay, 1987; Mokros and Tinker, 1986). Ponte (1987) points out the importance of the global-visual properties of graphs that allow an expert to see at a glance the general features of a relationship. Drawing on these two aspects, Stevenson (1990) proposes that graphs are, therefore, systems of symbolic representation with global-visual properties. Thinking of graphs in this way helps to establish a focus on these two aspects of graphs, which pose particular difficulty in students' learning of graphical concepts.

While there are many types of graphs such as bar graphs and pictographs, the study reported in this thesis is concerned (mainly) with Cartesian graphs. These graphs are drawn using a Cartesian coordinate system, in which the position of a point is determined by its relation to reference lines called axes that are normally at right angles to each other. Descartes (1596-1650) proposed the idea that the relationship between two variables could be visualised by representing the corresponding set of values on a coordinate system. The graph (a line or a curve) can be expressed algebraically as an equation of two variables. Therefore, pairs of values of the algebraic variables in any equation can be represented or interpreted as points on a line or a curve. The curve that is drawn in a coordinate system for a particular equation is the graph of the equation. Conversely, by picturing or representing the related sets of values of two variables as ordered pairs of numbers on the x-y plane, the relationship between the two variables can be visualised through the graph.

A line or a curve drawn in the Cartesian coordinate system for a particular equation or function is a graph of the equation or function. The properties or features of a function can be seen globally by drawing a graph of it. In fact the graph of a function carries with it more immediate information about the function as a whole than a table of value does, which takes much more time to read and assimilate. Graph plotting is useful to introduce students to solving an equation, or to estimate the positions of local maxima or minima. Accurate graph drawing becomes less useful as a student advances in his or her work. Algorithmic and algebraic methods provide alternative approaches to the solutions of equations, and local maxima and minima can be found by methods such as examining the sign of the gradient at points around the point of zero gradient. More important is to

have a good idea of the appearance of the graph of a function. This requires the ability to focus on the major features, such as the domain and range, roughly where the roots may occur, whether points of zero gradient are local maxima, minima or neither, and the occurrence of asymptotes and discontinuities. A graph, which is plotted from a table of values for only a comparatively short interval on the x-axis may miss some of these important features of the function. Taking into account a large interval is tedious and may still not produce the complete graph of the function. A complete graph of a function is a graph that shows all its important behaviour such as its y-intercept, zeros, relative extrema, and end behaviour (Demana et al., 1993). A sketch graph can, by distorting the scales on the axes, concentrate on these major features. However, to acquire the skill of graph sketching requires the ability to have some kind of expectation of what the graph of the given function or equation will look like (Stevenson, 1990).

It is obvious that students have to learn to recognise the different forms of the equations and their corresponding general shapes of graphs. Certain standard forms of equations give rise to certain standard shapes of graphs, for example straight line, parabola, sine curve, and exponential. To understand the connections between equations and their graphs, it is helpful to draw some members of a “family” of equations. The example below (Figure 2.01) shows some members of a family of parabolas, identical except for horizontal translation.

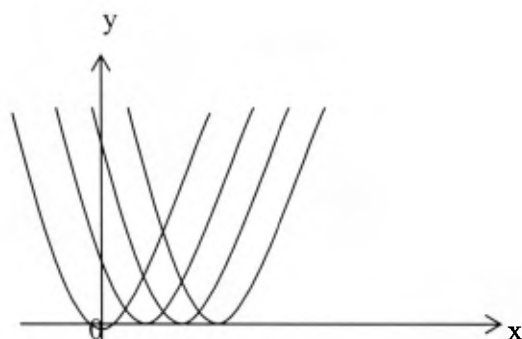


Figure 2.01 Graphs of a family of parabolas

However, the process of generating each individual graph manually is tedious. The amount of effort and time need to construct them will probably exceed the benefit to be gained from graphical representation (Philipp et al., 1993). It is here that the use of

graphing technology is thought to be able to offer students an alternative way of generating graphs and investigating the different shapes of graph associated with forms of equation (Goldenberg, 1988; Shumway, 1990; Vonder Embse, 1992; Kieran 1993, Fox, 1998; Hennessy et al., 2001).

This study is concerned only with graphs drawn in the Cartesian coordinate system. This means graphs of polynomial equations (including linear, quadratic, cubic quartic and quintic equations); trigonometric equations; logarithmic and exponential equations are investigated. These graphs may be graphs of functions or may not be graphs of functions, as in the case of circles, ellipses, etc.

2.2 Misconceptions and difficulties in graphing

It has been noted that there are salient features of the graphing domain such as scale, gradient and symbolism that cause misconceptions and confusions among students. Goldenberg's (1988) seminal work on graphical representations shows that graphs have ambiguities of their own, and that students often make significant misinterpretations of what they see in these representations. Thus, in order to understand the ways in which the graphing calculator supports (or does not support) students' development of graphing, it is imperative to look at the kinds of difficulties that students encounter in graphing *without* technology and the probable reasons for these difficulties, and to see whether these misconceptions and difficulties persist in a technology-supported learning environment. It will be useful here to distinguish between difficulties and misconceptions as there may be a particular misconception as the source for an observed difficulty. The distinction provided by Leinhardt et al. (1990) is used here: misconceptions are erroneous features of student knowledge that are repeatable and evident. Difficulties are learning problems related to particular features of graphs and the graphing activities. However, since difficulties and misconceptions *are* intertwined, they are discussed here together.

Since this study is concerned only with Cartesian graphs, only those misconceptions and difficulties that are thought to be within the scope of the study are examined here, namely: linearity, continuity, representations, scaling, and variable. However, these

concepts are not mutually exclusive, and do overlap with each other. The role of graphing technology in confronting these misconceptions and difficulties is also considered.

2.2.1 Linearity

Several studies have pointed out students' predisposition toward linearity in both abstract and contextualised situations, where students tend to produce linear graphs when asked to connect two given points (Dreyfus and Eisenberg, 1983; Markovits et al. 1983). Markovits et al. (1983) explained this as stemming from students' over-generalisation of the special property of linear functions, that a line is uniquely determined by two points. The propensity toward linearity is also evidenced on translation tasks. For example, Zaslavsky (1987, cited in Leinhardt et al., 1990) found that students used only two points out of the three labelled points of a parabola to determine its equation. They erroneously used them to calculate the slope of the straight line through these two points and substituted the calculated value into the leading coefficient of the equation for the parabola.

Leinhardt et al. (1990), drawing on Karplus, attributes students' tendency towards linearity to a belief in the inherent accuracy of straight lines, and to the form of learning that students have been exposed to. They criticise the extensive use of activities based on joining dots with straight lines that start as early as pre-school, and continue with plotting of points on the Cartesian coordinate system where a figure or picture is the end product of the exercise. The practice of introducing students exclusively to straight line graphs as an introduction to graph plotting also contributes to the linearity tendency. Leinhardt et al. claim that students may continue to exhibit the inclination to over-generalise the properties of other families of functions in conjunction with linear functions. This claim is supported by studies such as Tall (1987) and Orton (1983), which also suggest that the conventional introduction of the coefficient of the leading term in the linear algebraic expression as the parameter governing the slope results in students often making the same generalisation incorrectly for parabolas and higher order polynomials. Dugdale (1993) argues that students with insufficient algebraic knowledge have changing perceptions as they develop and refine new concepts. She gives an

anecdotal evidence of a pair of students who seemed to be progressing well in determining equations of polynomial graphs in a technology-supported learning environment. They recognised the effects of the various terms in a graph and had worked out on their own the idea that the constant term is the y-intercept and successfully matched a variety of graphs of polynomials of degree 6 or less. Nevertheless, they erroneously thought that the constant term was the vertex of the given parabola.

The ease of graphing provided by graphing technology means that graphing does not necessarily have to be confined to linear equations at the beginning of the learning sequence. However, it is interesting what forms of difficulties and misconceptions still persist when a graphing calculator is used (see section 2.3).

2.2.2 Continuity

The concept of continuity conjures up the notion of infinity. This is an intangible quantity and students have difficulty in grasping the meaning of the “uninterrupted” line of a continuous graph. Several studies show that the difficulty arises from the tendency to see only the marked points on such a graph (Janvier, 1983; Kerslake, 1981; Mansfield, 1985). For example, Kerslake instructed students to plot and join the points of a set of ordered pairs to form a straight line. When asked about the total number of points on the line and between any two points, several students gave the actual number of points that they had plotted as the total number of points on the line. Only 6% of the fourteen year old students had an idea that there were infinitely-many points on the line and still fewer of the same age group correctly responded for the number of points between any two given points. Several students thought that there was no other point between the two points, and a significant number (41%) said there was just one, probably the midpoint. Although some students recognised that there were many points on the line, the physical constraints of actually drawing the points seemed to restrict somewhat their conception of the total number of points. This has some relation to interpreting a point as having a physical mass instead of being an abstract entity, as reported in other studies (e.g. Mansfield, 1985).

Students' early experiences with graphing may have contributed towards their tendency to focus on individual points. In particular, it is claimed that the early experience with graphing, which is heavily dominated by point-plotting activities, leaves many students with the misconception that curves are not continuous (Herscovics, 1989; Leinhardt et al., 1990; Demana et al., 1993). Arguably the early encounter with graphing in the form of graphing discrete quantities may have resulted in some students developing a fixation⁴ that graphs are made up of identifiable discrete points. Demana et al. (1993) observed that the continuity of the line is never explicitly taught even at Grade 8 level. There is no mention that a line is continuous or infinite or that a point is on the line if and only if its coordinates satisfy the equation.

The propensity towards individual points hinders students from developing a coherent understanding of the line concept. As argued by Leinhardt et al. (1990) although lines are accepted as a legitimate part of graphs, they only serve to connect the points rather than having a meaning in their own right, and this is seen to be in conflict with viewing a graph as an object or as a conceptual entity. It also obstructs from viewing the graphs globally, which is essential in understanding the gradient concept (Kerslake, 1981) and the idea of transformation (Goldenberg, 1988). For example, Kerslake showed that only a third of the fourteen-year old students sampled realised that the gradient of the straight line was the same at all points on the line.

It is yet to be explored whether the presence of graphing technology can facilitate students to understand the idea of continuity at middle school level. The graphing calculator enables the user, within the limits of its readability, to trace the points (by using the TRACE cursor) on the line with the coordinate values displayed on the screen. The question remains whether students recognise that it is impossible to trace every point that lies on the line and between any two points on the line.

⁴ Fixation here refers to the related phenomenon functional fixedness described by Gestalt psychologist where when a particular procedure is exercised, it can become fixed making it difficult to think of the problem situation differently (Hiebert and Carpenter, 1992).

2.2.3 Representations

Making connections between the three different representations: algebraic, tabular and graphic is seen as crucial in understanding about graphing. Moschkovich et al. (1993) however, propose that this should include thinking simultaneously from an object and process perspectives. “From the *process perspective*, a function is perceived of as linking x and y values: For each value of x , the function has a corresponding y value” (p. 71). Having the process conception of a function is understood as being able to, for example, substitute a value for x into an equation and calculate the resulting value for y or being able to find a solution to an equation by choosing any point on its graph. On the contrary, “from the *object perspective*, a function or relation and any of its representations are thought as entities - for example, algebraically as members of parameterized classes, or in the plane as graphs that, in colloquial language, are thought as being “picked up whole” and rotated or translated” (p. 71). For example, having an object conception of a function means that one could recognise that equations of lines with the form $y=3x+b$ are parallel. According to Moschkovich et al., understanding connections between these perspectives enables one to view a graph or an equation both as a whole and as being made up of many individual points or ordered pairs.

Janvier (1987) calls for attention to the psychological processes involved in going from one mode of representation to another, which he termed “translation”. The complexity of translation differs in relation to the direction of the movement and the sequence of translations involved. For example, the translation from equation to graph and the translation from graph to equation are different in that the former is carried out indirectly: as equation to table to graph. Arguably the translation from graph to equation is more difficult because it necessarily involves an extensive repertoire of pattern recognition. In contrast, graphing an equation is a matter of executing a standard algorithm (from generating a table of values, to ordered pairs, and to plotting these as points and joining them with a line) (Leinhardt et al., 1990). Studies within the realm of linear equations indicate that flexibility to move and make the connection between algebraic and graphical representations is necessary to fully understand the straight-line concept. Moschkovich et al. (1993) assert that students’ partial understanding of the straight-line concept can be attributed to their misconceptions arising from a failure to

come to grips with the “Cartesian Connection”. That is, the ability to interpret the meaning contained in the statement: “A point is on the graph of the line L if and only if its coordinates satisfy the equation of L ”. For example, students can treat the algebraic and graphical representations as two different domains. Although they may refer to “ m ” in the equation $y=mx+c$ as the slope, they may not ascribe it to the “slope-related graphical properties” to equations with varying values of “ m ” (see section 6.1 for details). Similarly, the student may refer to the “ c ” value of the equation as the y -intercept, but may not know that the point $(0,c)$ lies on the graph of the equation despite using the term y -intercept to refer to a property of the graph. Students may also not realise that one can change one of the parameter while leaving the other constant and thus generate a family of lines with specific properties. This resonates with the problem of “fixation” reported by Bell et al. (1987) of how some pupils concentrate on only one aspect of some given data and do not coordinate this with other aspects, resulting in difficulties such as interpreting gradients. Also possible is a fixation on changes in the y variable when the relationship between x and y should have been considered.

Students also have difficulties in making connections between tables and graphs. Olsen (2000) points out that the form and constitution of a table of ordered pairs and its graph are quite different. In the table, each ordered pair consists of two numbers whereas on the graph, each point is a single geometric object. Students often do not make the connection that each point stands for an ordered pair (Goldenberg, 1988). The tabular and graphical representations in themselves pose an ambiguous situation. The table is a sampling of some of the ordered pairs whereas the graph contains all the ordered pairs (Olsen, 2000). The fact that the table has a finite number of ordered pairs and the graph has both the calculated ordered pairs and a line or curve drawn through the points results in students not seeing the meaning of the line or curve through the points.

The “table-graph-ordered pair” anomaly can persist in a graphing technology environment. The points plotted on the graph do not necessarily correspond to the ordered pairs in the table generated by the graphing technology being used. However, within the graphing calculator environment, operating in the TABLE Mode enables the user to define the ordered pairs in the table and the points to be plotted on the display screen. Using the TRACE cursor of the graphing calculator, the user can trace the

corresponding ordered pairs on the “plotted points” graph and thus better understand the relationship (Vonder Embse and Engebresten, 1996). In addition, the graphing calculator also provides the user the facility to toggle the “points” graph with the corresponding continuous graph. This feature seems to have the potential to directly show students the equation-table-graph translation.

2.2.4 Scaling

Scaling requires attention to the axes: their scales and units. It involves choosing the length of the unit interval and whether both axes have the same scale. Some student misconceptions and difficulties have been traced to a confusion with scaling. Understanding of graphs requires students to know which features of the graph do not change when the scale changes and which features do change when the scale is altered. Some students thought it is legitimate to construct different scales for the positive and negative parts of an axis (Kerslake, 1981), while some students believe that the scales on the x and y axes have to be equal (Cavanagh and Mitchlemore, 2000; Goldenberg, 1988). In the interpretation of a graph, the inclination and shape largely depends on the choice of scales in the coordinate system. Students have to be able to differentiate which features are tied to the graph itself (for example, the x-intercept, the y-intercept) and which features are tied to the coordinate system on which it is created (the slope and consequent appearance of the graph). The Concepts in Secondary Mathematics and Science (CSMS) study (Kerslake, 1981) found that students have difficulty in recognising the effect of changing scales on the appearance of a graph. When asked which two straight line graphs in figure 2.02 show the same information, only 46%, 63% and 68% respectively for 13, 14 and 15 year olds were successful.

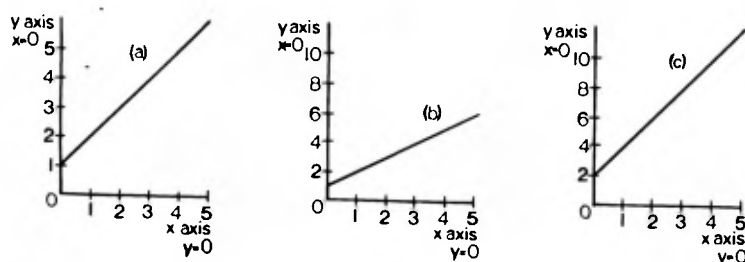


Figure 2.02 Item on scaling in the CSMS study

Changes of scale are one of the main sources for graphical visual illusion (Goldenberg, 1988). Such visual illusion becomes particularly critical when using graphing technology, since the technology easily produces many different graphical representations of a single function or equation (Goldenberg, 1988). For example, Vonder Embse and Engebresten (1996) describe a situation where a pair of perpendicular linear graphs does not appear at right-angles on the calculator screen because of unequal scales. Goldenberg (1988) discusses how changing one of the axes can suggest to students that a graph has shifted up or down, or to the left or right. This is discussed in detail in section 2.3.1.

However, it is the capability of the graphing calculator to generate the same graphs easily in different “view-windows” that makes the effects of scaling salient. Hector (1992) suggests that students continually work with “scale” when they look for complete graphs. They also see that parabolas can be made “fat” or “skinny” by varying the x scale. In conventional hand graphing, there is usually just one graph of a particular equation and drawn by choosing the appropriate scale within the boundaries of the graph paper. With graphing technology, students can easily choose to generate the same graph drawn on different scales. This allows easy exploration of the invariants of the graph such as the slope, the intercept, the maxima and the minima but also potential ambiguities.

2.2.5 Variable

Graphing necessarily involves the concept of variable but Kuchemann’s (1981) work highlights the fact that students can often manipulate the letters in an equation without an understanding of the concept of variable. For example, they do not see the differences among the letters a, b, c and x in the equation $y=ax^2+bx+c$. Goldenberg (1988) suggests that because they are all letters, which may assume various values, gives a connotation that they are all variables. Depending on the focus of a graphing activity, the understanding of the variable concept is sometimes unnecessary (e.g., in scaling). However, if the focus of the activity is on graph transformation, this inevitably calls for understanding about variables. Recognising which entity in the equation determines the general shape of the graph and which entity governs the transformation of the graph, including its position in the graphical space, calls for an ability to discern between the

variable and the parameter(s) in an equation. This is significant in the case of polynomials as the parameters also affect the shape of the graph. Goldenberg (1988) warns that students who explore the effects on the graph of, say, $y=ax^2+bx+c$ as they vary the values of a , b and c may actually misconceive that these values are the variables.

Goldenberg cautions that many graphing technologies do not differentiate between variables and parameters, which may further obscure rather than clarify this difficult concept. Many graphing technologies, including the graphing calculator, generate graphs automatically from left to right, traversing over a set of values of x without any student intervention, and only leave the student to manipulate the equation's parameters. The variable x appears not to be understood as the independent variable by the students, but rather the parameters as they explore the effects of varying the values of a , b , and c on the graphical output. To what extent the variable concept influence students' understanding of graphing concepts in a technology-supported learning environment remains unclear.

Summing up, some of students' difficulties and misconceptions arise within the knowledge of graphing domain itself. However, what emerges from this review is that the way students are taught, with emphasis on point plotting and rarely on the global view of a graph, can exacerbate the problems. According to Dick (1992), using graphing calculators will bring attention to the need to address these misconceptions. However, the use of graphing calculators can engender another set of misconceptions related to the artefacts of the tool itself. This will be discussed further in section 2.3.2 on errors related to use of graphing calculators.

2.3 The use of a graphing calculator in students' learning

Penglase and Arnold (1996), in a comprehensive review of research on graphing calculator use in mathematics education, raised two critical questions:

- “1. In what ways can graphics calculators be used to maximise learning and achieve learning?

2. What teaching practices and what types of learning environments best complement their use in order to bring about maximum benefits for students?" (p. 59).

Disappointingly they pointed out that their review of research on graphing calculator use could not provide definitive answers. Indeed, they noted some conflicting findings. Graphing calculators can facilitate the learning of functions and graphing concepts, and the development of spatial visualisation skills. They can promote mathematical exploration and encourage more graphical investigation, and examination of the relationship between graphical, algebraic and numerical representations rather than just emphasising on algebraic manipulation and proof. Other studies, however, indicate manipulative techniques in the learning of functions and graphing concepts are still essential, that the use of graphing calculators may not facilitate the learning of particular pre-calculus topics, and that some "de-skilling" may result from their use.

This review on graphing calculator use focuses on student' learning of graphiing concepts and issues of pedagogy. "Common" errors in graphing calculator use are discussed here as they have implications for students' learning.

2.3.1 Graphical concepts

There is a dearth of studies on the use of graphing calculators at lower and middle secondary school levels (12-16 year olds). Most studies have involved upper-secondary (advanced) level mathematics or precalculus students (e.g., Ruthven, 1990; Giamati, 1991; Rich, 1991; Becker, 1992; Doerr and Zangor, 1999; 2000), or university students (e.g., Emese, 1993; Shoaf-Grubbs, 1995; Adams, 1997; Mesa 1997; Smith and Shotsberger, 1997; Borba and Villarreal, 1998; Hollar and Norwood, 2000). Most of these focus on the use of graphing calculators to facilitate students' learning about functions. Where appropriate, some of these studies will be referred to, since some of the findings and issues relating to the learning of graphical concepts have broader implications for graphing calculator use at the middle secondary school level. There appears to be a general agreement among researchers that students' understanding of graphing concepts is enhanced by the use of graphing calculators, in particular students' understanding of the relation between functions (equations) and graphs (e.g., Ruthven,

1990; Rich, 1991; Shoaf-Grubbs, 1995; Smith and Shotsberger, 1997). However, there remain some aspects of graphical concepts which students do not learn effectively with graphing calculators, for example, graphing skills (Vazquez, 1991; Steele, cited in Penglase and Arnold, 1996). Notably most of these studies compared students using a graphing calculator and with students who are not (e.g., Ruthven, 1990; Giamati, 1991; Shoaf-Grubbs, 1995).

Ruthven (1990) studied 87 upper secondary school mathematics students. 47 students using a graphing calculator outperformed the 40 non-users on symbolisation items that required examining certain graphs and describing them algebraically, a task that has been noted to be more difficult than in the reverse direction. Students in the calculator group were more successful in identifying the family of curves to which a curve belonged and in refining the algebraic expression to better fit a given graph. Ruthven's analysis distinguished three types of approaches on the symbolisation items: analytic-construction, numeric-trial, and graphic-trial. The first of the three approaches refers to using existing mathematical knowledge and information available in the given graph to formulate the algebraic expression. The numeric-trial approach refers to formulating a symbolic conjecture based on some coordinates of the given graph and modifying it repeatedly if necessary. The third approach uses the graphing facility of the calculator to repeatedly modify the algebraic expression and compare it with the given graph. Students seemed to favour the analytic-construction approach but it was noted that students who used the graphic-trial were more successful than those students who were unsuccessful with the analytic-construction. Although Ruthven does not differentiate which of the three approaches demonstrates the most complete understanding of the graphing concepts involving translation, he does show how the graphing calculator can support the graphic approach to problem solving. Indeed, Ruthven asserts that the regular use of a graphing calculator facilitates the "specific relationships between particular symbolic and graphic forms, as it is through such relationships that the calculator itself is operated,..." (p. 447).

Asp et al. (1993), cited in Penglase and Arnold (1996), carried out a study involving six Year 10 classes, reporting positive results for graphing calculator use in linear and quadratic graphing. This research is interesting in that it distinguishes the role of the tool

from the instructional process. They compared the effects of the use of graphing calculators with those of the “ANUGraph” software package within a unit of work on linear and quadratic graphing. The same two teachers taught the three classes that used the graphing calculators, and the other three classes that used ANUGraph. The focus was on the interpretation of graphs relating to real situations. Both groups were observed to have improved significantly in interpreting graphs with intersections, maximum or minimum values, and matching graph shapes to algebraic forms. The calculator group demonstrated greater improvement in reading and plotting points than the computer group. However, Asp et al. remark that practising activities by hand was still essential. Unexpectedly, the post-test and the post-interviews with students revealed that those students using the graphing calculators experienced more difficulty than their computer counterparts in mastering the facilities of the tool. Anderson et al. (1999) also assert that graphing calculators require new skills for them to be used effectively. However, classroom observations conducted by Paton and Christofides (1995) on much younger children indicate otherwise. Their anecdotal evidence for children at the primary school level showed that these children were able to remember the main commands (e.g., “ON”, “OFF”, “CLEAR”, “ENTER”) and the positions of the keys. However, the level of operations might not be the same as for secondary students: it involves only basic level of operations, i.e. just sequences of one or two button pushes to observe changes on the graphic display on only one parameter.

The age range of the sample of students used by Vazquez (1991) is by far the closest to the age sample considered in the study described in this thesis. His study involved fifty-seven eighth grade students. Data was collected by means of a pre-test and a post-test about linear functions, the Card Rotation Test for spatial visualisation ability, homework assignments, and two questionnaires on attitudes toward mathematics and toward calculator use in mathematics instruction. His results show that the use of graphing calculators does not necessarily have a positive effect on the development of graphing skills for some group of students. Although there were no significant differences between the treatment groups and non-calculator groups in test achievement for linear functions, Vazquez concludes that the graphing calculator should be incorporated into the mathematics curriculum, given its positive effects in other areas such as increase in spatial visualisation skills and efficiency in carrying out homework assignments.

In another study of similar age group students (14 years old), Smart (1995) shows that the graphing calculators enable students to develop strong visual representations of equations of various forms. The value of this research is that it adopts a case study approach, which offers in-depth descriptions of students' learning processes with the graphing calculator. Smart illustrates how the strong visual images provided by the graphing calculator gave students a useful tool to predict whether the relationship of two given quantities was linear or not. In one example, a student investigated non-linear data that had the form $y = m \tan x$. The student used trial and improvement, often visualising in advance of drawing what the effect of a parameter change would have on the graph, and making use of the feedback from the graph to revise her guess, becoming more accurate each time. However, as the research involved only high-achieving students, the results should be treated with some caution.

It seems that how and when graphing calculators are introduced to students also has a significant impact on their learning. This is clearly illustrated in the study carried out by Giamati (1991), involving 126 pre-calculus students in high schools. The study investigated the effect of graphing calculator use, as students developed their graphing knowledge of stretches and shrinks, reflections, translations, and reciprocals of functions. Students used the graphing calculator to analyse the effects of various parameter changes on graphs of functions and relations. Giamati found that the control group (not using the calculator) was better able to sketch functions, exhibit understanding of translations, stretches and shrinks, and describe parameter variations than the group which had access to graphing calculators. Students who had partial or poor understanding of the relationship between graphs and equations were cognitively distracted by having to learn how to use the graphing technology. Giamati suggests that physically constructing tables for values of functions by hand is necessary to the development of students' understanding of the relationship between graphs and their equations. The reported unfamiliarity with certain characteristics of the graphing calculator may to some extent explain for its lack of effectiveness. Giamati says that those students who benefited from graphing calculator use were those who had started with a firm understanding of the relationship between graphs and equations, indicating that some pre-requisite knowledge of graphical concepts is necessary for understanding graph transformations. This research also suggests that access to graphic displays to observe and examine graphs

does not necessarily lead to understanding of how the various parts of the algebraic expression in the equation influence the graphs. The meaning that students draw from what is observed on the screen is crucial. No amount of looking at the graphic display is going to be meaningful unless students are able to make the links between the representations. This brings into question the role of the teachers in learning. It also brings into question the aspects of the learning situations which facilitate or inhibit students grasp of particular graphing concepts. Are aspects of the learning situations such as instructional approaches related to how students use graphing calculators? Also, what difficulties and misconceptions do students encounter when using graphing calculators in their learning? The discussion that follows next attempts to look into some of these issues.

2.3.2 Errors related to graphing calculator use

There has been very little research carried out that is specifically concerned with graphing calculator-related misconceptions. In general, the nature of misconceptions observed reflects the comments made by Steele (1993, cited in Penglese and Arnold, 1996) that students blindly accept whatever is displayed in the view-window. The student belief that the graphing tool automatically or “magically” produces the correct and complete graph of an equation (Goldenberg, 1988; Smart, 1995) is also evident from students readily accepting the initial graph shown in the default window as satisfactory (Steele, 1993; Ward, 2000; Cavanagh and Mitchelmore, 2000). Tuska (1993), found that students believe that the viewing window of the calculator display enough of the graph of a function for them to always determine the function’s behaviour at end points. The same finding is echoed by Smart (1995). Smart describes how many students accepted what they saw on the screen as the whole graph rather than a window through which only part of the graph might be viewed.

Students’ errors or misconceptions when using the graphing calculator may be related to the limitations of the tool itself, such as the limited number of pixels or arise from the ambiguities of graphs as described by Goldenberg (1988), which may result in visual illusions. Another error is a propensity to use only the default window or the built in view-windows of the graphing calculator. For clarity, these sources of errors are

examined under the following headings: visual illusions, view-windows and tool artefact. Some of the issues underlying each of these problems may overlap, for example, the issue of scaling.

2.3.2.1 Visual illusions

Visual illusions were described by Goldenberg (1988). Although his studies on graphing involved a computer software called the “Function Analyser”, it is worth noting some of the sources of illusions he reported as they also apply in the graphing calculator environment; some examples will be illustrated here by generating them with the graphing calculator. Particularly relevant is the illusion which he attributes to the interaction between the position and the orientation (direction) of the graph, and the shape of the window.

The first example illustrates the illusions encountered when looking at a family of linear equations $y=mx+b$. To the trained eye or someone who have developed some analytical expectations, the line “move up” as b increases, (or “move down” as b decreases), as in figure 2.03 and figure 2.04. Because of the infinite nature of the line where no discrete points are obvious, the line may appear to move from left to right as the constant term increases (as in figure 2.05 and figure 2.06) or from right to left if the line slope is positive.

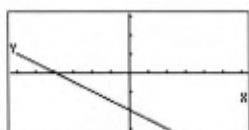


Figure 2.03 Graph of $y=-0.5x-2$

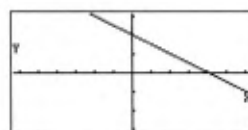


Figure 2.04 Graph of $y=-0.5x+2$

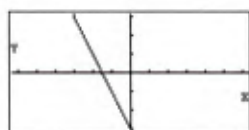


Figure 2.05 Graph of $y=-2x-3$

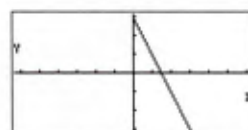
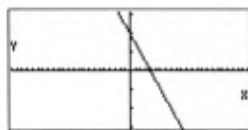
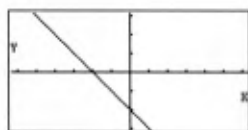
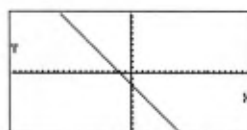


Figure 2.06 Graph of $y=-2x+3$

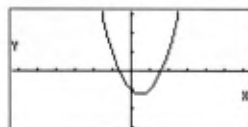
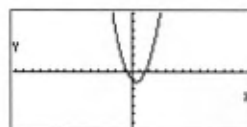
Figure 2.07 Graph of $y = -0.5x + 2$

However, when the x and y-axes are represented on different scales as in figure 2.07, the graph of same equation that appears in figure 2.04 resembles one that belongs to the family of equations represented in figure 2.05 and figure 2.06.

Goldenberg points out that the “zooming in” and “zooming out” operations, which facilitate the user to view “closer to” or “farther from” the graph, are also a source of confusion. Students expect that the shape of the graph will not be affected when they change both axis scales by the same factor. This may be so in the case of straight lines, as shown in figure 2.08 and figure 2.09. However, it appears that the line has moved “closer” to the centre of the window giving the illusion that it has moved higher, thus suggesting that the constant term has changed.

Figure 2.08 Graph of $y = -x - 2$ in the INIT windowFigure 2.09 Graph of $y = -x - 2$ when “zoomed out”

With a parabola, it appears that the shape changes when viewed close up as in figure 2.10 and from further away as in figure 2.11. The eyes have been deceived into thinking that the parabola has “narrowed”.

Figure 2.10 Graph of $y = x^2 - x - 1$ in the INIT windowFigure 2.11 Graph of $y = x^2 - x - 1$ when “zoomed out”

The position of the parabola can also alter its shape. In figures 2.12, the upper parabola appears narrower although they are translations of each other. Students can become confused with changes in the coefficient of x^2 .

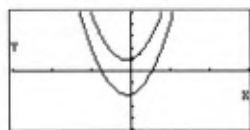


Figure 2.12 Graphs of $y=5x^2-2x+1$ and $y=5x^2-2x-2$

2.3.2.2 View-windows

Arguably, students' difficulties and misconceptions may be related to the fixed or finite size of the view-window of the graphing calculator. With graph paper, students can determine the graphical space (the dimension of the "window") which maintains the same scale values. However, the graphing calculator has a finite view-window: the dimension of the window remains the same but what changes is the scale. Students can miss this fundamental point as the graphing calculator automatically graphs in the view-window that has been set in the previous activity, which can be the default window or built in window, or some other view-window settings. Cavanagh and Michelmores' (2000) investigation involving 25 Grade 10-11 students all too clearly points to students' careless attention to scaling. Only 16% of the students immediately recognised that the graphs of $y=2x+3$ and $y=-0.5x-2.5$ (drawn in the built in "standard window" as shown in figure 2.13) did not appear at right angles on the screen because of the unequal scaling.



Figure 2.13 Graphs of $y=2x+3$ and $y=-0.5x-2.5$ in the "standard window"

Similarly, when given the same graph ($y=x$), one drawn in the default "initial window" and the other in a window where both the scale and the interval between the tick marks on the y-axis have been doubled (Figure 2.14), only 8% of the students recognised that the apparent differences was an artefact of the scaling. The remaining 92% of the students thought that the second figure represented the line $y=0.5x$.

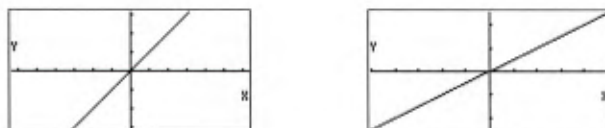


Figure 2.14 Graph of $y=x$ drawn in different view-windows

Cavanagh and Mitchelmore (2000) attribute the difficulty that students have in understanding the scale concept in the graphing calculator environment to students having an absolute understanding of scale rather than a relative understanding. “Absolute” means to interpret scale exclusively as either the measure of the distance between adjacent markings on the axes or as their value, whereas a “relative” interpretation of scale is as the ratio of distance to value. They argue that this might be a result of students’ lack of experience of manually drawing graphs with axes that are not scaled equally. They found that students in their study tended to use symmetrically scaled axes marked at regular unit intervals when asked to draw a graph by hand.

The ease with which the graphing calculator allows students to choose the windows, and to readjust in the scales seems to result in some students’ unthinking use or to careless treatment of view-windows. In a study by Ward (2000) involving eighteen high school students, he found that students used mainly three strategies to obtain the graph of an equation. 70% of the time students used the “press and pray” strategy: students simply stored the equation and immediately pressed DRAW or GRAPH to display the graph, taking little notice of their window settings. Ward observed that about 16% of the time, students used either the default window or the “standard window”, and very few manually set the window using the critical points in the equation. Ward details how the “press and pray” strategy resulted in a “visual conflict” when one student was asked to graph $y=3x+400$. The student was confused when the graph she drew was not as she predicted. She expected that the line would be quite steep “because the slope is 3”. From her previous experience, graphs “always showed up” in the “standard window” and appeared to be quite “steep” (as shown in figure 2.15). She was somewhat baffled when the graph of $y=3x+400$ did not appear on the screen. She adjusted the viewing window using the y-intercept as a guide resulting in a line that was more “horizontal” as shown in figure 2.16.



Figure 2.15 Graph of $y=3x$ in the standard window

Figure 2.16 Graph of $y=3x+400$

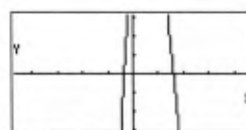
The problem of the critical use of the default viewing window has been reported in several studies (e.g., Steele, 1993; Ward, 2000; Cavanagh and Mitchelmore, 2000). Cavanagh and Michelmore (2000) report how Grade 10-11 students responded to a task asking them to manually draw a sketch of the graph of $y=0.1x^2+2x-4$. Only 28% of the students sketched a parabola. They all commented that the x^2 term indicated that the graph must be a parabola, and zoomed out until they saw the U-shaped graph. The remaining students drew a straight line as their sketch of the quadratic equation with 20% simply copying the straight line directly from the calculator screen using the default viewing window.

2.3.2.3 Tool artefacts

Two aspects are discussed here, the left to right “movement” of plotting and the limitations of the pixel-based screen. The left to right generation of graphs by the graphing calculator can result in a number of misconceptions. Ward (2000) describes how a student who was accustomed to displaying the graph in the default window, interpreted the graph of $y=2x^3-16x^2+12x+6$ in the following window setting.



Figure 2.17 The view-window setting

Figure 2.18 The graph of $y=2x^3-16x^2+12x+6$

The student predicted that the non-displayed portion of the graph existed off to the right of the screen reasoning that that is the way the calculator draws the graph, from the left and continuing to the right. Thus the student rules out the possibility that the missing piece of the graph could be positioned to the left of the screen. Goldenberg (1988) also warns how the left-to-right movement can cause misconceptions. When the graph being drawn automatically sweeps over a domain in x , and students being left in active control

only over the parameters in the equation, can mislead students into thinking that the parameters *are* the variables.

The other confusion that students have using any graphing technology, including graphing calculators, is related to the physical appearance of the lines on the screen. Each graph is composed of a series of pixels (small rectangular shapes) on the screen, resulting in the line not being represented as smooth. The different patterns of pixels corresponding to different slopes produce different degrees of “jaggedness” on the graphs. The study by Moschkovich et al. (1993) is informative here (although it deals with a computer environment). They found some students misinterpreting the relationship between the jaggedness of the straight lines and the gradient of the line (e.g., compare the graph in figure 2.16 and figure 2.18). They gave an example where students used a graphing program to graph $y=x+2$, $y=2x+2$, $y=3x+2$,... and were asked to state what they observed. Instead of observing that the lines were increasingly steeper and all passed through (0, 2), they commented on the “insignificant” fact that some of the lines were more jagged than others, according to the magnitude of the coefficient. These observations cause students to see things that were not intended (line jaggedness) and not see things that were intended (point of intersection on the y-axis).

These examples of errors in graphing calculator use have a bearing on the warning of Leinhardt et al. (1990) about when technology is used in graphing. They warn that a too strong reliance on the graphing tool might obscure students from understanding the underlying patterns and principles that drive the production of the graphs. This implies that hand graphing is still an important aspect that needs to be considered when students learn graphing. Indeed, Demana et al. (1993) assert that considerable “point plotting” is necessary but warn that excessive emphasis may interfere with students’ understanding of the continuous nature of graphs. Giamati (1991), suggests that physically constructing tables for values of functions by hand remains necessary to the development of students’ understanding of the relationship between graphs and their equations. Many others have raised the issue of how much paper and pencil skill remain important when graphing calculator is integrated into students’ learning (e.g., Dunham and Dick, 1994; Waits and Demana, 1996; Tan, et al., 1997). However, the observations made by Ward (2000) (admittedly for a small student sample) suggest that hand graphing and calculator

graphing are two independent processes. Not all of the students in Ward's study who sketched the graph of the equation before generating it with a graphing calculator used the sketch as a guide for setting the domain and range of the view-window. Ward suggests that the opposing processes in generating the graphs in the two graphing environments might account for why students had compartmentalised the procedures for how to graph with paper and pencil, and how to graph using a graphing calculator. When graphing using paper and pencil, the students set the scale first, that is, they labelled the tick marks on the y-axis and used the y-intercept as a guide, which in turn defined the bounds (y-min and y-max) on the axes. On the other hand, when graphing an equation using a graphing calculator, students specified the bounds (y-min and y-max) on the axes first using the value of the y-intercept, and only then chose a scale.

Summing up, the difficulty that students have in graphing with the graphing calculator is related to the finite view-window of the graphic screen. Students seem unaware of how the finite view-window influences the graphical space and therefore the scales. There is a need to get students to be aware of the view-window settings when they use a graphing calculator to generate the graphs. Students also have to be able to choose the appropriate view-window on which to draw the graphs that they are investigating. Most important, they need to know the reasons underlying their choices of the view-windows, for example, whether it is to look at the y-intercept, or the gradient.

2.3.3 Issues of instructional approach

The literature on graphing calculators which describes the role of instructional approach is limited. Research has been confined to high school and college/university levels. These studies focus either on the effects of different teaching styles on graphing calculator usage, or the effectiveness of graphing calculator when they are integrated with various teaching approaches. The studies do not always consider graphing in detail, but they may provide insights and have general implications for students' learning with the graphing calculator at the middle school level.

An investigation carried out by Emese (1993) among students in a university differential calculus course looked at the effects of graphing calculator use when integrated with

what was termed as “guided discovery” style of teaching based on lecture and discussion. This included the use of worksheets designed to lead students towards the discovery or understanding of relationships or techniques. Using a pre-test/post-test design, it was found that there were no significant differences in achievement between these students and two other groups of students: one taught with the same approach but without the use of graphing calculators, and another taught with the traditional approach. However, students who had been taught using the guided discovery approach with the aid of graphing calculator indicated that they were inspired by this style of teaching.

A study by Strait (1993) compared the effectiveness of two teaching approaches, deductive and inductive, that incorporated graphing calculators in the learning of functions and analytic geometry among college students. The deductive approach involved a sequence of “rule, example, practice” whereas the inductive approach involved a sequence of “example, rule, practice”. Strait found no significant differences between the two groups in their procedural skill development or in their conceptual understanding, but students taught using the deductive approach demonstrated higher levels of factual knowledge. Since the study had no control group, it is unclear whether the graphing calculator had any influence on either of the teaching approaches. Another study by Adams (1997), however, investigated whether the use of graphing calculator had any such influence. Besides including a control group, she also included a non-calculator group that followed only the new instructional approach. Adams compared the performance on the function concept for four groups of college algebra students. The first group ($n=15$) used the graphing calculator only in a usual teaching approach (textbook and topics presented by their instructor), the second group used a specially designed assignment without the graphing calculator ($n=21$), and the third group ($n=16$) had both the specially designed assignment and access to the graphing calculator. The control group ($n=19$) did not use calculators or any specially designed assignment. The specially designed assignment, which they called the “conceptual change” assignment, was based on a model of learning through cognitive accommodation where the learner is led towards a situation of cognitive conflict. The first group outperformed the second and third groups (in that order). Adams attributed the poorer performance of the third group to having to operate under two new situations for their learning. It must, however, be noted that only one of the six teachers involved in the research was familiar with

graphing calculators and none had applied the model of learning that was used in their teaching of mathematics, suggesting that care must be taken in interpreting Adam's results.

The research reviewed so far does not detail the role of the teacher in students' learning when using the calculator. A recent study by Lawler (2000), involving two groups of a total of 46 students in an introductory algebra class at college level, assesses the importance of the instructor. The purpose of the study was to see whether "written scaffolds" increase students' achievement in some linear functions using the graphing calculators. By written scaffolds, Lawler means teacher interventions given in written form, and they are viewed as sustained scaffolds within a cognitive apprenticeship framework. The study did not show any significant differences for the experimental and control group in post-test achievement. However, the analysis reveals that written scaffolds and verbal scaffolds provided by the teacher were an effective method to make connections between the multiple representations of the linear functions, suggesting that the form of intervention does influence the learning of particular concepts. This is discussed in further detail in section 2.5.

Research on the effect of different instructional approaches is inconclusive. Lawler's (2000) study indicates that particular forms of teacher intervention have an impact on students' development of particular concepts, suggesting that looking at different instructional approaches and trying to generalise the results in terms of performance may obscure some salient contributions that each approach has to offer. Therefore, contrasting the effectiveness between different instructional approaches on some aggregated performance measure may not be informative in trying to discern in what ways the graphing calculator can be used in students' learning. The research described in this thesis has attempted to assess the particular effects of teacher interventions for students' development of graphing.

In addition, from the viewpoint that technology is an inherent part of our lives, distinguishing between graphing calculator use and non-use adopted, as many of the cited studies do, may not be useful in terms of informing us what we should do in educational practice. Students' difficulties and misconceptions when using a graphing

calculator in their learning points to the fact that when using technology, two other aspects must be considered: the role of the technology and the role of the teacher.

2.4 The role of technology in students' learning

The role of technology in students' learning, can be assessed overall from how well students master the technology. The level of mastery has consequences on the nature and quality of students' engagement (Guin and Trouche, 1999). In any technology, its set of "levers" represents a sort of reification and encapsulation of sequences of actions previously carried out by the user. In the case of graphing calculator, the levers can refer to the buttons which are used to call certain menus and execute required commands, for example, drawing a graph without the need to compute a table of values, or indeed needing to know what is entailed in producing a graph. The question remains open as to what extent knowledge and understanding of the internal structure and processes of the tool or machine must be required on the part of the user. Certainly this would depend on the objective and the type of task. For certain tasks, a "functional knowledge" of the tool or machine as described by Dorfler (1993) is sufficient: knowledge which takes the tool merely a "black box". This again poses the question of the extent to which the user must have a knowledge and experience of using the respective tool, that is a knowledge of the tool's modes of working and the effects that can be achieved with it. Functional knowledge of a tool almost always embodies a "constraint-support structure" of the tool's limitations and capabilities. The constraint-support structure embodied in the tool affects learning (Demana et al., 1993; Dion, 1992; Kaput, 1992; Goldenberg, 1988) as this is what gives rise to, for example, visual illusions and misconceptions. Following Kaput (1992), it can be argued that the distinction between constraints and support is not useful because whether a feature is regarded as one or the other, is contingent on the relation of the user's intention with those of the designer of the tool, and the contexts of its use.

With specific reference to the graphing calculator and symbolic calculator⁵, Guin and Trouche (1999, 2000) argue that students need to develop the instrumental genesis of the

⁵ Combines two modes of calculation: exact calculation and the approximate calculation of graphing calculator, e.g., TI-92.

tool in order to manipulate it. By instrumental genesis they refer to the knowledge needed to appropriate the artefact as an instrument in mathematical activity, and build schemes to control the uses of the artefact. They draw this idea from the theoretical approach of “instruments” by Verillon and Rabardel who distinguish the difference between the artefact (material tool) and the instrument (a psychological construct). The instrument is not given and users have to develop it themselves. The artefact becomes an instrument when the subject has been able to appropriate it for himself/ herself and has integrated it with his/ her activity. Guin and Trouche (1999; 2000) characterise the instrumental genesis into two distinct phases or instrumentation levels. The first instrumentation level is the discovery phase of various commands, their effects and their organisation, with few references to mathematical knowledge. The second instrumentation level is an organisation phase involving an awareness of the effective constraints and potential uses of the calculator, where students refine their previous strategies and techniques and develop a reasonable scepticism of the calculator results. Accordingly, at this level, students develop a conscious attitude towards all available information that is not restricted to the calculator only but which tries to achieve mathematical consistency between all information including text and calculation by hand. Guin and Trouche (1999, 2000) view the teacher’s role as crucial in guiding students towards the acquisition of the instrumentation process.

In Guin and Trouche’s (1999) study, their observation of student behaviour in the instrumentation process involved a class of tenth-grade (15-16 year old) students, and they found that weaker students often gave up the idea of understanding the calculator commands’ meanings and responses. Students develop avoidance strategies, including random trials and “zapping” to other commands in the same menu. As the students in this study were learning about the graphing calculator simultaneously with an introduction to the function concept, the researchers noted the conceptual difficulties that students encountered interfered with manipulations and interpretations. They also noted students’ difficulties with the algebraic and graphic registers (language and notations) necessitating them to devote time for exercises between these two registers with regard to functions, equations and numbers. Nevertheless, they reported that students became aware of the potentialities of visualisation, expectation and verification offered by the interplay between several registers afforded by the graphing calculator. Guin and

Trouche's study indicate that some mathematical knowledge underlies the instrumentation process, for example, the manipulation of "Factor" or "Expand" requires that students give explicit mathematical meanings to these commands. Guin and Trouche's study also lends support to the findings of Giamati (1991) and Adams (1997). In Giamati's (1991) study, it was students who already had a conceptual understanding of the relationship between graphs and equations who benefited from the use of graphing calculators. Adams (1997) concludes that students must have a basic understanding of the concept of function to be able to understand the reasoning behind the operation of the graphing calculator, in order to use it as a tool for learning. Otherwise students will see the graphing calculator as a "machine for doing mathematics" which provides a visual display of functional equations.

The instrumentation process raises two issues. One is the issue of when and how to introduce the technology in students' learning. Second is the issue of the mathematical background that students have – this will be discussed later.

The way that the graphing calculator is integrated into the curriculum was considered by Gomez and Fernandez (1997). In their study involving pre-calculus students in their first year of a university course, they introduced the graphing calculator to the study of functions (in which the emphasis was on graphical representation and problem solving) in three phases: introduction, adaptation and consolidation. In the introduction phase, the tasks merely involved learning how to use the graphing calculator. In the adaptation phase, some tasks were designed for graphing calculator use and some tasks took advantage of the graphing calculator possibilities. The consolidation phase involved mostly tasks taking advantage of the graphing calculator possibilities. The three phases were developed during three consecutive semesters. The results show significant differences in achievement (on students' final grades in their calculus course) between the calculator group and the non-calculator group only at the consolidation phase. The researchers attribute the difference to the fact that during this phase, the graphing calculator was used to create new learning opportunities through mathematical investigation, exploration, and emphasising relationships between representation systems. They also attribute it to the change in the visions that teachers had about mathematics, and its teaching and learning as a consequence of the way the graphing

calculators were integrated into the curriculum. Had teachers' visions not changed, it is questionable whether the same result could be obtained. The researchers conclude that the way the graphing calculator was integrated into the curriculum influenced students' achievement, and they suggest that students have to complete each phase before proceeding to the next. Gomez and Fernandez's study does not detail how students appropriated the graphing calculator in their learning processes in each phase, that is, how they acquire the instrumentation process described by Guin and Trouche (1999, 2000). The findings do, however imply that the instrumentation process is an evolving one. Hubbard (1998), who surveyed high school students on their use of graphing calculator noted that more "hands-on" time for students with the graphing calculator increased their understanding of the functions and capabilities of the graphing calculator. Cavanagh and Mitchelmore's (2000) investigation on students' misconceptions in interpreting graphing calculator displays, lead them to conclude that greater exposure to graphing calculators might lessen the appearance of misconceptions. From an analysis of the effects of graphing calculators on students' solution approaches, van Streun et al. (2000) conclude that only prolonged use of graphing calculators leads to enrichment of the solution repertoire of students, and a better understanding of the mathematical concepts involved.

In order to interpret how the graphing calculator functions as a tool for learning, Berger (1998) suggests that we consider the distinction made by Jones (1993) between two principal effects of working with a graphing calculator: effects *with* the technology or effects *of* the technology. Jones describes the process of working with technology as one in which the student plans, implements and interprets the solution whilst the technology performs the techniques. The result of the use of technology is based on its potential to afford greater and easier access to multiple representations (algebraic, numerical, graphical) of concepts and processes (Kaput, 1992; Hector, 1992; Demana et al., 1993; Reusser, 1993). It assumes that there will be cognitive "residue" from learning with graphing calculators because presenting the same mathematical concept in both algebraic and graphical forms helps students advance the learning of the concept. However, Jones rejects this notion and contends that it is working *with* the tool that gives the student the potential to work at a much higher level than may be possible without the graphing calculator. Drawing from Jones's idea of working *with* technology, Berger (1998)

integrated the graphing calculator into an existing curriculum of a first year textbook-based university course. Berger attributes the lack of evidence for a “cognitive reorganisation” role of the graphing calculator as being due to the way it was integrated into students’ learning: simply as an add-on to the existing curriculum. She identifies cognitive reorganisation as those effects which may occur as a consequence of using the technology. It must be pointed out that students in her study were directed to use the graphing calculator to verify and support analytic results, thereby the role of the graphing calculator was deliberately narrowed.

In a study by Hennessy et al. (2001) on first year university students, they reported that the graphing calculator served as a “catalytic, facilitating and checking role” towards a deepening understanding of graphing. The researchers describe the graphing calculator as a “critical participant” in students’ learning. The activities that were used were specially designed to exploit the graphing calculator facility, attempting to develop an understanding of graphing and its rationale through exploration and data handling. The researchers conclude that the graphing calculator’s features, such as immediate translation between representations, helped to structure thinking and transform mathematical activity. They detail how one student reported that the facility of the graphing calculator to draw the graph, generate the table and trace the values resulted in her spending more time investigating the relationships and ideas behind the graph.

How do the type of tasks that accompany the integration of technology influence the role of technology in students’ learning? Ehrmann (1995) cautions that if technology is used simply to carry the same old thing, the same result is obtained. To obtain different results new thinking must be added with new technology, implying that learning activities have to be transformed. Some researchers suggest that new learning tasks need to be designed such that students can develop the use of tools in line with the reorganisation metaphor, involving structural changes in students’ cognitive activity and social interaction (Dorfler, 1993), rather than using the tool along the lines of the amplifier metaphor⁶ (Pea, 1987) or the incrementalist view⁷ (Schofield, 1995). The consolidation phase of Gomez

⁶ With technology many more calculations can be carried out in shorter time and with higher accuracy without change in the quality of what we do.

⁷ That the purpose of technology is to help teachers and students perform the work they already do more easily, efficiently, or effectively.

and Fernandez's (1997) study seems to have aspects of the reorganisation metaphor in that the graphing calculator could be described not only as a tool with which to think, but also as a tool which helps thinking to develop (Pea, 1987). Similarly, in Hennessy et al.'s (2001) study, learning tasks were designed to exploit the reorganisation potential of the graphing calculator.

Mesa's (1997) study on eight students at university level gives another insight on students' learning tasks when technology is used. If the task was related to previous knowledge, the students assigned a verification role to the graphing calculator, but if the task did not relate to previous knowledge, the students assigned an exploration role to the calculator.

It seems that in order to examine the role of the graphing calculator in learning one must look at how students appropriate the graphing calculator to construct mathematical meaning from mathematical tasks. Doerr and Zangor's (2000) is one of the few studies that provides a detailed investigation into the role of the graphing calculator in the learning process. Their qualitative classroom-based study involved a total of 31 students between 15 and 17 years of age in two pre-calculus classes. The tasks used were based on modelling problems, which they claim provided a "rich setting" for studying the patterns and modes of graphing calculator use by the students (and the teacher). They identified five categories of patterns and modes of calculator use by the students: computational tool, transformational tool, data collection and analysis tool, visualising tool and checking tool. Their study reveals the emergent role of the calculator. For example, they describe how the shift in the role of the calculator from a graphing tool to a visual checking tool supported students' thinking about the idea of the non-uniqueness of an algebraic representation for an exponential or trigonometric graph. Initially, the students used the calculator as an efficient graphing tool to explore the relationship between graphs and equations, that is, they sketched the graphs after they had graphed the equations on the calculator. The teacher then encouraged the students to use their knowledge of transformations to sketch a graph or find an equation in situations where students had to rely more on their knowledge of transformations and of the shape of the parent function in order to sketch the graph on paper. Here, the students used the graphing calculator to check the graph they had sketched by comparing it with a

calculator-generated graph. They traced the graph on the screen through given and expected points. The researchers attribute the emerging role of the calculator here to the teacher's confidence, flexibility of use and awareness of the limitations of the calculator. And it must be pointed out that the students in this study had been using their graphing calculator for over a year before the study begun. The students in Hennessy et al.'s (2001) study also had some proficiency in using graphing calculators before the start of the study. It is evident that the appropriation of learning tasks that accompany graphing calculator use is closely linked to how well students are acquainted with the technology.

Whichever way technology is positioned in student's learning, the technology itself will necessarily influence classroom functioning and mathematical learning. It impacts at the symbolic level by changing the representational medium in which mathematics is expressed, and at the interactivity level by changing the relationship between the learners and the subject matter, and between the learners and teacher (Balacheff and Kaput, 1996), causing changes in what and how students learn (or do not learn). Farrell (1996), observed that students' (and teachers') roles and behaviours changed when graphing calculators were used even within the traditional pre-calculus curriculum, with the students assuming the roles of task setter and consultant (e.g., able to advise and help when called upon to do so) when technology⁸ was in use. She reported that students used more symbolisation (employing symbols to represent information) compared to students who had no access to technology. Students' activities were less didactic, although they still used a great deal of didactic behaviours including absorbing, recalling and imitative rule following. They investigated more when technology was used in their learning activities. These findings clearly contradict those of Berger (1998), who also used the graphing calculator as an "add-on" to an existing curriculum. The reason for the contradiction may be explained by the way the students appropriated the graphing calculator. Berger dictated the limited use as a checking device: to verify and support the results of their paper-and-pencil analysis. Thus the graphing calculator was not recognised as a tool with which to think.

⁸ Students could use both computer software and graphing calculators. 43% of the time it was graphing calculators alone; 27% of the time it was computers alone; and 30% of the time it was both the graphing calculators and computers.

Ruthven (1996) also cautions that meaningful calculator graphing is far from being an automatic process, rather, it requires an exploratory process. To sketch the graph of some specified expression requires first that the expression has to be transformed into a format acceptable to the calculator's syntax, and then the user has to determine the appropriate range for the axes or select a view-window offered by the calculator. The features of the graph sometimes require that the user has to repeatedly modify the view-window settings, which is not a straightforward process for novices. This seems to support Ward's (2000) observation of students' misconceptions when they used the graphing calculator. When students rely heavily on using the "press and pray strategy", with default viewing windows and ZOOM features to obtain graphs of equations, it gives the impression that they are doing less mathematical thinking⁹. Students' use of technology is also related to their mathematical disposition. Henningsen and Stein (1997) describe that "having a mathematical disposition is characterised by such activities as looking for and exploring patterns to understand mathematical structures and underlying relationships; using available resources effectively and appropriately to formulate and solve problems; making sense of mathematical ideas, thinking and reasoning in flexible ways: conjecturing, generalising, justifying, and communicating one's mathematical ideas and deciding whether mathematical results are reasonable" (p. 525). This description resonates with the higher order thinking¹⁰ that Resnick (1987) proposed as necessary for students' successful reasoning processes.

Disposition seems related to mathematical ability. Students' mathematical ability has not been properly accounted for in many graphing calculator studies. There is limited evidence in that the graphing calculator impacts students of differing mathematical ability in different ways. Guin and Trouche (1999) observed that the "weaker students" in their study found the calculator commands to be an inhibiting factor for learning. As mentioned earlier, they developed various avoidance strategies. Lagrange (1999) describes a situation where the "able" high school students in his study decided

⁹ In general, mathematical thinking is an act of sense making and rests on the processes of conjecturing, generalising, specialising and convincing (Goos et al., 1999). It can also be seen in the type of thinking that is needed to solve unfamiliar mathematical problems necessarily involving some degree of independence, judgement, originality and creativity (Polya, 1981).

¹⁰ "High order thinking order thinking is nonalgorithmic, tends to complex, yields multiple solutions, involves nuanced judgement and interpretation, the application of multiple criteria, uncertainty, self-regulation of the thinking process and imposing meaning, and is effortful" (Resnick, 1987, p. 2-3).

immediately on the need to use the “zooming” facility of the complex calculator (TI-92) compared to other students who took considerable time over this decision. The study by Averbeck (2001) on students’ learning of the function concept and the role of the graphing calculator showed that differences between college students’ symbolic manipulation skills impinged on their flexibility in working with the graphing calculator’s different representations of functions. Although he stopped short of describing how the graphing calculator influenced the learning of the two group of students, it is probable that the two groups of students used the graphing calculator differently. Studies involving technology generally also indicate that students of different knowledge background benefit differently from using technology, implying that the role of the technology may be different in the students’ learning (Dalton and Hannafin, 1987; Underwood and Underwood, 1990; Hativa and Lesgold, 1996).

Students need to be “active learner” in a process when technology is used (Demana and Waits, 1988, cited in Farrell, 1996). By active learner it is meant that students take an active role in their learning: contributing their own ideas, trying their own solutions and even challenging the teacher. The role is one of questioning rather than merely accepting (Nickson, 1992). The use of graphing calculators encourages active learning (Fox, 1998). However, students’ actions are tied to the classroom mathematics culture in which students have learned to act in accord with the normative rules or instruction. For example, Cobb et al. (1989) describe how students who have adapted to “calculationally” oriented instruction, in which the emphasises is on the application of calculations and procedures for deriving numerical results, will tend to approach mathematical discussions with a focus on getting answers. Such students have difficulty with mathematical reasoning as they view the process as unimportant (Thompson et al., 1994). Conversely, students who have adapted to a conceptually oriented instruction will likely engage in sharing their understandings in meaningful discussions (Cobb et al., 1991) and view mathematical reasoning as an important aspect of learning (Thompson et al., 1994). In the LAMP and RAMP project (cited in Trickett and Sulke, 1988) it was observed that students had difficulty in initiating their own work. As they seldom get the chance to use their own initiative, students had become very dependent on their teacher for directing their learning. The students saw the teacher as having total control and responsibility for leading and controlling the work in the classroom. The research by

Porzio (1999) also provides evidence that students “behave” as they are taught, and that the addition of graphing calculators does not necessarily improve the learning of calculus. The direct telling of instructors about the connection between symbolic and graphical representations during instruction deprived the students of the opportunity to develop the type of reflective abstraction (as described by Dubinsky, 1991) needed for knowledge construction.

Summing up, it seems there are many facets to the role of technology in students’ learning. It is clear that students need to learn how to use the technology and acquire some of the instrumentation aspects described by Guin and Trouche (1999, 2000). The constraint-support structure of the technology influences students’ learning. And since mathematical knowledge underlies the constraint-support structure, students’ mathematical background must influence the role of the graphing calculator in their learning. Arguably, distinguishing the kinds of mathematical knowledge, or experience with the technology that underlies the constraint-support structure may be useful in helping students to cope with the demands of learning to use the technology. The role of the graphing calculator is related to how the graphing calculator is positioned in students’ learning. Used as an add-on to the existing curriculum as in Farrell’s (1996) study, a positive change in students’ learning process was observed, but used as an add-on as described by Berger (1998) with its uses narrowly prescribed, was counter-productive in students’ learning. The role of the graphing calculator in students’ learning is an emergent one. In trying to look at the extent to which the graphing calculator facilitates students’ learning, it is useful to identify the threshold point in which the graphing calculator begins to play a mediating role in learning. Using the graphing calculator as a *mediational tool* means that students use the calculator to help them to appropriate mathematical concepts, that is students engage thoughtfully (Salomon, 1993) or consciously (Berger, 1998) with the tool. It is understood that thoughtful or conscious use denotes being able to explain *how* one generates an object (e.g., graph) with the graphing calculator rather than merely producing the end product itself. Using the graphing calculator as a mediational tool means using it to direct one’s own mental processes, suggesting that one would use it to test out conjectures, to make guesses, to extend, and to articulate what one is thinking. The graphing calculator, in other words, is

used as means of focussing one's attention and directing one's mental operations toward the solution of problem.

2.5 The role of the teacher with technology

The use of technology in mathematics classrooms raises several areas of concern for teachers, including curriculum issues and classroom dynamics. In particular, the altered dynamics of a technology-based classroom may shift the role of the teacher to that of a facilitator and a guide. These roles may be problematical or even disagreeable for teachers who have become accustomed to teacher-centred classrooms. In part, this is equally true for the implementation of any form of curriculum innovation. The process of becoming acculturated to new forms of mathematics teaching confronts teachers' basic epistemological perspectives, values, beliefs and practices.

The relationship between teacher's knowledge and pedagogical strategies and their use of the graphing calculator is currently hardly examined. Many graphing calculator studies (Drijvers and Doorman, 1996; Graham and Thomas, 1997; Mesa, 1997; Borba and Villarreal, 1998; Porzio, 1999; Hollar and Norwood, 2000) do not report or describe the role of the teacher in the classroom although some indicated the importance of teacher intervention when graphing calculators are used (Guin and Trouche, 1999, 2000; Lagrange, 1999; Mok, 1999). In a study of teacher's roles when graphing calculator technology was used in a high school pre-calculus class, Farrell (1996) found that there was a shift in teacher's role from task setter and explainer to that of consultant and fellow investigator when technology¹¹ is used. The role as a fellow investigator in which the teacher becomes a true participant in the problem solving process highlights the challenges that lies ahead for teachers when technology is used. However, Farrell's study shows that that teachers still maintained their role as a manager (tactical, director, authoritarian) and resource provider (a system to explore, giver of information) while assuming the new roles described above. The shifts in teacher's role were also accompanied by a decrease in exposition (lecture) and an increase in investigation and group work. It was also reported that each of the six teachers that were observed

¹¹ When technology is used, 43% of the time it was graphing calculators alone; 27% of the time it was computers alone; and 30% of the time it was both the graphing calculators and computers.



maintained an individual style in that they did not do the same things with the new tool. But as the study involved only a few teachers, the generalisation of the results of this study should be treated with caution. Two of the teachers in the Farrell's study lectured very little and involved students in much exercise, consolidation and practice regardless of technology use. This has two implications: one is that teaching styles could give skewed results when comparing teachers' teaching activities when technology is used or is not used. Secondly, it implies that changing teachers' instructional practice is not as easy as has been argued by many (e.g., Lubinski and Jaberg, 1997; Even and Lappan, 1994).

In contrast to the findings above, the research by Simmt (1997) found that teachers did not change their methods or approaches when using graphing calculators. In her study, six teachers were observed in their classrooms as they taught lessons on graphing of quadratic equations. Teachers used the graphing calculator as an extension of the way that they always taught the unit, with the graphing calculator used as a device to provide a graphical display. Students were expected to observe and generalise about transformations of parabolas from these graphical displays. Although there was some evidence that teachers used the calculators to facilitate guided-discovery activities, generally they did not use the calculators to facilitate and/or encourage the students to conjecture and prove or refute ideas. The fact that the teachers in this study did not shift their roles may be explained in part by the short time that they used the technology as compared to the teachers in Farrell's (1996) study where the observations took place near the end of a year long course (Doerr and Zangor, 2000). More importantly, Simmt's (1997) study suggests that the teachers' views of mathematics learning are also manifested in the ways they choose to use the technology. This also concurs with other non-technology studies where teachers' images of mathematics influenced their teaching orientation (e.g., Thompson et al., 1994) and consequently students' learning orientation (e.g., Cobb et al., 1991; Thompson et al., 1994).

The relationship between teachers' view of mathematics learning and their instructional role during implementation of graphing calculators has been revealed in some studies (e.g., Tharp et al., 1997; Herring, 2000). Tharp et al. (1997) found that teachers who held a less rule-based view of mathematics learning were more inclined to view

calculators as an integral part of their instruction than those teachers who held a rule-based view. They define rule-based teachers as those who view mathematics learning as that involving manipulation of symbols and memorisation of facts, as opposed to the view that mathematics learning is based on reasoning about relationships and patterns. They also found that rule-based teachers were more likely than non-rule based teachers to use lectures and a procedural format of instruction. The researchers reported some attempts of rule-based teachers to use the exploration/discovery approaches but the teachers found it frustrating and threatening when students discovered new uses of the calculator on their own. The teachers consequently returned to the lecture mode in order to control the use of the calculator and avoid their own embarrassment. This confirms the general point that implementing new technology continually confronts teachers with many uncertainties. A lesson might not go as planned, especially when students discover other aspects of the technology that are not expected. It challenges the teacher with an ever-expanding knowledge base that is outside the realm of expertise for many (Rist, 1991). On the other hand, non-rule based teachers featured more inquiry learning using the calculator, and students were less restrained to use their calculator. Doerr and Zangor (2000) suggest that the teachers in Simmt's (1997) study could be considered as rule-based teachers as they were described as preferring algebraic solutions to mathematical problems and having a view of mathematics that strongly favoured "traditional" approaches to problems. The findings of Tharp et al. (1997) are also evident in Herring's (2000) case study on two teachers whose teaching styles are different. The teacher who aimed for an instrumental understanding (as defined by Skemp, 1978; see below), believed students should work out problems analytically first and then use the graphing calculator to support their answer. The other teacher, who taught for a relational understanding (Skemp, 1978) believed that the graphing calculator should be used to facilitate the learning of algebraic concepts through experimentation.

The way that teachers appropriate the graphing calculator in their teaching also depends on their philosophies and beliefs about the technology (Fleener, 1994; Simonsen and Dick, 1997; Milou, 1999; Herring, 2000), and it has been observed that teachers do not significantly change their beliefs about calculators over time (Schmidt, 1999). Smith and Shotsberger (1997) report that a teacher who felt that a calculator's role was to verify answers and not a means to solve problems, used the calculator for class demonstrations

and for verifications of answers only. Doerr and Zangor (1999, 2000) describe how the beliefs of one teacher in their study were reflected in her pedagogical strategies in the classroom, and enacted in her interactions with the students. They found that the teacher's confidence, flexibility of use, and her awareness of the limitations of the graphing calculator resulted in using it to justify results, for visual checking, and for graph checking. Non-calculator strategies were also included where appropriate. It is unclear as to what kind of learning environment the students in that study had experienced prior to the experimentation phase. Classrooms are known to be intrinsically stable settings with well-established cultures, social dynamics and work-related agendas, all rooted in the established curriculum in such a way that use of technology does not necessarily bring about changes (Amarel, 1984). Given that the teacher in Doerr and Zangor's study has twenty years of teaching experience, arguably she was highly competent in the content knowledge of the domain, enabling her to successfully implement the graphing calculators in her mathematics instruction.

It is widely acknowledged that teachers' content knowledge has an impact on their instruction (e.g., Shulman, 1986; Fennema and Franke, 1992; Norman, 1993; Cooney, 1999; Hudson and Borba, 1999). As Hudson and Borba (1999) suggest, teachers' subject knowledge is important if they are to work "effectively in more open-ended ways" (p. 22). However, the "deficiency model" that points out what teachers lack in mathematical knowledge is insufficient to explain how they teach. Norman (1993) asserts that it is equally important to examine the quality of the content knowledge held by teachers. His review of a number of studies shows that teachers have variable degrees of understanding of the mathematical concepts they teach. Many teachers were uncertain in applying routine arithmetic procedures and had rather scanty conceptualisations of rational number. Norman further suggests that we describe the quality of teacher's content knowledge in terms of the "instrumental" and "relational" as defined by Skemp (1978). Instrumental understanding can be defined as knowing how to perform the methods and procedures to complete a task. It might entail an algorithmic application of rules or formulae without a clear conception of why the particular application works for solving the problem. Relational understanding can be defined in terms as knowing both how to complete a task and why the approach works. It also means an understanding of the relationships among the relevant concepts involved. Norman reports that he found

many teachers to have a primarily instrumental understanding of function. Cooney et al., (cited in Cooney, 1999) provide extensive evidence that pre-service teachers can lack fundamental understandings of school mathematics although they have completed advanced level mathematics. Their survey revealed that the teachers had difficulty in recognising the graphs of exponential or logarithmic functions. They were comfortable solving equations but less so when asked to identify and analyse different kinds of graphs. Cooney et al.'s teachers seemed to exhibit a somewhat instrumental kind of understanding of the mathematics they possessed. Norman (1993) raises the question of whether the kind of limited understanding that students have in some areas may be perpetuated by the kind of understanding that teachers have. As suggested by Entwistle (1987), the quality of teachers' explanation influences not only students' interest but also their ability to develop accurate concepts.

The literature on students' errors when using the graphing calculator clearly points to the importance of the role of the teacher when technology is used. The few studies (e.g., Ward, 2000; Cavanagh and Mitchelmore, 2000) in this area do not describe the manner in which the teacher intervened when students encountered the errors, but they do make suggestions for what teachers could do to prevent students from making the errors, such as "...allowing students to display graphs in a variety of view-windows to demonstrate why some linear equations may not appear 'steep' " (Ward, 2000, p. 38). How teachers respond to student errors and difficulties has implications on their learning (Lepper et al., 1990; Mc Kendree, 1990), whether misconceptions are removed (Chi, 1996) and whether students remain "on task" or decline into proceduralized thinking and unsystematic exploration (Henningsen and Stein, 1997). The type of feedback that teachers give has implications for keeping students "on task" and the kinds of understanding that students derive (Merrill, et al. 1995). Merrill et al. categorised three types of errors that students in their study committed when using technology in their learning: syntactic errors, semantic errors and goal errors. They describe syntactic errors as those arising from the notational syntax of the technology. They say that these kind of errors are difficult for students to repair and that students may learn little by repairing this type of errors themselves, rather than being told how to do repair. They warn that the difficulty of repairing syntactic errors entails a risk of floundering. Semantic errors are conceptual errors involving erroneous choices or applications of a command function. Ensuring that

students recognised what was incorrect in their solution was found to be helpful in helping students recover from such errors. The last category, goal errors, consists of errors concerning setting or fulfilling tasks goals. Here they argue that helping students to maintain and reflect on the goal structure is necessary. This categorisation of errors and how to deal with them suggest that teachers need to have a high level of technical competence of the technology in use. Abuloum's (1996) survey of 43 teachers and 1,687 students at middle school level found that teachers with high level of training in the use of graphing calculators tended to have students with high achievement scores. Abuloum's study also suggests that the quality of knowledge of the functioning of the graphing calculator invariably exerts influence on the type of interventions that teachers provide for students.

Much of the literature points towards the way teachers define their role when technology is used is a function of their views on mathematics learning and on the use of technology in learning. The quality of teachers' knowledge also influences the way teachers teach, and the types of intervention that they can provide for their students. It seems therefore that for successful integration of graphing calculators, teachers' roles need to be aligned to the expected role of the calculator in students' learning.

2.6 Conclusion

Most of the graphing calculator studies considered in this chapter used some kind of research instrument to gauge student's understanding of some mathematical concepts including graphing concepts. This concentration on the end product of learning essentially misses the learning processes that occur, including the difficulties that students face when using the tool and the student-teacher interactions that forms an important aspect of learning. The few studies (e.g., Farrell, 1996; Doerr and Zangor, 2000) that have taken account of the learning processes have described either the role of the calculator or the role of the teacher in students' learning, and have not accounted for understandings of mathematical concepts that students develop. These studies also did not take into consideration the different mathematical background, and in particular the mathematical attainments of the students. A methodology that compares different instructional methods may obscure the salient features that each instructional method has

to offer in students' learning. Therefore two critical questions associated with graphing calculator use remain elusive. The first question is to what extent and in what ways can graphing calculators be used to help students in the learning of graphical concepts? The second question is, what teaching practices contribute to students' learning of graphical concepts with graphing calculators?

CHAPTER 3

THE THEORETICAL FRAMEWORK

This study attempts to detail students' development of graphing concepts with the graphing calculator by taking into account both learning processes and outcomes. The joint contribution of the teacher and the tool is a central concern. This chapter lays down the theoretical framework for this study. It is built upon Vygotsky's notion of knowledge construction, specifically his notion of zone of proximal development (ZPD). The chapter begins with the rationale for adopting Vygotsky's model. General ideas of Vygotsky's perspective on knowledge construction are discussed. Then follows a discussion on Vygotsky's notion of ZPD and the different interpretations that has been made of it. Two broad interpretations can be distinguished and these form the basis for the two learning models used in study as described towards the end of this chapter. The chapter concludes with the research questions of the study.

3.1 The Vygotskian perspective on knowledge construction

Vygotsky's model of knowledge construction is used in this study as it helps to account for the following issues in the context of learning of graphing with graphing calculators:

- (i) the interaction process in classrooms, in particular the student-teacher interaction,
- (ii) the use of language, educational tools and symbol systems, and
- (iii) the development of knowledge in teaching and learning.

Vygotsky's emphasis on knowledge as situated in culture, in time, and its internalisation by individual through interactions with others is representative of the situations in schools. For Vygotsky, learning is a social process that takes place in a participatory framework. Children develop as they participate in cultural practices by interacting with others. Learning occurs as the child moves from the social level to the individual level. Initially, the child experiences active problem solving at the social level, or interpsychological level (between people) and then the child gradually functions more independently at the individual level, or intrapsychological level (within the individual),

where concepts are internalised. Therefore in Vygotskian terms, all higher mental functions or processes (such as memory and voluntary attention) are learned and not biologically developed (Lerman, 1998a), and develop from action to thought (Schmittau, 1993; Sharp and Gallimore, 1991).

Although constructivists do acknowledge the importance of social interaction in learning, their accounts located conceptual change primarily within the individual, which seem to trivialise the contribution of social interaction. Lerman (1998a) argues that if social and other interactions are viewed only as creating perturbations that do not necessarily contribute to individual learning or internalisation, then there is no mechanism for the intersubjective to become intrasubjective. On the contrary, the Vygotskian approach argues that “the particular interaction pattern is the learning opportunity, and the form and content of this interaction constitutes the form and content of conceptual development” (Washescio, 1998, p. 231-232). This provides insights into how students develop *and coordinate* procedural understanding and conceptual understanding, or Skemp’s (1978) instrumental and relational understanding. How rote learning¹² takes shape and how students develop a mastery of the rhetoric of mathematical knowledge¹³ (Ernest, 1999) can also be explained from this perspective.

Vygotsky’s ideas offer extensive ways to describe the learning processes considered in this study as he emphasises the importance of imitation, practice, gesture, routine, language, conjectures and refutations, and novelty in the construction of knowledge. Notably, he criticised thinking of imitation in terms of a mechanical process. For him, a learner can imitate only to the extent that it is within his/her developmental level. Vygotsky’s focus on social processes led him to examine representational systems, or what he terms psychological tools or “signs”, that are needed to participate in such processes. Among the psychological tools he mentions are “language; various systems for counting; mnemonic techniques; algebraic symbol systems; works of art; writings;

¹² Rote learning is tied closely to the context in which the knowledge is learned and therefore can be assessed and applied only in those contexts that appear similar to the original. It is characterised with procedural competence and performing routines (Hiebert and Lefevre, 1986).

¹³ The rhetorical forms of mathematical knowledge are the conventions and standard written mathematical expressions of school mathematics, including employing standard methods of computation, transformation and proof, standard notations, technical language, accepted forms of spatial organisation of symbols and figures (Ernest, 1999).

schemes, diagrams, maps and mechanical drawings; all sorts of conventional signs...” (1981, p. 137). He considers these psychological tools to be social in that they are culturally invented, not invented by each individual or discovered by the individual’s independent interaction with nature. These psychological or cultural tools are “carriers” of established meanings. They enable individuals to create imaginary models of objects and to operate with them (Davydov and Zinchenko, 1993). Vygotsky considers language as the most important psychological tool because it not only imparts a unique quality to human thought but it also provides the basic medium for teaching and learning.

For Vygotsky, higher mental functioning is mediated by both psychological tools (signs) and “technical tools” (tools). Signs are internally oriented, aimed at mastering oneself; tools on the other hand are externally oriented, aimed at mastery over nature. Signs function as an “intermediate link” (1978, p. 39-40) that enable humans to “control their behaviour from outside” (1978, p. 40) because the sign “possesses the important characteristic of reverse action (that is it operates on the individual, not on the environment)” (1978, p. 39). Words and signs are “those means that direct our mental operations, control their course, and channel them toward the solution of the problem ...” (1986, p. 107). Tools and signs not only expand human consciousness and production, but they also make demands on the user: their use has to be learned and practised and their limits explored. However, Vygotsky posits that it is not the tools or signs in and of themselves which are important for cognitive development, but the meanings encoded in them. The meanings attributed to the signs depend on the usage of the signs by the individuals engaged in a shared activity. In Vygotsky’s words, “it is the functional use of the word, or any other sign, as means of focussing one’s attention, selecting distinctive features and analysing and synthesising them, that plays a central role in concept formation” (1986, p. 106). This extraordinary concept of mediation, that sees words and other signs as a necessity for new and higher forms of psychological processes, can also be related to various perspectives on how mathematical understanding is developed, for example, through the “articulation”¹⁴ of one’s ideas (Carpenter and Leher, 1999), and “thinking in the reverse”¹⁵ (Dubinsky, 1991).

¹⁴ Articulation involves the communication of one’s knowledge, either verbally, in writing, or through some other means like pictures, diagrams, or models (Carpenter and Leher, 1999).

¹⁵ Thinking in the reverse does not necessarily refer to undoing existing internal process but rather to constructing a new process which consists of the reverse of the original process of knowledge construction (Dubinsky, 1991).

Vygotsky's emphasis on the necessity of the use of signs and symbols as mediators of sociocultural participation provides an interesting perspective on technology, such as the graphing calculator that is the focus of the present study. These technical tools are also displays of symbol systems and psychological signs, and therefore can be viewed as mediators of sociocultural participation. Psychological tools in Vygotskian terms increase intellectual power and also transform the intellectual processes in that they enable cognitive development. Rejecting this form of mediated learning implies disregarding the ways in which many of our competencies develop as well as the ways in which the social reproduction of bias and belief can occur (Confrey, 1995).

Vygotsky's investigations of concept formation explicitly suggest that acquiring mathematical concepts demands instruction. He distinguishes between two different but interrelated types of concepts: spontaneous or everyday concepts and scientific or theoretical concepts. Spontaneous concepts are unsystematic, tightly linked to particular contexts (Daniels, 1996), emerging from the child's own reflection on everyday experiences (Kozulin, 1986). Scientific concepts are in general organised in hierarchical systems (Daniels, 1996) originating in the structured and specialised activity of classroom instruction (Kozulin, 1986). For Vygotsky, a mature concept is achieved when the scientific and everyday versions have merged (Lave and Wenger, 1996). Vygotsky views concept development as more than merely a routine exercise that can be trained but as a "complex and genuine act of thought" (1986, p. 148). The appropriation of meaning for scientific concepts is not a simple matter of internalising external systems of meaning. Vygotsky asserts that internalisation is the "internal reconstruction of an external operation" (1978, p. 56).

Thus, the appropriation of meaning for scientific concepts is a developmental and constructive process. Scientific concepts undergo a development within the child and this is characterised by a highly complex interaction between already existing concepts and new concepts, on the one hand, and by the individual's active constructional activity on the other hand. The appropriation of scientific concepts is accompanied by a reorganisation of the existing conceptual structure of the child. Thus, cultural tools and sign systems derive their meaning from the child's constructional activity. Since this constructional activity is characterised by an interaction with already existing concepts,

construction processes will vary between individuals, explaining why different children will use cultural tools such as graphing calculators in different ways.

Vygotsky's concept of zone of proximal development (ZPD) provides a useful way to model instructional processes in school and the assessment of student's intellectual abilities. For Vygotsky, instruction is effective only when it proceeds ahead of the child's development. In his well-known words, "the only good kind of instruction is that which marches ahead of development and leads it; it must be aimed not so much at the ripe as at the ripening functions" (1986, p. 188). According to this, instruction should focus on those intellectual capabilities that have not yet developed but are in the process of development i.e. those which lie in the zone of proximal development (ZPD). He defines the ZPD as "the distance between the actual developmental level as determined through independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (1978, p. 86). The "actual developmental level" denotes the actual or current level of capability of the child. Vygotsky believes that a child can function beyond this level of actual development when assisted by a teacher or collaborating with a more capable peer. The term potential development describes the "enhanced" level of development (Jones and Thornton, 1993) under guidance. The actual level of development can be viewed as the lower boundary of the ZPD and the potential development level as the upper boundary of the ZPD (Jones and Thornton, 1993).

The ZPD is thus conceived in terms of developmental progression from other-regulation to self-regulation (Brown and Ferrara, 1985), and the added capacity that a child has when supported in performance by a teacher or more skilled peer (Wilson et al., 1993). Some view the ZPD as a kind of "force field" that the child carries around (Lerman, 1998b), a dynamic region of sensitivity to instruction (Rogoff, 1991) with the instruction process creating the ZPD (Hedegaard, 1996; Lerman, 1998b). The child's zone can be "bridged" by opportunities for action with external support. This notion of bridging includes interaction with manipulative materials (Taylor, 1993) and technological tools and devices (Wilson et al., 1993).

Vygotsky also asserts that two children having the same level of actual development may differ in terms of their levels of potential development, that is their capability to learn

under teacher's guidance varies. This implies that the ZPDs will be different for these two children although working in similar interactive settings. He also insists that the child with the larger ZPD will perform better.

Thus, Vygotsky also recognises the fundamental asymmetry of the teacher-student relationship (Cobb, 1995; Lerman, 1998b), that is, the difference between the learner's understanding and the teacher's understanding. He sees positive value in the role played by the teacher's understanding, including the teacher's specific content knowledge, during the learner's construction of new knowledge. The contribution of the teacher in bringing to the learner existing cultural knowledge is not seen as opposing the individual's construction of knowledge but as enabling it. The construction of knowledge proceeds in two directions, toward the induction of the individual into cultural practice, in this case the practice of mathematics, and toward empowering the individual as an autonomous thinker. The ZPD is "the place at which a child's empirically rich but disorganised spontaneous concepts meet the systematicity and the logic of adult reasoning. As a result of this meeting, the weaknesses of spontaneous reasoning are compensated by the strengths of the scientific logic" (Kozulin, 1986, p. xxxv). The ZPD is thus an articulation of bridging the gap between the top-down (the mathematical knowledge brought to the learner from the teacher/peer) and bottom-up (the mathematical knowledge that the learner brings to the interactions) components. This allows the learner to develop new concepts and a new organisation of knowledge. Thus the organised system of concepts and theories that an individual is taught serves a critical function in the reorganisation and transformation of the individual's formal knowledge.

The ZPD provides a framework in which a learner may be successfully encouraged to enter a new task to gain competence with the assistance of a more able adult or peer. The emphasis of leading and guiding towards increasing competence means that the tasks do not need necessarily need to be related to the learner's prior activities or competence. This is useful in introducing a new domain that the child has not explored before. Moll (1990) and Cazden (1981) describe the zone as making conceivable the idea of "performance before competence". Thus lack of familiarity or competence with algebraic and computational techniques need not prevent a student from exploring complicated calculator-generated graphs and from being engaged in the problem-solving

activities that this entails. Vygotsky's idea of intellectual development through ZPD leaves open both the degree to which mediated assistance is successful and the extent to which successful mediation is generative of novel knowledge. This is a useful idea for this study as it suggests that some areas in the learning of graphical concepts that up to now been confined in the more advanced years (as prescribed in the Malaysian Mathematics Curriculum) could be implemented at a much earlier age.

Vygotsky conception of the ZPD indicates his emphasis on the importance of orienting and guiding learners toward the essential features and relations in the object of learning. Learning environments are important components of instruction but students cannot be thrown into settings, for example with technological tools and left to themselves. Aspects of pedagogy, the ways and means to facilitate learning as a meaningful activity, have to be considered. "Activity" for Vygotsky denotes personal or group involvement, intent and commitment, things which are not reflected in the usual meanings of the word in English (Crawford, 1996). For Crawford, learning in the ZPD requires students to engage in strategic learning behaviours such as developing arguments and explanations, and providing justifications which are communicated both explicitly and implicitly. The learning environment encourages students to engage in these strategic learning behaviours. Goals of learning activities as well as expectations of accomplishment are clearly articulated. The teacher's or more able peer's capacity to initiate and maintain incrementally changing environments is partly a function of their beliefs about the learners' capacities for learning and partly about their own efficacy as instructors.

A criticism of Vygotsky's ideas is their potential for teachers to become authoritarian (Confrey, 1995). However, Vygotsky's insistence on focussing instruction in the ZPD implies that teachers must know where children are in terms of their actual development. This allows them to be in a better position to know when more help is needed and when to step back and allow children use their own initiative – a process that Wood (1988) refers to as "contingent teaching".

Tudge (1990) suggests a more dynamic conception of the ZPD in which the zone extends to all those around the learner. A more systemic view of the ZPD also suggests the dimension of a "shared ZPD" for students (Crawford, 1996). Crawford criticised the general interpretation of the ZPD as merely an individual construct constituted within a

mathematical activity. This reduces it to a dyadic interaction. The shared ZPD is a social construct in mathematics classes. In her words, the shared ZPD entails the “possibilities and probabilities of mathematical activity and development” (p. 53) within a collaborative peer group involving “problem definition, argument, strategic decision making, risk taking, experimentation and self-evaluation”(p. 56).

The concept of ZPD gives suggestions as to how instruction and diagnosis should be structured in school. For meaningful learning, the ZPD of the child should be created through instructions (Hedegaard, 1996; Lerman 1998b) based on each child’s potential developmental level. In other words, “pedagogy should be oriented not toward yesterday but towards tomorrow in child development. Only will it be able to create in the process of education, those processes of development that are present in the ZPD” (Davydov and Zinchenko, 1993). Secondly, the view of the ZPD as a dynamic zone that is continuously evolving suggests that instruction and learning tasks need to be continuously reviewed. Attempts to “operationalise” the ZPD in instruction have led to several different interpretations of the ZPD and these are elaborated below.

3.2 Different interpretations of the zone of proximal development

The most prevalent interpretation is “scaffolding”, a term coined by Wood et al. (1976). In this interpretation, the role of the teacher is more like a tutor or a guide. In their study, Wood et al. outlined the scaffolding role of the tutor as follows:

- (i) simplifying the task of the learner;
- (ii) accentuating relevant features for the learner;
- (iii) demonstrating or modelling attempted solution;
- (iv) recruiting the learner’s interest in the task;
- (v) maintaining the learner’s pursuit of a particular objective; and
- (vi) reducing the frustration of the task without creating tutor dependence.

The tutor structures and models the appropriate solution to the problem, engages the child in this solution as the tutor monitors the child’s current level of skills, and supports or scaffolds the child’s extension of current skill and knowledge to a higher level of competence. The instructional approach explicitly calls for support for the initial performance of tasks to be later performed without assistance. Wood et al.’s learning model involves the notion of fading, i.e. reducing assistance as the child progresses.

Clay and Cazden (1990) caution against perceiving the scaffold in the ZPD as simply being eliminated as the child's competence grows. The scaffold provided by the teacher continues because the level of difficulty of the learning tasks increases. Thus, the scaffold is, as they put it "always at the edge of the cutting edge of the child's competencies, in his or her continually changing zone of proximal development" (p. 219).

Relating to the scaffolding notion to the ZPD, Greenfield (1984) describes the five characteristics of the scaffold (as it is known in building construction) that can be applied to defining teacher intervention within a child's ZPD in an "informal" learning situation. The characteristics are:

- (i) it provides a support;
- (ii) it functions as a tool;
- (iii) it extends the range of the worker;
- (iv) it allows the worker to accomplish a task not otherwise possible; and
- (v) it is used selectively to aid the worker where needed.

For Greenfield, it is important that the teacher structures an intervention based on what he or she knows the learner can accomplish. Independent discovery through a process of trial and error learning should be avoided at the early stages of learning.

The ZPD has also been interpreted as guided participation (Rogoff, 1991) or cognitive apprenticeship (Brown et al, 1989; van Oers, 1996; Forman 1996). In apprenticeship learning, skills are learned in a community of practitioners through observation, coaching and successive practice. After observing an expert execute an activity, the learner tries it with teacher coaching. The teacher provides reminders which are gradually removed as the task is mastered. This is similar to Lave and Wenger's (1991) notion of legitimate peripheral participation where learning is seen as increasing participation in a community of practice. "Legitimate peripheral participation" is used instead of apprenticeship to distinguish it from craft apprenticeship which gives a more restrictive connotation (Lave and Wenger, 1991). In cognitive apprenticeship, the goal is to engage the learner in purposeful, constructive activities that facilitate development and explore new thoughts, and skills.

The learning processes of scaffolding and cognitive apprenticeship in the ZPD can be described as assisted performance according to Gallimore and Tharp (1990). They outline six ways of assisting performance: modelling, contingency, managing, feeding back, instructing, questioning, and cognitive structuring. They portray progress through the ZPD in a model of four stages starting from assisted performance, to self-assisted behaviour, to internalization and automatization, and a final recursion back through previous stages for improvement in performance. Learning progression in the ZPD in Gallimore and Tharp's interpretation involves a sequence of instructions or tasks. It necessarily involves a hierarchy of stages.

All the above interpretations of the ZPD focus on providing opportunities for carefully structured, dyadic interaction whereby the teacher attempts to enhance each individual learner's progress by providing support for tasks beyond their individual capability. Learning is modelled as movement from assisted to unassisted performance. These interpretations give an impression of a unidirectional conception of development where the child acquires more advanced, adult sanctioned practices in the zone. Implicit in this is a highly structured learning environment with specific tasks and the role of the teacher being akin to a guide. This didactical view has been criticised by Crawford (1996) as it fits more closely to the transmission model of learning in which an expert teaches a novice. The teachers assume that the students share their needs, goals and intentions in a lesson. This can result in students viewing their role as knowing what the teacher expects and reduces them to memorising and practising how to reproduce the accepted information and procedures. Noss and Hoyles (1996) views this phenomenon as the didactical paradox, didactical situations compel teachers to tell students what to do but this same process deplete the learning environments of any cognitive substance.

A more dialectical view of the ZPD suggests that there is a tension between the needs, expectations and goals of teachers and students (Crawford, 1996). Interaction between the teachers and students also change the teachers' perspectives. Crawford (1986) in her work, comparing aboriginal and non-aboriginal Australian student's cognitive processes, concluded that the way students are positioned by the teachers in the classroom and the way in which students position themselves in relation to the tasks and to each other influence learning outcomes. These "positionings" are derived from their various experiences outside school. Guidance within the ZPD is bi-directional because the

learner's changing task performance must guide the teacher's instruction as much as the teacher's actions guide the learner (Brown et al., 1996).

Newman et al. (1989) also share the dialectical view of the ZPD. They caution against misinterpreting the tasks in a ZPD by ordering them from simple to complex or structuring later tasks that require mastery of earlier tasks. They criticise this as deriving from traditional views of learning, such as Gagne's (1967) notion of learning hierarchy. Instead, they proposed that learning should be based on the "social distribution" of tasks, that is divisions of labour for a sequence of tasks between the teacher and the child. They emphasise that the goals of the activity should be clear for the learner and the teacher but that carrying out the components of the task should be flexible. The components of the task that the expert (teacher) undertakes may be "higher" level or "lower" level depending on what the expert sees appropriate. The expert may undertake the more difficult components, leaving the lower level operations to the novice. Alternatively, the expert may allow the novice to execute the difficult component of the task and give support by handling the routine details that might otherwise distract the novice from the higher levels of thinking. In this way learning is seen as the negotiation of meaning in the construction zone where the focus is the students' and the teachers' mutual appropriation of each others' contributions. Both the students and the teacher manifest "powerfulness" and "powerlessness" at different times in an activity (Lerman, 1998b). This seems a useful idea for when technology is involved in a classroom where both the teacher and student are novices. Technical competence is not seen as a necessity for the teacher to maintain control and the student's confidence. Teachers in the ACOT (Sandholtz et al., 1997) project observed that children learn about and master technology far more rapidly than adults. And Selinger (2001), quoting Cuban, insists that using computer technology in Vygotsky's terms should be seen about supporting students working together on self-directed activities to create understanding.

Viewing learning as a dialectical situation suggests an exploratory learning environment with the teacher more as a facilitator or counsellor (Newman et al., 1989). For Hedagaard (1996), a learning activity is intended to develop the ZPD of the class as a whole where each student acquires knowledge through activities shared between the student and the teacher, and among the students themselves. She believes that theoretical knowledge must be acquired through exploratory activity.

The notion of ZPD thus provides an approach to teaching and learning. The different interpretations of ZPD reflect a continuing dialectic or tension between teacher guidance and student-initiated exploration. Two major teaching and learning approaches emerge from the two broad interpretations in the ZPD. The first interpretation of the ZPD conceives learning as occurring in didactical situations where learning tasks are structured. The second interpretation takes a more dialectical view of learning where learning tends to be less directed, in other words, students assume more control of their own learning trajectories. Learning tasks are thus less structured and more exploratory in nature. Based on these two broad interpretations, two learning models are proposed in this study: the *Structured Learning Model* (StrucLM) and the *Interpretative Learning Model* (InterLM). The Structured Learning Model requires a didactic and structured approach to using the graphing calculator. The Interpretative Learning Model requires an exploratory learning approach where both the teachers and students play a more active role in interpreting how the graphing calculator functions as a tool for learning.

3.3 The learning models for this study

In this study, the approach is to emphasise the social environment of the instructional resources, that is the graphing calculators. Contrary to the cognitive/constructivist approach of looking at the student-machine interaction as one where the student constructs knowledge on his own using the machine, this study views the machine as a mediator between the teacher/student and the student, and the teacher as a mediator between the student and the machine. Thus learning is seen in terms of the social interactions between the teacher/student and machine and between the student and the machine. Objects, events or concepts gain their meaning through their use and the machines are viewed as helping to create “functional learning environments” (Newman, 1987). The term functional means that learning activities must have a function or purpose from the learner’s point of view. The role of the teacher is both to help the student appropriate the machine as a tool for learning, and to be an adaptive expert “translating” the student’s ideas into the terms of the machine.

The purpose for using two learning models in this study is to provide a framework to model different ways of using the graphing calculator in the process of learning graphical

concepts. It is also to effect different kinds of student-teacher interactions when graphing calculator is integrated in students' activities.

To implement the two learning models, it is helpful to think of the learning situations at two ends, "structured" and "open" of a continuum. This can also be thought of as having direct instruction, explanation or exposition at one end and investigation, discovery or inquiry at the other end. Bliss (1994) discusses them in terms of "exploratory" and "expressive" modes of learning. The former involves the students in exploring ideas about a topic or example presented by the teacher and the latter involves the students in exploring their own ideas. It is useful to reflect on the delineation between exposition and discovery approaches. Dean (1982) categorises exposition methods as lecture, deductive or inductive. The lecture method necessarily involves three phases beginning with definitions of expressions or symbols which form the subject of the lecture; the second phase is explanations of the definitions, and the final phase involves relating the different components. The deductive method, as the name suggests, involves the teacher solving a problem by explaining a series of logical, agreed rules of inference. In the inductive method, the teacher considers a number of particular examples and focuses the pupil's attention on the common properties which are present in all these examples to make a generalisation from them. The method of exposition of Dean has many aspects of the direct instruction as analysed by Jaworski¹⁶ (1994) and the whole-class teaching as criticised by Goulding¹⁷ (1997).

The other extreme method of instruction is discovery or inquiry. However, Elliot and Adelman (1975, cited in Jaworski, 1994) contrast between inquiry and discovery. The teacher guidance in both methods adopts not telling or explicitly indicating pre-structured learning outcomes. In their view, the teacher in the inquiry approach is more "refrained" (not telling) than in the discovery approach. Jaworski (1994) proposes that the fundamental difference between discovery or inquiry can be viewed in terms of

¹⁶ "Typically the teacher introduced the mathematical content of a lesson using exposition and explanation (teacher talk), usually in front of the classroom (using blackboard and chalk). Pupils were then given exercises through which they practised the topics introduced by the teacher." (p. 8)

¹⁷ Whole-class teaching is interpreted as the class being taught as if it were an individual. It sees learners in behaviourist terms in which the teacher's role is to mould and strengthen students' responses until they are correct. The teacher gives explanations, and demonstrates examples with expectations that students follow the sequence of the teacher's ideas and decisions, and master them at approximately the same rate.

Ernest's two metaphors of the problem solving process: discovery as seeking a destination and inquiry as emphasising the journey. Dean (1982) distinguishes four methods of teaching under the heading of discovery teaching: directed discovery, guided discovery, exploratory discovery and free discovery. The directed discovery method contains only a small element of discovery and requires very little initiative from pupils. They are being pushed along a fixed path with the teacher directing the investigation and continually giving instructions to pupils who are not certain about the next step. In guided discovery, pupils are not directed stage by stage. The teacher begins the lesson with a statement or question which suggests the right way of thinking about, and investigating a situation. Exploratory discovery structures the learning activity by providing the objects or ideas which the pupils should use but does not give any instructions even as to the aim of the lesson. Free discovery comes from the natural curiosity of the pupil, about any object or ideas. It is totally initiated by the student but teachers give encouragement and provide suggestions when they think pupils will learn more in one particular direction. Dean summarises the sequence from directed to free discovery as a transformation towards:

- (i) increase in pupil initiative,
- (ii) decreasing probability of pupil discovery,
- (iii) reduction in the homogeneity of pupils' attainment,
- (iv) the teacher's role shifting from director to adviser,
- (v) the teacher's image changing from one who knows everything to one who has to search for information,
- (vi) decreasing efficiency in knowledge transmission, and
- (vii) increasing difficulty in assessing and planning progress towards a defined end point.

This characterisation is useful in assessing the learning models proposed in this study as it implies that the type of task, the nature of teacher intervention, and the method of presenting the task to the students have to be considered and differentiated. First, it needs to be pointed out that the expressive learning model of Bliss (1994) that seems synonymous with the Dean's exploratory discovery is rejected in this study. This model is in opposition to Vygotsky's idea of learning, in which teachers are central in the learning process. Also, the wholesale application of the notion of exposition, direct-instruction, and whole class instruction is rejected as it neglects Vygotsky's emphasis on the interaction and activity necessary in the learning process.

In general, the Structured Learning Model and the Interpretative Learning Model will take on some aspects of Dean's (1982) categories of directed discovery and the guided discovery, respectively. The Structured Learning Model views student learning as being pushed along a structured path where the teacher will direct the investigation by continually giving instructions to students who are uncertain about the next step. In the Interpretative Learning Model, students are not directed stage by stage. The teacher begins the lesson by proposing some accessible starting points, using statements or questions that suggest some ways of thinking about, and investigating a graphical concept.

The learning tasks used in this study can be categorised in terms of their openness. The tasks in the InterLM are more open in three ways: in terms of the process, the end products, and the ways to develop the tasks. Nohda (2000) says that processes are open when there are multiple correct ways of carrying out the task or solving a problem. By open end products, Nohda refers to multiple correct answers to a problem. When ways to develop are open, there is a flexibility for extending and developing new problems by changing the conditions or attributes of a tasks. Tasks in the open learning approach can also be seen as having well-structured objectives but ill-structured procedures (Magidson, 1992).

With respect to teacher intervention, this study models itself on the work of Hoyles and Sutherland (1989). They make a distinction between process type interventions and goal type interventions which can be read as more closed forms of intervention versus more open forms of interventions. Process type interventions are more closed in the sense that they are more structured and directed towards encouraging students to reflect on and predict the process or problem solving procedure. Some examples of intervention questions of this type are: "Have you thought about this? How about rearranging the points? What happens when the value changes?" Goal type interventions are directed towards encouraging students to reflect on the goal of their activity. They are more open in the sense that students are encouraged to generate meanings for themselves. Some examples of intervention questions of this type are: "What are you doing? Why are you doing it? Can you tell me about it? Did you find where you went wrong?" Hoyles and Sutherland categorise both process and goal interventions as "reflectional" or "metacognitive". It is relevant here to point out that they categorise another type of

teacher intervention: directional. Directional intervention types are those that influence and/or change the focus of the students' attention and consists of six subcategories: nudge (e.g., "Do you want to clear the screen?"), method (encouraging students to use a suitable method of problem solving), building (encouraging students to apply a particular piece of previously learned material or knowledge), factual (supplying a particular piece of new information which is necessary for student to continue, or reminding students of a piece of information), powerful idea (introducing a 'new powerful idea' or concept), and mathematical idea (introducing a new mathematical idea).

The StrucLM and InterLM differ in terms of the use of whole class instruction. In the structured learning approach, at the beginning of the lesson, the teacher explains the tasks to be undertaken and the mathematical ideas to be explored followed by individual question and answer about those ideas during the tasks. Also, the necessary operations and syntax of the calculator required for executing the tasks are explicated at the start. In the interpretative learning approach, students carry out the learning tasks with a minimum of whole class instruction, which is restricted to presenting new mathematical terms or ideas necessary to proceed in the activity, or necessary calculator operations and syntax. Therefore students in the StrucLM are each in a "teacher-led" setting and students in the InterLM are in an "activity-led" setting.

The StrucLM conceives learning as occurring in didactical situations in the ZPD. The tasks are designed to confront students with a particular mathematical idea and to ensure that students use appropriate mathematical ideas. The teacher's role is as a guide, suggesting tasks and presenting alternative interpretations of problems. The teacher uses process type questions in the interventions. The calculator is used to support whatever concept teachers want students to develop or discover. Both the teacher and the calculator are seen as providing the scaffolding for the student.

On the other hand, the InterLM adopts a more dialectical view of learning in the ZPD. The learning activities are less structured and more exploratory in nature allowing pupils the opportunity to sometimes explore some of the graphical ideas on their own with the graphing calculator. The role of the teacher is more like a facilitator. The intervention strategies used in the InterLM are goal directed with commentaries and follow-up questions that put responsibility back on to the students. Students in the InterLM are

expected to take on problem for themselves with minimum whole class instruction. Students are encouraged to reflect on what they want to achieve. In this directed (with clear objectives) exploratory approach, it is expected that students will (re)construct mathematical ideas by choice and not as a result of the teacher's insistence (Noss and Hoyles, 1996). The calculator is adopted as a tool for students to explore and develop mathematical ideas.

The present study does not aim to investigate the differences between the two learning models per se. Rather, the two learning models are used to provide a framework to model different ways of using the graphing calculator in the process of learning graphing concepts. Arguably, students will request different teacher interventions, in the two distinctively different learning environments. This will bring forth the salient features or types of interventions that facilitate or constrain students' learning. Merely comparing the two learning models is not insightful as it obscures the salient characteristics of the interaction processes that each model has to offer, which could point to both the teacher's role and the role of the graphing calculator in assisting student's development of graphing concepts.

Summing up, the two learning models proposed in this study are intended to provide different ways of using the graphing calculator and thus identify the salient features of teacher intervention that assist students' learning with the graphing calculator. This gives insights on to what extent and in what ways graphing calculators assist the learning of graphical concepts.

3.4 The study

The review of literature shows that although there are some studies specifically intended to explore the effectiveness and limitations of the graphing calculator as a tool for teaching and learning of graphing, the role of the calculator and the role of the teacher remain unclear. The role of the teacher is important but simply contrasting between different instructional methods is not useful. Also it is equally important to look at both students' learning processes and the mathematical thinking that emerges from the use of the graphing calculator.

Thus, the aim of this study is to explore in what ways and to what extent graphing calculators can be used by 14 year old students in their learning about graphical concepts. The influence of the graphing calculator is investigated using two learning models, the Structured Learning Model (StructLM) and the Interpretative Learning Model (InterLM). The StructLM assumes a more didactical view of teaching and learning. Students are taught in a teacher-led approach using structured tasks. The InterLM takes a more dialectical view of teaching and learning. Students learn through an activity-led approach that is based on open learning tasks with the graphing calculator.

This study is concerned only with graphs drawn in the Cartesian coordinate system. The graph types considered are polynomial equations (linear, quadratic, cubic and quartic equations), trigonometric equations; logarithmic equations and exponential equations. These graphs of equations may or may not be graphs of functions.

3.5 The research questions

This study attempts to investigate the following research questions.

1. To what extent do students of different mathematical attainment develop graphical concepts with the graphing calculator according to each learning model?
2. What characterises the transitions in students' understanding of graphical concepts with the graphing calculator?
3. What forms of teacher intervention emerge when graphing calculators are integrated in students' learning?

CHAPTER 4

METHODOLOGY

This chapter begins with the design of the study, and follows with the design of the lessons used in the study in section 4.2. In section 4.2, differences between the lessons in the Structured Learning Model and Interpretative Learning Model are highlighted. The design of the assessments used in this study is presented in section 4.3. Section 4.4 presents the findings of the pilot study and its implications for the main study. The next section describes how the main study was implemented. In this section how the two teachers were selected and trained to carry out the study is discussed. A brief description of the selection of the students in the study and how the videotaping was carried out is rationalised. The data collection method is discussed to include how the test instruments were administered throughout the study. Section 4.6 describes the development of the analytic framework, beginning from construction of students' and teachers' profiles to the selection of students for inclusion in the analysis. The chapter concludes with a description on the analytic framework used in this study to examine the kinds of mathematical understanding that students developed when using the graphing calculator.

4.1 The design of the study

The aim of the study is to track the students' knowledge development in each learning model. The graphical concepts in the learning models were based on the graphical concepts contained in the Malaysian Secondary School Curriculum (Pusat Perkembangan Kurikulum, 1989b, c; 1990a, b). This was necessary to ensure that parents would find it worthwhile to consent for their children to participate in an independent research project carried on outside the formal schooling hours, given the high emphasis that Malaysian parents placed on the academic achievement of their children in general. If the study was totally separate from the curriculum, it would be difficult to get support not only from the parents but also to solicit cooperation from the school to carry out the study.

The learning tasks were designed according to the two learning models proposed in this study to provide a framework to model different ways of using the graphing calculator in the learning of the graphical concepts. Three aspects were considered in differentiating the learning models: the nature of the tasks, the role of the teacher, and the method of presenting the tasks. The differences between the two models are shown in the table below:

Table 4.01 Differences between the Structured and Interpretative Learning Model

Aspect considered	Structured Learning Model	Interpretative Learning Model
Nature of the tasks	Structured	Loosely structured, more open
Role of the teacher	Didactic/ guide/ coach	Dialectic/ facilitator
Method of presenting the tasks	Teacher-led	Activity-led

The tasks in the StrucLM were more structured, whereas the tasks in the InterLM were more loosely structured with more degrees of freedom for students to choose, for example, their own numerical values as demonstrated in Table 4.03. The design of the tasks was to warrant the teacher to play a more didactic role as a guide in the StructLM and a more dialectical role as a fascilitator in the InterLM. The StructLM and the InterLM differ in terms of the use of whole class instruction. In the StructLM, students were guided by the teacher in their learning activities in a teacher-led setting, whereas in the InterLM, students carried out the learning activities with restricted teacher instruction, guided by the tasks outlined in their worksheets in an activity-led setting.

The learning tasks were organised into a series of fifteen lessons structured according to the graphical concepts found throughout the curriculum that spans in the secondary school (from ages 14-17). The lessons were sequenced following the order of the graphical concepts found in the curriculum as part of the aim is to see what and how students learn with the graphing calculator, and whether students could learn material beyond what was outlined for the school year.

As the study used two learning models to provide a framework to model different ways of using the graphing calculator in the learning of graphical concepts, two groups of students were selected to learn the graphical concepts. Each group followed one of the learning models throughout the whole study period, with a teacher being given responsibility to implement each of the learning model. Only one type of learning model was used for each group in order to:

- (i) ensure that each teacher could concentrate on one role only, either as a coach/guide or as a facilitator described according to each learning model, and
- (ii) enable the researcher to focus on the activities and interaction that were taking place in each of the learning models. It would be difficult for the researcher to distinguish and to focus on each of the two learning approaches if they were taking place simultaneously.

The two groups of students in this study were in the same school. This is important because different schools means different school context including ethos, location, resources, leadership, etc. which can exert an important influence on learning (Broadfoot, 1996). Secondly, since the graphing calculator is not used in Malaysian classrooms, it is expected that the teachers are novice graphing calculator users, and therefore it is anticipated that they would find it problematic to manage the new classroom situation with learning activities incorporating the graphing calculators. Having both teachers in the same school could provide opportunities for the two teachers to share ideas on and discuss issues arising from the calculator use and therefore provide collegial support that could boost their confidence (Thomas et al., 1995) and have a positive attitude towards the research (Ainley and Pratt, 1993).

As this study is aimed to model how students of different mathematical attainment learn graphical concepts with a graphing calculator, each group of students has to be representative of low, medium and high mathematical attainment. This is discussed in detail in section 4.6.3.

4.2 The design of the lessons

The fifteen lessons in both the learning models were sequenced according to the order of the graphical concepts outlined in the Malaysian Secondary School Curriculum

(spanning over four years of secondary school) from the basic idea of Cartesian coordinates (including naming the coordinates) to the idea of the Cartesian Connection, the straight-line concepts, and graphs of non-linear equations as shown in table 4.02 below. The table also shows comparison between the lessons that were constructed in both learning models. The lessons have the same learning objectives and in many lessons the same learning activities but the nature and approach of the tasks are different in each learning model, as illustrated in Table 4.03.

Table 4.02 The learning activities in the learning models

Lesson	Learning activities in	
	Structured Learning Model	Interpretative Learning Model
Lesson 1	Representing information in different ways. Examining the idea of mapping, ordered pairs/ Cartesian coordinates and Cartesian graph.	Representing information in different ways. Examining the idea of mapping, ordered pairs/ Cartesian coordinates and Cartesian graph.
Lesson 2	Familiarizing with some conventions of the calculator (e.g. using SET UP command). Using the calculator and graph paper to plot points and writing them correctly as ordered pairs. Exploring the “finite” viewing window. Differentiating between positive and negative values for x and y, and origin.	Familiarizing with some conventions of the calculator (e.g. using SET UP command). Using the calculator and graph paper to plot points and writing them correctly as ordered pairs. Exploring the “finite” viewing window. Differentiating between positive and negative values for x and y, and origin.
Lesson 3	Exploring more commands, e.g. [AC/ON], <OFF>, [EXIT], [SHIFT], [EXE] and <QUIT> when plotting points.	Exploring more commands, e.g. [AC/ON], <OFF>, [EXIT], [SHIFT], [EXE] and <QUIT> when plotting points.
Lesson 4	Examining the effect of changing scales and view-windows. Differentiating between the INIT and STD window. Relating between the scales and view-windows. Using the view-windows including moving between different view-windows, storing and recalling.	Examining the effect of changing scales and view-windows. Differentiating between the INIT and STD window. Relating between the scales and view-windows. Using the view-windows including moving between different view-windows, storing and recalling.
Lesson 5	Drawing lines on the graph paper and on the calculator. Examining number of points passing through a line.	Drawing lines on the graph paper and on the calculator. Examining number of points passing through a line.

	Examining effects of pixels. Writing equations of vertical and horizontal line.	Examining effects of pixels. Writing equations of vertical and horizontal line.
Lesson 6	Examining the effect of view-window/scales on pixels. Looking at the connection between points, graph and equation (using paper and pencil). Examining the straight line graph as a continuous graph (using the tracer). Examining the variable concept (using paper and pencil). Examining the graph of $y=x$ in the RUN Mode. Examining the connection between the equation and the points that lie on the line.	Examining the effect of view-window/scales on pixels. Looking at the connection between points, graph and equation (using GRAPH Mode and TABLE Mode and Dual Screen). Examining the straight line graph as a continuous graph (using the tracer and G-CON and G-PLT). Examining the range and variable (using the TABLE Mode). Examining the graph of $y=x$ in the GRAPH Mode. Examining the connection between the equation and the points that lie on the line.
Lesson 7	Examining the intercept concept and the meaning of number in $y=x+\text{"number"}$. Drawing the graph of $y=x$ in the GRAPH Mode and TABLE Mode. Drawing graphs of $y=x$ using the G-CON and G-PLT facility. Looking at the connection between points, graph and equation (using GRAPH Mode and TABLE Mode and Dual Screen).	Examining the straight line graph that passes through the origin. Investigating the effect of varying the values of m in $y=mx$ using the DYNA Mode.
Lesson 8	Examining graphs of $y=mx$ using the GRAPH Mode and DYNA Mode. Drawing graphs of $y=mx+c$ using the GRAPH Mode. Examining the properties of the straight lines with different facilities.	Exploring the graph of straight line that intercepts the y -axis. Examining the general equation of the straight line as $y=mx+c$. Differentiating between the x and y -intercept.
Lesson 9	Drawing straight-line graphs on the graph paper and choosing the appropriate scale and range. Differentiating between the x and y -intercept. Determining whether a point lies on the line or not.	Examining the properties of the straight line using different modes and Dual Screen facility.
Lesson 10	Computing the gradient. Determining the equation of a straight line passing through any two points, and given gradient and passing through a point. Exploring effects of visual illusions (on gradients) with the calculator.	Drawing straight-line graphs on the graph paper and choosing the appropriate scale and range. Differentiating between the x and y -intercept. Determining whether a point lies on the line or not.
Lesson 11	Examining the properties of parallel and perpendicular lines. Examining visual illusions (on proximity	Computing the gradient. Determining the equation of a straight line passing through any two points, and given

	and perpendicularity) with the calculator by using the Zoom and other view-window settings.	gradient and passing through a point. Exploring effects of visual illusions (on gradients) with the calculator.
Lesson 12	Distinguishing the shape of the graph of $y=x$ and $y=x^2$. Examining some families of graphs of quadratic equations. Examining the interaction between the scale and the shape of the graph.	Examining the properties of parallel and perpendicular lines. Examining visual illusions (on proximity and perpendicularity) with the calculator by using the Zoom and other view-window settings.
Lesson 13	Looking at the connection between the different quadratic equations and the properties of the graph. Drawing quadratic graphs with the calculator.	Examining the families of graphs of $y=ax^2$. Exploring some families of graphs of quadratic equations. Examining the interaction between the scale and the shape of the graph.
Lesson 14	Examining the algebraic form and the shape of the graph. Exploring graphs of cubic, quartic and reciprocal equations with the calculator and using its facilities including Zoom and view-windows. Exploring graphs using the CONIC Mode.	Examining the algebraic form and the shape of the graph. Exploring graphs of cubic, quartic and reciprocal equations with the calculator and using its facilities including Zoom and view-windows. Exploring graphs using the CONIC Mode.
Lesson 15	Exploring graphs of equations of various forms. Using the calculator “sensibly” to draw the complete graph.	Exploring graphs of equations of various forms. Using the calculator “sensibly” to draw the complete graph.

↔ arrow depicting location of the same activity in the two models


Lessons 1-6 are two corresponding sets of lessons in each learning model where the concepts contained in the learning activities run concurrently. Students were initially introduced to the calculator and simple graphical concepts at the same time as they worked on the learning tasks. Included in the lessons, therefore were tasks to induct students to use the conventions and commands of the graphing calculators and to contrast between the mathematical conventions and the calculator conventions. This was necessary to gauge the difficulties that students will have in familiarising with the calculator commands and conventions, and to what extent the difficulties have an impact on their learning of mathematical conventions involved with the graphical concepts. The lessons were also designed to address issues of misconceptions in graphing and errors in graphing calculator use derived from the literature review. Three main issues were considered: issues of scaling, pixels and hand graphing. In the former, the tasks were

designed to confront students with the differences between the two view-window-settings: the default INIT window (with a square coordinate grid) and the built in STD window (with rectangular coordinate grid). The lessons were also designed so that students could work in various view-windows thereby encountering the scaling concept and how the view-window settings affect the readings on the graphing calculator and the appearance of the line. With regard to pixels, some activities were designed to ensure that students were aware of the effect of pixels on the appearance on their graphs. Hand graphing activities were also included as it is reported this was necessary for a concrete understanding of graphing (e.g. Giamati, 1991; Demana et al., 1993). These issues were also addressed in the subsequent lessons.

In Lesson 7 and 8, the activities differed in each learning model as they followed different progression of concepts to conform to the conditions of structured and open tasks, which resulted in variation of the calculator functions and facilities. In the InterLM, it seemed a natural progression from Lesson 6 that investigates the line $y=x$ for students to pursue with the exploration of the concept of $y=mx$ in Lesson 7 using the DYNA Mode of the calculator and to proceed with exploration of graph that intercepts the y-axis in Lesson 8 and families of straight lines in Lesson 9. For this reason too (natural progression of exploration), students in the InterLM were exposed to the GRAPH Mode facility, the TABLE Mode facility and the Dual screen facility much earlier in Lesson 6 when they were looking at the connection between points, graph and equation whereas these facilities were introduced in StrucLM in Lesson 7 when the task involves drawing the graph of $y=x$.

Table 4.03 illustrates the differences in the nature of the learning tasks according to the two learning models. The task in Structured Learning Model is extracted from Lesson 8 and the task in the Interpretative Learning Model is extracted from Lesson 7 where the objective of the task is for students to recognise the effect of varying the values of m in the equation $y=mx$. Included are examples of suggested interventions that teachers could make either process type or goal type questions to warrant a more didactic or less didactic intervention.

Table 4.03 Example of tasks and teacher interventions in the two learning models

Structured Learning Model	Interpretative Learning Model
<p>TASKS</p> <p>Draw graphs of relationships with $y = \text{number} \times x$.</p> <p>Some examples to start with are $y = 2 \times x$, $y = \frac{1}{2} \times x$ and $y = (-1) \times x$.</p> <p>Then predict what the graph of $y = 4 \times x$ will look like; graph it to check your prediction.</p> <p>Begin your investigation with the GRAPH Mode. Repeat your investigation in the DYNA Mode. In this mode, specify the parameter (or in the graphic calculator terms the dynamic variable) range from – 3 to 4.</p> <p>What can you deduce about the value of the number/ parameter in the equation?</p> <p>What is the common characteristic of this family of graphs?</p>	<p>TASKS</p> <p>Try to reproduce as close as possible the “Starburst” picture below.</p> <p>You may find it helpful to explore the DYNA Mode before you attempt to draw this figure in the GRAPH Mode</p> <p>It might also be helpful if you record the lines you have used.</p> <p>What do you notice about the “number” in front of the variable x?</p> <p>Write a report on how you obtain the Starburst picture.</p> 
<p>INTERVENTIONS</p> <ol style="list-style-type: none"> 1. Look at the direction of the line when “the number” is positive. 2. Look at the direction of the line when “the number” is negative. 3. What happens to the “steepness” of the line as the value of “the number” increases (or decreases)? 4. Are graphs having different “numbers” parallel to each other? Are graphs having the same “number” parallel to each other? 	<p>INTERVENTIONS</p> <ol style="list-style-type: none"> 1. What line did you begin with? Why did you choose this line to begin with? 2. What is the equation of the line that you chose? Why did you choose this equation? 3. What is affected/ happens as the line gets steeper/ less steep? 4. How is the direction of the graph affected? <p>What happens when the “number” in front of the variable x takes a positive value? What happens when the “number” in front of the variable x takes a negative value?</p>

The StrucLM follows a “curriculum model” which permits the lessons to be presented in closer adherence to the order of the graphical concepts outlined in the curriculum thereby warranting the calculator functions that are used to be according to what is thought logical and appropriate for the concept. For example, the intercept concept was introduced in the form $y = x + \text{“number”}$ in Lesson 7 after examining the graph of $y = x$ in Lesson 6, which calls for the GRAPH Mode facility, the TABLE Mode facility and the

Dual screen facility. Investigation of $y=mx$ with varying values of m followed in Lesson 8 where the GRAPH Mode facility fitted with the nature of the task which appropriately followed with introduction of the DYNA Mode.

Lessons 9, 10 and 11 in the StrucLM are corresponding to Lessons 10, 11 and 12 in the InterLM as illustrated in Table 4.02. In Lesson 12, the StrucLM guides the students to examine the quadratic graphs from distinguishing between the shape of $y=x$ and $y=x^2$ to examining different families of quadratic graphs where the parameters are specified, consistent with the “closeness” of the tasks. This is continued in Lesson 13. For the InterLM, the “openness” of the tasks sees it a natural progression to explore graphs of different forms of quadratic equations by varying the parameters beginning from $y=ax^2$ to $y=(x+a)^2$, $y=x^2+ax$, and $y=(x+a)^2+b$, which could be carried out in one lesson (Lesson 13). In both learning models, the lessons were designed to confront students with problems of visual illusions.

Lessons 14-15 are again two corresponding sets of lessons in each learning model where the concepts contained in the learning activities run concurrently but the pedagogical approaches differ.

4.3 The design of the assessment

The aim of the study is to chart the development of students' knowledge. The assessment used in this study follows an *ipsative* method of assessing the students' performance (ipsative means self-referential or in “relation to itself”), thereby comparing the current performance of the student to his/her previous performance to examine how much progression occurs. Two types of assessment instruments were prepared: a pre-test and post-test to assess students' overall concept development and intermediate assessment. The intermediate assessments contained a series of five assessment tests to check the reliability of the outcome of students' performance in the post-test. It was used for diagnostic purposes in assessing students' progress in the development of graphical concepts, as well as identifying students' difficulty in specific graphical concepts or use of calculator functions.

The pre-test (see Appendix 12) was a paper-pencil test designed to gauge the level of mathematical knowledge (algebra, number, graphical) at the beginning of the study. The questions related to the number concept such as decimals and fractions, operation on numbers, number line and negative numbers regarded as pre-requisite knowledge for learning of graphical concepts were tested. Questions relating to algebra including algebraic manipulation (with varying degrees of difficulty), substitution, squares and square roots were assessed to see how it would impact students' understanding of graphical concepts. The graphical concept questions in the pre-test was designed to gauge the extent of graphical knowledge that students had prior to the study, and to assess the level and the types of graphical concepts that students had not encountered prior to the study.

The post-test (see Appendix 13) was a paper-pencil test designed to assess students' progression in terms of what kinds of graphical concepts they had developed. Therefore some of the graphical concept questions found in the pre-test were also used in the post-test. In addition, in the post-test, some of the graphical-concept questions were designed to test specific graphical concepts, for example the idea of gradient. Students had to explain, defend or justify a solution or process they had employed in "solving" the problems or tasks to enable the researcher to determine the level of sophistication of their knowledge and to discern the kinds of mathematical understanding students had developed with the graphing calculator. A few of the graphical concept questions were selected from tests used in other studies (mostly from the CSMS studies, Kerslake, 1981) to assess students' graphical concepts and to provide base-line comparisons that could illuminate the potential of graphing calculator in students' learning of graphical concepts.

The intermediate assessment consisted of five tests (Assessment 1, 2, 3, 4, 5) that in general were paper and pencil tests (Appendix 7, 8, 9, 10, 11 respectively). However, included were tasks to assess whether students could use the graphing calculator facilities to facilitate their development of the graphical concepts. For example, Question 1 in Assessment 1 requires the students to draw the triangle with the following points: (3,6), (-1,-1), (6,-2) on their graph paper *and* on their calculator. As each assessment was to be administered at different stages of students' progression in their learning activities, the test items in each assessment were based on upon the graphical concepts found in the related lessons.

4.4 Pilot Study

A pilot study was carried out before the main study to trial the design and implementation of the study. The study was carried out in a regular school, that is a normal day school. Students were pre-tested before the study and formed into two groups with each group following one of the learning models. Two teachers who taught upper secondary level were chosen by the school administrator to implement each of the learning models separately to each group of students. Schooling time is divided into two sessions in Malaysia, one in the morning session (7.45 a.m. – 1.15 p.m.) and the other in the afternoon session (1.15 p.m. – 6.45 p.m.). Most lower secondary students (Secondary 1 and Secondary 2) follow the afternoon session whereas most of upper secondary students (Secondary 3 to Secondary 5) follow the morning session. Thus students in the pilot study were taught outside their normal schooling hours but the two teachers worked within their normal schooling hours.

The pilot study was structured around each teacher's free time, that is, time when they were not teaching according to their timetable. The teachers were taught how to use the graphing calculator and explained how to implement the learning models. Each teacher was provided with a teaching module prepared by the researcher, designed to serve as a guide for the teachers on the scope of graphing concepts that could be covered with the graphing calculators, the corresponding learning activities that could be carried out, and general suggested interventions (see Table 4.04 p. 86), based on the learning models. Each student was provided with a graphing calculator, a simplified and translated version of the calculator manual¹⁸ sufficient for their purpose and some worksheets containing learning tasks.

4.5 Outcomes of the pilot study and implications for the main study

The pilot study showed that involving a classroom of twenty-five pupils, thought to lend the study a more natural setting resulted in two main problems. First, the video

¹⁸ Unlike the simple 4-operation calculators, it is not possible simply to pick up a graphing calculator and find out how it works just by pressing the buttons. The original manual of the graphing calculator was thought to be too massive and complicated. Combined with the fact that it is in English, it would have been overwhelming for students.

recording of the students identified as case subjects was very difficult. Since the calculator is a very small device, some room was needed to enable the camera operator to manoeuvre the camera to capture how students were using the graphing calculator, for example, what buttons the students were pressing and the display on the screen of the calculator. Second, the large number of students limited the teacher's interaction with the students selected for the case study, both in terms of duration and the number of interventions, which is important for this study. The interaction time and frequency was further reduced when the teacher was occupied with other students that were not subjects of the study. Therefore a major modification had to be made in the main study by limiting the number of students in each learning model group to twelve instead of twenty-five students.

The pilot study also showed that it was chaotic to carry out the study within teachers' normal schooling hours. It limited time for organisation of the class, and teacher training and discussion, and thereby teacher's knowledge and skills to implement in the manner it was envisaged, and possibly the teachers' commitment to the study. It was necessary that in the main study the two teachers who participated are volunteers, who were willing to implement the study outside their normal school schooling hours. This also presupposes that they both had the same level of commitment and motivation for taking part in the study, as teacher variables, for example teacher efficiency and enthusiasm, are factors that may affect the outcome of any learning experiment (Ary et al., 1990).

It was observed in the pilot study that teachers tended to follow rigorously the specified guidelines, using only the topics explicitly mentioned and not applying the guidelines to topics of their own choosing. Therefore the teaching modules were modified for the main study to provide a more comprehensive overview of the possible topics and examples of tasks involving them. Teaching Module for the Structured Learning Model (Module A) (Appendix 2) and Teaching Module for the Interpretative Learning Model (Module B) (Appendix 3) each was designed to contain fifteen lessons, in which each lesson was planned for approximately one-hour. The modules include direction on the incorporation of the graphing calculators into the learning activities, including the introduction of the different facilities as well as the necessary commands and syntax of the graphing calculator required for each task.

The teacher implementing the InterLM in the pilot study reported some difficulty in conducting the lessons in the manner envisaged by the study, citing that it was difficult not to intervene in didactical manner as students tended to wait for teacher's direction. This teacher also reported that she was anxious that students might lose interest in the activities. Therefore, to assist the teacher in the main study, the teaching modules were modified to include the kinds of interventions that teachers could use when implementing the learning activities. Instead of the general intervention questions suggested for use in the pilot study (see Table 4.04), where in the Structured Learning Model, the teacher is encouraged to intervene using directed, process type questions and in the InterLM, the teacher is encouraged to intervene using goal-type questions that prompt the student to reflect on what they are doing, thinking or planning, more specific intervention questions were suggested in each lesson (for examples, see Table 4.03, p. 81).

Table 4.04 General intervention questions suggested in the pilot study

Structured Learning Model	Interpretative Learning Model
Have you thought about this?	What are you trying to achieve?
How about rearranging the points?	What are you doing?
What happens when the value changes?	What have you done so far?
Would it help to change only the value of c?	What have you tried?
What other values have you tried?	Did you find where you went wrong?
Does this remind you of anything?	Is there any other method?
Why....? Or because ...? (to extend students work)	Is that all there is?
Is this the same as...?	What do you know about.....? (at start of a piece of work)

In light of the teacher's comments, every lesson was accompanied with corresponding students' worksheets in the main study. The students' worksheets also proved to be very useful in the pilot study, where both the teachers in the pilot study viewed them positively in terms of reducing preparation time and maintaining classroom control. It was observed that the worksheets provided a useful resource for students to refer to the learning tasks and that they saved time for the teacher to write down the tasks or for students to copy them. It also enabled the students to move back and forth between activities that interest them or were within their conceptual grasp. Indeed, Graham (1992) noted that when students were given complex calculators with only a manual, they were distinctly less eager about the technology. Students were critical of its

idiosyncratic logic and less able to exploit its special features. Therefore the worksheets for the main study were modified to include more details to provide some form of guidance and autonomy in learning as well as indirectly aiming to maintain students' enthusiasm for their access to technology. These worksheets were designed to correspond with the learning tasks found in the respective teaching modules. These worksheets would also provide good data at the end of the study for the analysis.

The pilot study showed that the simplified manual of the graphing calculator was a good starting point for students to advance to the original manual. Therefore copies of the original manual were also made available in the classroom so that students could refer to them if they wished.

The pilot study showed that students reacted more strongly when they realised that were singled out for videotaping. They, for example, covered their work or drew away from active participation, or tended to avoid making "mistakes" to maintain their self-esteem. Other students teased the subjects under observation and influenced them to act out causing some disruption. Therefore, in the main study, it is important to avoid students recognising they were specifically videotaped.

4.6 Main study implementation

The main study was carried out in a "SMART" School. The students were 14 year olds in their second year of their secondary schooling (Secondary 2). This age group of students was selected because they have not been taught formally about Cartesian graphs in school prior the study.

The first two sections below describe how the teacher selection and the teacher training process were carried out. Following this, how the twenty-four students were selected to participate in the study and a final six students in each learning model were selected was detailed. The process of videotaping was described and the data collection method, detailing how the test instruments were administered and how the lessons were conducted were outlined.

4.6.1 Teacher selection

Since the study was carried out for an intensive four-week period and conducted outside formal schooling hours, it restricted participation of upper secondary level teachers as students in Secondary 2 who participated in this study followed the afternoon schooling session where teachers are mostly college-trained teachers, which means that their mathematical qualification is equivalent to a high school diploma. The two teachers in this study had almost the same number of years of teaching experience (three years), so they were comparable in terms of classroom control, pedagogical skills and competency. The only drawback was both these teachers had no experience in the teaching of graphical concepts beyond what is prescribed in Secondary 2 in the curriculum. However, the teacher who implemented the InterLM is a graduate teacher, which means she is qualified to teach up to Secondary 6 level (equivalent to “A” level mathematics). Nevertheless, both the teachers could be considered as novice, in terms of subject matter and also the use of graphing calculators. Studies have shown that there are major differences between novice and expert teachers in several aspects related to subject matter knowledge and pedagogical content knowledge (e.g., Robinson et al., 1992). Expert teachers have a greater repertoire of learning activities and flexible routines (Leinhardt and Greeno, 1986), are more organised and connected (Robinson et al., 1992) and therefore, they may have better mastery of tasks such as the management of activities and establishment of behaviour routines, the teaching of content and awareness of students’ problems and situations as compared to novice teachers. However, the voluntary (teachers) and cooperation (school administrators) factors seem to outweigh the expertise factor, which could be dealt with teacher training.

4.6.2 Teacher training

On the first encounter, the researcher discussed with the two teachers the purpose of the study, the respective learning models and the learning activities associated with each of it. The researcher also made clear to the two teachers involved that the aim of the study was not to compare the effectiveness between the two learning models and tried to ensure that teachers did not misconceive that the study was indirectly attempting to evaluate their own competence. The researcher pointed out to the teachers that since the two learning models were implemented in two classes the idea of comparison was not

sensible. An introduction to the graphing calculator use followed. They were then given the choice of the learning model that they preferred to try out. This was thought a necessary aspect for successful implementation of the learning models as teachers have to agree with the goals and strategies implicit in the teaching approach (Schmidt and Kennedy, cited in Hatfield and Bitter, 1994; Schofield, 1995) and willingness to confront their own teaching practice (Carey et al., 1995). They were given both modules so that they could see the scope of the tasks, and compare and contrast both the learning models. The researcher subsequently met with each teacher individually three times of about two hours each time over a span of two weeks to discuss and share ideas on using the graphing calculators and implementing the study. All the learning activities in the teaching module were discussed and tried out together. The teachers learned to use the graphing calculator during these sessions.

The researcher then met both the teachers together and discussed both learning models. As the learning models depended to the manner the learning models are being implemented, during the training sessions, videotapes of the pilot study were used to highlight the kinds of interventions that were envisaged in each learning model.

The researcher also met the teachers everyday about one and a half hours before each session began to discuss and run through the learning tasks to be covered and the manner in which the instruction was to be carried out.

4.6.3 Selection of students

A teacher assigned by the principal selected the twenty-four students. These students were representative of low, medium and high mathematical attainment. (The term attainment is used here instead of ability to avoid the connotation of learning difficulty that is predetermined by heredity or environment.) Two groups were formed with each group using one type of learning model. Each group had students whose mathematical attainment ranged from high to low as based on their mathematics examination results. The following mathematics examination results were considered: the mid year and end of year results of the previous year of schooling (Secondary 1), the mid year result of the current year of schooling (Secondary 2) and the *Ujian Penilaian Sekolah Rendah* (UPSR) examination result. (The UPSR examination is a public examination taken by

all students at the end of their primary schooling.) Six students, two from each of the low, medium and high mathematical attainment were identified by the teacher in each group for detailed observation. Only six students were selected for detailed observation due to the constraint of only two video cameras and two camera operators afforded throughout the study.

4.6.4 Videotaping in the classroom

Two video cameras, manoeuvred by two camera operators, were used to capture as much as possible of all the selected subjects' verbalisations and teacher interventions during the learning activity. This enabled the researcher to capture and analyse important episodes including non-verbal data such as gestures, indicating degree of involvement and interest shown by the subjects, students' actions on the graphing calculator (e.g., what buttons they pressed), and on other materials such as graph paper and views of the calculator screen. A third video camera (camcorder) was fixed in so as to capture a view of the whole classroom. The two camera operators employed to handle all the video cameras freed the researcher to observe the classroom activity and to help the teachers when requested. The camera operators were also instructed to "zoom in" on students' actions with the graphing calculator, and displays on the graphing calculator screen, as well as capture any learning episodes that might unexpectedly arise and were thought to be significant for the study.

4.6.5 Data collection method

The pre-test was administered to all twenty four students before the start of the study and the post-test was administered to all students at the end of the study. A series of five sets of assessments tests - Assessment 1, 2, 3, 4, and 5 were also administered to all students at different stages of their learning activity as shown in the "flowchart" figure below.

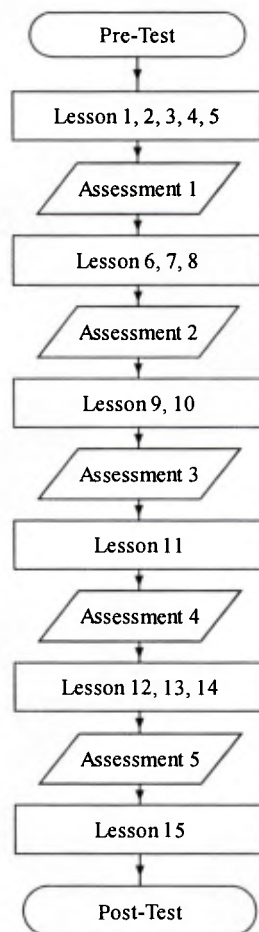


Figure 4.01 Flowchart of instrument implementation

Each teacher taught by using the respective teaching module prepared by the researcher. Each group of students used a corresponding worksheet detailing the learning tasks. Students were each given a “learning pack” containing a graphing calculator, a simplified graphing calculator manual, and the set of worksheets (according to the learning model they were designated). The calculator manual and the worksheets were all in Bahasa Malaysia, the national language and the medium of instruction in Malaysian schools. The learning pack was given out at the beginning of the study, and remained with the students throughout the study, and was collected at the end of the study.

Although group work was not central to this study, the sitting positions of all students were organised in such a way (see figure 4.02) that students of different mathematical attainment had the opportunity to interact with each other. This is to ensure that low

attaining students were not neglected in classroom interactions and have opportunity to participate in them. It was found that the low achieving student in Bennett and Cass's (cited in Good and Brophy, 1997) study was ignored or chose to withdraw from active participation when placed together with two high achievers. The arrangement shown in figure 4.02 was necessary to enable the video cameras (and the camera operators) to record the activities of the six students, and to give the illusion that the whole class was being videotaped (to avoid students recognising they were specifically videotaped). Students were designated to these sitting positions on the first day and were told to remain in these positions throughout the study.

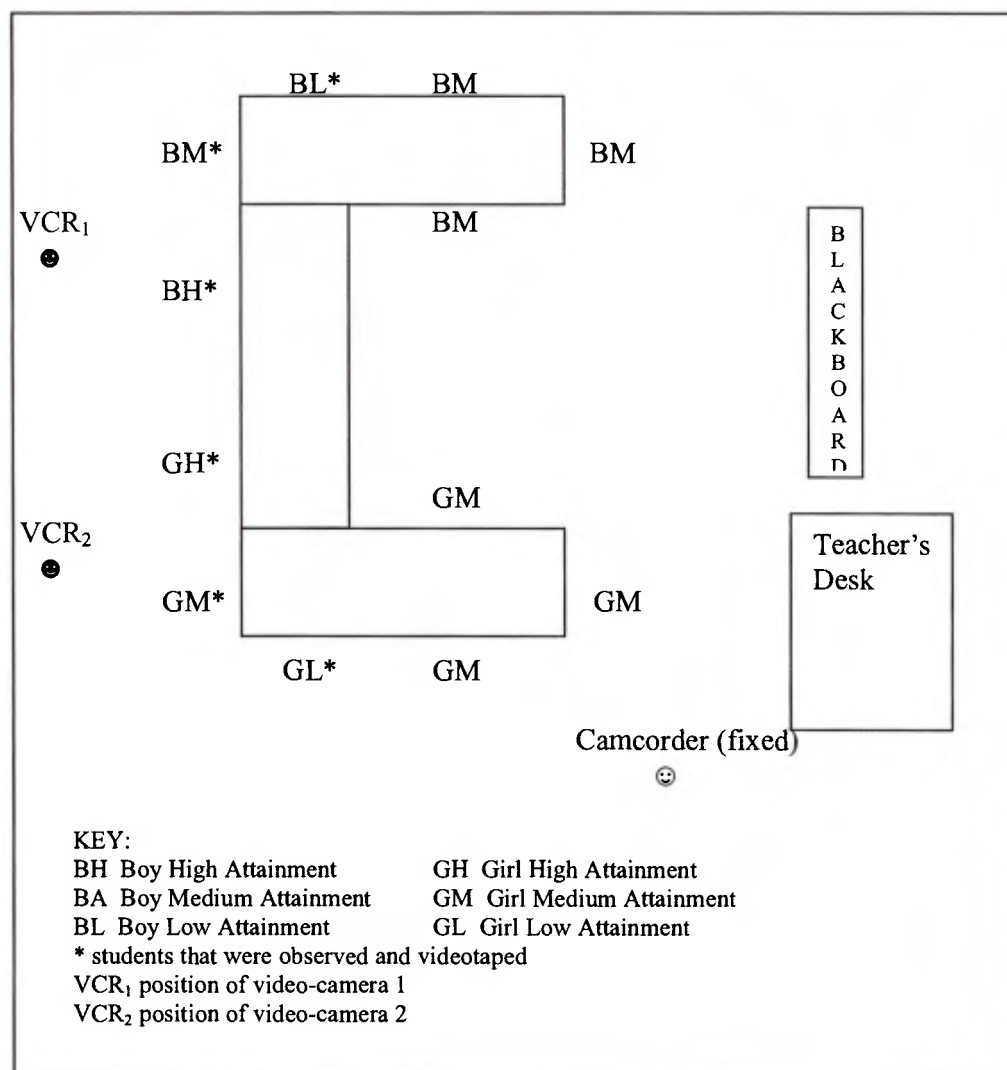


Figure 4.02 The layout of the classroom and students

At the beginning of the lesson, the teacher in the StrucLM gave whole class instruction to students. In the InterLM, whole class instruction was given only when it involved new information which was necessary to enable students to begin the activities found in their worksheets, for example the meaning of new terms used and the calculator's syntax. Students worked on the tasks individually or with their peers. The teacher went round to observe what students were doing and intervened when requested by students or when students were found to be "off-task". The teacher was reminded to intervene at random. After working about thirty minutes or so, the teacher in the StrucLM led a whole class discussion of students' interpretations and problem solutions of their tasks. The discussion was geared towards focussing on the mathematical concepts students were expected to discover or develop in their learning activities. In the InterLM, the teacher intervened only when requested by the students and the discussion was geared towards helping students to clarify the mathematical concepts they construed from their learning activities.

The classroom activities were observed and videotaped throughout the study. Students' worksheets were collected at the end of the study to provide evidence of their graphical concept development. They were however not told that their worksheets would be collected at the end of the study so that they did not copy their peers' work for the sake of completion of the questions in their worksheets.

In the classroom, the researcher maintained the position of "an active observer", present in the class at all times. Since the study was done outside formal school hours of the students and teachers, it was possible to arrange the classes to run consecutively. The researcher was introduced to the class on the first day and students were told of the role of the researcher as working together with the teacher in a developmental project involving the use of the graphing calculator. In this way, the researcher could interact naturally with students during the learning activities to find out what they had understood and it also maintained the self-esteem and respect of the teachers who were novices in the subject and calculator use.

The students were also assured that this was not an assessment on their performance, that the whole exercise was to explore the ways in which calculators could assist students'

understanding of graphs. Students were encouraged to voice the problems that they encountered when using the tool.

The researcher also acted as a support for the teacher, assisting with any issues that arose related to graphing calculator use, for example, the functions of the buttons and the features of the calculator. When requested by the teacher, the researcher provided suggestions in dealing with problems that arose in the learning activities, in particular clarifying ideas on the implementation of the learning activities.

4.7 The analytic framework

The analytic framework follows an iterative design, developed through repeated viewing and analysis of the videotape transcriptions and the analysis of the case studies themselves. This section describes how the case studies emerged as the basis to characterise students' development of graphical concepts with graphing calculators. The first section describes the preliminary data analysis, and follows with the selection of the case study students. The next section describes how the iterations of the analysis reveals the aspects that characterise the transition in students' understanding of the graphical concepts with the graphing calculators and how these were subsequently used to describe and interpret students' development of graphical concepts with graphing calculators. This section concludes with a description of the use of *epistemological case studies* to characterise the processes in which students' appropriated the graphical concepts with the graphing calculators.

4.7.1 Preliminary data analysis

For each student (six from each learning model selected for the videotaping) and each teacher, a profile was constructed by transcribing the videotape, beginning from Lesson 1 to Lesson 15. The student profile included descriptions of what he/she did with the graphing calculator during his/her learning activity, how the tasks were carried out, the difficulties encountered and how they were resolved. The teacher profile outlined what each teacher did throughout each lesson, how and when he/she intervened the students in the lessons and what he/she did when students encountered difficulties in their learning

activities. The student and teacher profiles were used to map out what ensued in each student's learning activity throughout the 15 lessons.

4.7.2 Selection of case study students

An iterative process of repeatedly viewing and analysing the videotape transcripts was employed to select three students from each learning model as case study students in the analysis. The selection was based on

- (i) a good video data where the video captured important aspects of what students did during their learning activity (e.g., the sequence of button push on the calculator) including a substantial length of student-teacher interaction.
- (ii) a good fit of the researcher's three attainment categories of high, medium and low attainment. The mathematical attainment criteria included the students' mathematics examination results¹⁹, their performance in the pre-test, and observations of how they conducted themselves in their learning activities (i.e. the differences and difficulties they exhibited in carrying out the learning tasks and their mathematical dispositions during learning).

The high attaining students in this study had consistent high scores across the three mathematics examination results, demonstrated the highest computational and algebraic knowledge facility in the pre-test compared to the medium and low attaining students, and exhibited many characteristics of higher order thinking during their learning. Medium attaining students had average scores across the three examination results, demonstrated a fair computational and algebraic knowledge in the pre-test, and exhibited some characteristics of higher order thinking during their learning. Low attaining students had the lowest scores across the three examination results compared to high and medium attaining students, performed relatively poorly in the computational and

¹⁹ The following mathematics examination results were considered: the mid year and end of year results of the previous year of schooling (Secondary 1), the mid year result of the current year of schooling (Secondary 2) and the *Ujian Penilaian Sekolah Rendah* (UPSR) examination result (The UPSR examination is a public examination taken by all students at the end of their primary schooling.) (see p. 95).

algebraic items in the pre-test, and exhibited a lot of unsystematic “investigations” when carrying out their learning tasks and were sometimes off-tasks.

4.7.3 The case studies

Iterations of the analysis reveal four major features that inform students’ development of graphical concepts with the graphing calculator, namely, how students use the graphing calculator, students’ behaviour during the lessons, teachers’ behaviour in the interventions and the teacher-student interaction pattern in general. These categories are described in detail.

(i) how students use the graphing calculator

Two categories of behaviour were used to guide the analysis: graphing calculator as a mediational tool or as a static, display tool. Using the calculator as a mediational tool involved the student using it to test out conjectures, to make guesses, to extend, and to articulate what he/she is thinking in relation to the tasks. Using the calculator as a static, display tool involved the student using it simply to “do mathematics” (Adams, 1997), for example, to reproduce an object (e.g., graph, table) on the display screen. However, how the students use the graphing calculator depended on the students’ knowledge of the *functionality of the calculator* and their existing mathematical knowledge. Knowledge of the functionality of the calculator indicates whether the student is proficient in using the calculator, and includes a knowledge of which calculator button to press and what sequences of button presses are needed to retrieve the required menu to perform a desired operation. It also involves an expectation of what will be displayed on the screen and a knowledge of the effect on the graphic display when certain buttons or commands are executed. Also included in this category is an awareness of some aspects of the potentiality and limitations of the graphing calculator.

(ii) students’ behaviour during learning

This includes their mathematical dispositions, which involved identifying whether students exhibited behaviours of unsystematic exploration in which they failed to make systematic and sustained progress in developing mathematical strategies or understanding (Henningsen and Stein, 1997). Other aspects included noting how students behave in the student-teacher interaction, and the extent of their active participation such

as contributing ideas, challenging the teacher, and more question-asking rather than merely accepting (Nickson, 1992).

(iii) teachers' behaviour in the interventions

Several types of teacher behaviour were identified, namely

(a) teacher's responses to students' difficulties in terms of the mathematical language that teachers use, the quality of mathematical knowledge that the teacher imparts (through explanation) and the use of curriculum scripts. Curriculum script refers to a set of subtopics and example problems that are consistent with the standard materials that have to be covered. It could refer to an ideal template such as a standard solution procedure for a problem that tutors wish the students to learn (Graesser et al., 1995; Chi 1996). The extent to which teachers assess the student's missing knowledge pieces, address student's misconceptions, provide explicit explanations or specific directives to student's queries, elicit self-explanation from the student, or give long didactic explanations (Chi, 1996) seems to impact students' understanding.

(b) teacher's response to students' difficulty in relation to the functionality of the graphing calculator. Merrill et al.'s (1995) notion of syntactic errors and semantic errors in a computer learning environment was also observed in the graphing calculator environment. Syntactic errors arise from not following the calculator's syntax, or its rules for entering expressions. They also arise from using the wrong calculator terms or language. Semantic errors consist of errors made on calculator's commands that mostly are contained in menus. Included are errors that arise from unawareness of the potentiality and the limitations of graphing calculator.

(c) teacher's incomplete intervention. This refers to teacher intervention that did not continue to the end that answered students' or teachers' questions in the interaction appropriately, or steered students into the direction of solving the difficulties encountered in the task or calculator use.

(iv) the teacher-student interaction pattern in general

This involves identifying whether the interaction structure involves a series of continuous stream of exchanges between the teacher and the student which Chi (1996) refers as tutorial.

In view of the analysis iterations and for manageability, this study will use *epistemological case studies* to characterise the development of graphical concepts with the graphing calculators. In other words, the case studies are presented with reference to the graphical concepts of continuity, gradient, linear graphs, and non-linear graphs, chosen for the analysis as they encompass what students were learning in their learning activities. These concepts are interesting to investigate as not only are they basic concepts for students' future learning about functions but they have also been noted to be problematic in students' learning of graphs (see section 2.2).

Each epistemological case study details a cross-sectional analysis of how selected students of different mathematical attainment (low, medium and high), developed the graphical concepts with the graphing calculator. Presenting the data using epistemological case studies helps to model students' learning in general with the graphing calculator and to examine how teacher interventions impinged on the students' understanding. The cross sectional analysis enables examination of the extent to which the graphing calculator facilitates or inhibits students' development of the graphical concept under analysis. It also enables a global reflection on the types of intervention that each student experiences during his/her learning, and a characterisation of his/her overall learning pattern with the graphing calculator. How students of different mathematical attainment used the graphing calculator in their learning can also be characterised. The pattern of teacher intervention can be also be characterised through the examination of each student's case study.

Table 4.05 below charts the number and the types of case studies that will be presented in the analysis.

Table 4.05 A summary of the types of case studies

Epistemological case study	Students' progression in	
	Structured Learning Model	Interpretative Learning Model
Case study 1: The concept of continuity	High attaining student	High attaining student
	Medium attaining student	Medium attaining student
	Low attaining student	Low attaining student
Case study 2: The concept of gradient	High attaining student	High attaining student
	Medium attaining student	Medium attaining student
	Low attaining student	Low attaining student
Case study 3: The concept of linear graphs	High attaining student	High attaining student
	Medium attaining student	Medium attaining student
	Low attaining student	Low attaining student
Case study 4: The concept of non-linear graphs	High attaining student	High attaining student
	Medium attaining student	Medium attaining student
	Low attaining student	Low attaining student

Each epistemological case study will be presented using the same structure. The first part describes the graphical concept and the background of the concept. This includes the issues and difficulties surrounding its learning of the concept. There is a brief account of how students of different mathematical attainment performed in the related test item(s) in the post-test. Where relevant, students' performance in the related test item(s) in the pre-test is also presented to illustrate students' progress.

The second part of the case study details the expected learning outcomes using the graphing calculator. The rationale for the design of the mathematical tasks is first briefly described. This follows with an outline of the mathematical tasks in both the learning models with an indication of how the graphing calculator is thought to assist in understanding the concept.

The third part details how each case study student appropriated the graphical concept with the graphing calculator. The following aspects are considered in detailing how each case study students uses the graphing calculator in their learning activities.

- (i) How he/she performed in the related item in the post-test or assessment tasks.
This is to gauge what kinds of understanding that he/she had acquired by the end

of the study. The reasoning he/she provided in his/her answer is used as an indication of the type of understanding (full, partial or no) of the graphical concept.

- (ii) How he/she derived the understanding he/she exhibited in his/her post-test or assessment tasks are then substantiated. The student's written work is examined to find evidence of what he/she did in his/her learning activities. Some episodes of video transcription of the student's interaction with the graphing calculator during the related learning activities are analysed. Where relevant, included are the teacher's intervention and the student-teacher interaction. In the analysis of the videotape transcriptions, specific issues are examined to highlight how they contribute (or do not contribute) to the case study students' development of graphical concepts.
- (iii) Student's existing mathematical knowledge is examined to see what mathematical knowledge underlies the understanding of the functionality of the calculator, and how it influences the student's approach to carrying out the graphing tasks and his/her interaction with the teacher.

In summary, the main focus of the case studies is an analysis of the kinds of mathematical understanding that 14 year-old students developed while using the graphing calculators. In particular, it will examine the transition that students make from instrumental to mediational use of the calculators throughout the four graphical concepts, beginning from continuity, to gradient, linear graphs and non-linear graphs.

CHAPTER 5

CASE STUDY 1: THE CONCEPT OF CONTINUITY

The background of this topic is given section 5.1, which also presents the performance of the three selected students in each learning model on the related test item in the pre-test and post-test. Section 5.2 describes the expected learning outcomes using the graphing calculator. It begins with a brief outline of the mathematical tasks on continuity in each learning model. It follows with an explanation on how the graphing calculator is thought to assist in understanding the concept of continuity. Section 5.3 describes how the case study students in the StructLM appropriated the concept of continuity with the graphing calculator, with a conclusion on their development in the concept in section 5.4. Section 5.5 details similarly for case study students in the InterLM, with a conclusion on their development of continuity in section 5.6

5.1 Background

The understanding of the concept of continuity requires an ability to recognise that there are infinitely many points that lie on a line, and indeed lie between any two points on the line. It necessarily involves the concept of infinity, which conjures notions of “limitless” or “countless”; an intangible, abstract quantity. One possible explanation of students’ difficulties in understanding the concept of continuity may lie in the primary experience of discrete quantities. Some students may have developed a fixation (Hiebert and Carpenter, 1992) that graphs are made up of many identifiable discrete points that are “exact” and can be read off. This fixation prevents them from also recognising the equation of the graph as an algebraic statement that defines all the points on the graph (in other words the graph consists of the sets of points whose coordinates (x, y) satisfy the equation of the graph).

Understanding the concept of continuity may perhaps be more difficult if students do not have a clear number sense. Number sense is defined in terms of understanding number magnitude, number proximity, order of numbers, properties of operation on numbers, and multiples of 10 (Dworkin, cited in Sowder, 1992). This means recognising the order

of numbers (in this case real numbers) on the number line, and having the notion of place value, for example that 3.446 is greater than 3.44.

The following issues were examined in the pre and post-test (Question 8): whether students when plotting points and joined these points to form a straight line, they were aware that

- (i) infinitely many points may also lie on the line, or
- (ii) between any two points on two points on the line.

About 70% of 14 year olds in the CSMS study (Kerslake, 1981) gave some finite number for in case (i), and only 6% gave the correct answer. In case (ii), 18% of the 14 year olds gave the answer “none”, 41% gave the answer “1”, 22% gave some finite answer and only 4% said “infinite”. Tables 5.01 and 5.02 below depict the responses of the students in the StrucLM and the InterLM respectively. Responses in italics indicate answers in the post-test and non-italic indicates answers in the pre-test.

Table 5.01 Students' performance on continuity in the StuctLM

Name of student	Concepts/ tasks			
	Plotting the points (1½, 4)	Plotting the points (4,6, 10,2)	Number of points on the line	Number of points between two points
Wan (High Attaining Boy)	Point plotted is on the line	Point plotted is on the line	Did not answer	Did not answer
	<i>Point plotted is on the line</i>	<i>Point plotted is on the line</i>	<i>Infinite</i>	<i>Infinite</i>
Ng (Medium Attaining Boy)	Point plotted is on the line	Interpreted the point as two separate points (4,6) and (10,2)	As some finite number (5)	None
	<i>Point plotted is on the line</i>	<i>Point plotted is on the line</i>	<i>As some finite number (7)</i>	<i>Gave the point (2½, 6)</i>
Amin (Low Attaining Boy)	Point plotted is on the line	Interpreted the point as two separate points (4,6) and (10,2)	Did not answer	Gave the point (2.5, 6)
	<i>Point plotted is on the line</i>	<i>Point plotted is on the line</i>	<i>As some finite number (8)</i>	<i>Gave the point (2.5, 6)</i>

Table 5.02 Students' performance on continuity in the InterLM

Name of student	Concept/ tasks			
	Plotting the points (1½, 4)	Plotting the points (4.6, 10.2)	Number of points on the line	Number of points between two points
Grace (High Attaining Girl)	Did not answer	Did not answer	Did not answer	Did not answer
	<i>Point plotted is on the line</i>	<i>Point plotted is on the line</i>	<i>Many</i>	<i>Infinite. Also gave the answer (2.5, 6)</i>
Pre (Medium Attaining Girl)	Did not answer	Did not answer	Did not answer	Did not answer
	<i>Point plotted is on the line</i>	<i>Point plotted is on the line</i>	<i>As some finite number (6)</i>	<i>As some finite number (1)</i>
Emy (Low Attaining Girl)	Did not answer	Did not answer	Did not answer	Did not answer
	<i>Point plotted is on the line</i>	<i>Point plotted is on the line</i>	<i>Many</i>	<i>Gave the point (3, 6)</i>

The tables show that in general, students in this study started with extremely limited knowledge of how to name the points on the line. Therefore, a big leap can be expected to learning the concept of continuity, which is notoriously difficult (Kerslake, 1981; Leinhardt et al., 1990; Demana et al., 1993). In StrucLM, only Wan, the high attaining boy seems to have understood the idea of continuity whereas in InterLM, only Grace, the high attaining girl seems to have understood the idea of continuity. So what makes Wan and Grace successful? Why were the other students not successful? The reasons for this disparity will be considered later.

5.2 Expected learning outcomes with the graphing calculator on continuity

The tasks were designed to assess the extent to which facilities of the graphing calculator could assist students in developing the concept of continuity: the GRAPH Mode by using the tracer or the TABLE Mode by using the G-CON and GPLT command.

In the StrucLM, the mathematical tasks on the concept of continuity involved students drawing the graph of a set of given values on graph paper, and deriving the mathematical equation for the graph. The students were told to draw the graph ($y=x$) in the graphing calculator and to trace as many points as they could on the graph. In the subsequent task,

students were asked whether the following points could be found on the line: (2.2, 2.5), (8, 8), (20, 20), (-2, 1).

In the InterLM, students were given a straight-line graph with a few points marked on the graph. They were asked to replicate the same graph in the graphing calculator and subsequently to derive the mathematical equation for the graph. Using the mathematical equation ($y=x$), students were told to draw the graph in the GRAPH Mode and subsequently explore in the TABLE Mode.

A key functionality of the calculator in this respect is the “trace” facility. The tracing feature of the calculator enables the user to trail points along the line he or she has drawn with the display pointer (or tracer), and thus enables the user to see some of the ordered pairs that make up the line. As the tracer moves along the line, the value of the coordinate is displayed on the screen each time the cursor button of the calculator is pressed. By changing the view-window settings, many other points can be traced for the same graph or line. In the STD window, the calculator gives coordinates at up to 10 decimal places, for example -6.8253968254, 6.8253968254 for the graph of $y=x$. Besides the built in INIT window and the STD windows, students could specify other settings for the view-window. This was expected to confront them with the notion that naming each point on the line is inexhaustible as each time a new view-window is created or defined, different points could be traced.

Using the TABLE Mode of the graphing calculator enables the user to generate a table of values for the graph of the equation. The instant display should enable the student to see the connection between the equation of the graph, the graph itself and the coordinates that lie on the graph. Activating the “dual screen” feature in the SET UP enables the student to display the table and the graph side by side which could explicitly confront the student with the relationship between the equation and the coordinates on the graph. The G-CON and G-PLT commands available in the TABLE Mode allow the same graph to be drawn in two different forms, the former as a continuous smooth line and the latter as dotted line with each dot corresponding to the x and y values given in the table. These values depend on the range and pitch determined by the user. The pitch value defines the difference between the consecutive x values found in the RANG setting. The immediacy of displaying both graphs one after another in different forms with the push of the button

is expected to confront students with the fact that that the dotted graph and the smooth line graph are the same graph. The x and y values in the table are defined by the range of x values, and the pitch values. By changing the range and/or the pitch, students were expected to see that not only do the values of x and y change, but the number of given points on the line also changes. Hence, students were expected to see that it is impossible to name all the points on the line. Decreasing the pitch value will increase the number of dots plotted, the dots will become closer to each other and eventually seem to appear as a continuous line.

5.3 Analysis of case study students in the Structured Learning Model


WAN (HIGH ATTAINING BOY)


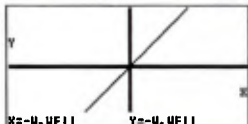
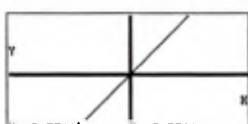


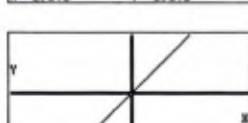
Wan was identified as a high attaining student both by his teacher and from the tests. Comparison of his pre and post-test results indicate that there was clearly a change in Wan's understanding of the continuity concept. His post-test showed that he recognised that there were infinitely many points on a line and between any two points on a line. This case study will focus on aspects of his development in relation to his use of the calculator. In particular, it will show the interplay between his understanding of the calculator's functionality and his own mathematical knowledge that seemed to support his emergent grasp of the continuity concept.

The trace facility

The following excerpt shows how Wan's understanding of the trace facility together with his number sense enabled him to structure his thinking about continuity.

Excerpt 5.01 Wan's understanding of the trace facility and number sense

Line No.	Action	Calculator Display/ Worksheet
1	This is the graph of $y=x$ he draws in his calculator.	
5	He begins to trace the graph. He looks untroubled by the notation E11 that accompanies the reading of the x and y coordinate.	

10		
12	He looks unperturbed that the tracer is not on the screen. He continues to press the cursor key ► confidently.	
15		
20		
25		
30		
35	Wan stops when $x=0$, $y=0$ and subsequently shows this to Amin and convinces his friend that this is the same graph of $y=x$.	

Wan's continual pressing of the cursor to obtain the tracer although the tracer was not initially shown on the screen, indicates that he knew that the tracer moves from left to right on the screen for the graph of $y=x$ (line 12). He was aware of the changing values of x and y displayed at the bottom of the screen and understood when the tracer would start to appear on the line. This also indicates that Wan understood how the tracer worked in relation to the view-window and the line.

Wan had chosen his own view-window setting using very large numbers. Wan's screen display indicates that he had defined the following values for his view-window:

```

View Window
Xmin : -6.3E+11
max : 6.3E+11
scale:1
Ymin : -3.1E+11
max : 3.1E+11
scale:1
UNIT|TRIG|STD|STO|RC|

```

Figure 5.01 A view-window with large values

He knew what the notation “E” stood for and how to interpret the exponential format of the calculator (for example $6.3E+11$ is equivalent to 6.3×10^{11}). This is not only an



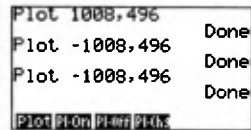
indication of his confidence in working with large numbers but also a confidence in how the calculator operates in general. Operating the EXP key for large numbers means he had changed the setting of the “Display” command in the SET UP.

Wan’s way of demonstrating to his peer Amin that “ $x=0$, $y=0$ ” (line 35) indicates that he knew this displayed graph was drawn on a view-window that is 10^{11} times bigger than the INIT window. He understood how the minimum and maximum horizontal values and vertical values defined the view-window, and how this influenced the appearance of the graph. He therefore had an expectation that the line $y=x$ drawn would appear similar to the line drawn in the INIT window, but the coordinates displayed when tracing would have the exponential notation.

The role of the view-window concept

Wan’s understanding of the functioning of the tracer could be related to his understanding of the view-window concept. The following excerpt shows the development and the extent of Wan’s understanding of the view-window concept. In particular, it shows Wan’s understanding of how the view-window affects the movement of the pointer (or tracer) on the screen. It also shows how Wan’s knowledge of the number concepts structured his activity, which progressed from simple cases of small numbers to cases of large numbers.

Excerpt 5.02 Wan’s understanding of the view-window in relation to the movement of the pointer

Line No.	Action	Calculator Display/ Worksheet
1	Wan changes the setting of the view-window.	
5	This is the screen of the calculator when he finished and he continues to key in the point he wanted to plot.	
10		
12	He is seen keying in the point (-1008,496). His screen shows he has plotted the other two points previously. He selects the same point he used previously.	
15		
17	He obtains this screen when he presses the EXE button. He does not	

20	look surprised at this display.	
22	He explores by moving the pointer. He tells Amin, "When I push once, I get 160000" and shows Amin the display.	
25		
27	He moves the pointer back and forth, up and down.	
30		
35		
40		
45		
50		
55	He then changes the setting of the view window.	

This excerpt sees Wan exploring using very extreme value for the x and y ranges and the scale. It is clear here that Wan was trying to establish how the view-window values affected the points plotted on the screen. From the display of the view-window whose values he was changing (line 1), it is clear that Wan was starting from these view-window values:

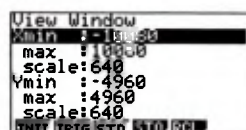



Figure 5.02 A previous view-window

Therefore, the current view-window he was working in is 1000 times larger. Wan keyed in the same point $(-1008, 496)$ (line 12). Selecting the same point indicates that Wan was not making a wild guess. He had an expectation of the outcome display.

When the calculator plotted the point $(0, 0)$, Wan was not surprised (line 17). It seems Wan expected that the calculator to plot the point nearest to the value he specified depending on the view-window setting. In Wan's worksheet, he described this kind of occurrence as a result of choosing a view-window that is "not suitable" ("tidak sesuai") for the points.

The excerpt also shows that Wan understood how the view-window values influenced the movement of the pointer (tracer) from one point to the next point on the screen. His remark to his friend indicates that he knew that by using the current view-window setting each press on the cursor key would move the pointer 160000 units (line 22). This means Wan was aware how the INIT window worked. In his earlier exploration with the INIT window, Wan might have discovered that the pointer moved a maximum of "63 presses" to cover the full length of x-axis from the origin and a maximum of "31 presses" to cover the full length of the y-axis from the origin. Examination of his worksheet indicates that Wan had explored with the following view-window setting and had deduced that each press on the cursor resulted in the pointer moving 0.5 units.



```

View Window
Xmin : -31.5
Xmax : 31.5
Yscale: 1
Ymin : -15.5
Ymax : 15.5
INIT (RIGHT) STO/DEL

```

Therefore, it is reasonable to assume that Wan understood that the total number of key presses remains the same no matter what the view-window settings are. What differs is the distance covered by each cursor key press, which is governed by the x and y values of the view-window. The subsequent actions show Wan moving the pointer along the x and y-axis in different directions (line 27 onwards) and then in another view-window whose x and y values were multiples of the INIT window except for the scale values.

Wan's sound number sense and understanding of how the same graph appeared in different view-windows enabled him to make sense of the task. The number sense enabled him to recognise the ordering or the position of the points traced on the line in relation to the other points traced in other view-windows. The graphing calculator helped Wan to articulate his thinking about the different points that could be traced on

the line. In his worksheet, Wan wrote that it was not possible to trace all the points on the line with the graphing calculator, reasoning that the movement of the tracer depended on the scale used. Two things seem to emerge here. First it indicates that Wan saw the tracer and the points on the line as the same object, that is, the position of the tracer is the point. Second, Wan used the term “scale” in a relative sense (the ratio of distance to value). However, when he was operating his calculator as shown in the excerpt above, the value of the scale that Wan chose indicates that he was using the term “scale” according to the calculator’s syntax, that is what the tick marks on each axis represent. Wan was able to discern between the two meaning of scale used in the calculator syntax and mathematical language.

Worthy of note is the fact that Wan wrote in his worksheet that the point (2,1) does not lie on the line $y=x$ because “it does not satisfy the equation”. It seems the teacher’s classroom explanation induced Wan to use these terms “satisfy the equation”. His understanding of how the x and y-coordinate changed along the line enabled Wan to follow the teacher’s classroom explanation about “any point (x,y) on the line”, which seems to move him to a higher level of knowledge.

In conclusion, it is possible to identify how Wan’s activity highlights the fact that understanding the movement of the tracer in different view-windows underlies the understanding of how the tracer moves on a line. This is closely linked to an understanding of how a point plotted in different view-windows will appear differently on the screen. What seems to emerge from Wan’s activity is the mediating role of the graphing calculator that enabled him to see the idea of continuity embedded in his learning activity. In other words, the calculator could be said to mediate the meaning of continuity for Wan by enabling him to structure and to modify the learning tasks according to his level of competency in the number notion. What was also noticeable was the lack of teacher intervention in his mathematical activity, which could be construed as further evidence of the mediational role of the calculator.

NG (MEDIUM ATTAINING BOY)

Ng was identified as a high attaining student by his teacher. However, the manner in which he carried out his tasks in his activity did not reflect the attributes of a high

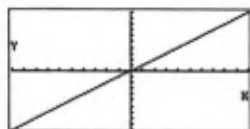
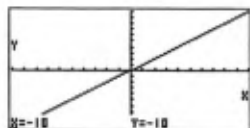
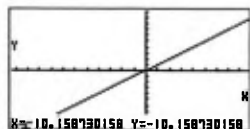


attaining student. His pre-test also indicates that he could not perform some algebraic manipulations, although he showed high competence in numerical computations and had a sound number sense. As his post-test shows, Ng did not understand that there are infinitely many points on the line. In fact he counted only the points that were requested to be plotted on the line. It seems that he did not really understand the question. He correctly plotted and named the point (2.5, 6) as another point that lies between (2, 5) and (3, 7) but his answer indicates that he saw this point as the mid-point.

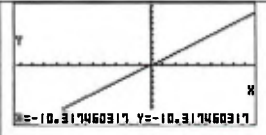
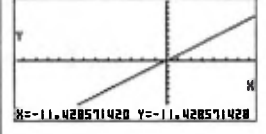
Ng's failure to understand the concept of continuity can be attributed to his lack of understanding of several aspects of the functionality of the calculator, and the teacher's interventions in his learning. The following extracts will try to illuminate the various aspects of Ng's performance which contributed to his lack of understanding. They centre around the interplay between his grasp of the calculator's functionality and the teacher's interventions.

Trace and the view-window

The excerpt below shows that Ng did not see the idea that some points on the line could not be traced, suggesting a failure to grasp the connection between the view-window and the trace function.

Excerpt 5.03 Ng's understanding of the view-window and tracer

Line No.	Action	Calculator Display/ Work Sheet
1	Ng is trying to trace the point (8,8) on the graph. He chooses to graph the line $y=x$ in the STD window. He presses SHIFT Trace to activate the tracer. He wrongly presses the cursor key ◀ resulting in the y-axis moving to the right. He continues to press this button a few times until the y-axis moved further to the right.	
5		
10		
11		
15		

20		
25	He gives up and stops to listen to teacher's explanation to whole class.	

In this excerpt, Ng's choice of the STD window indicates that he knew that (8,8) was not within the viewing region of the INIT window (line 1). However, the STD window does not allow the point (8,8) to be traced as this point could not be plotted on this view-window. It seems that Ng was not aware that the STD window does not read points in integers. This is surprising as when Ng was plotting the point (3,2) on the STD window, he wrote in his worksheet "the point (3,2) could not be plotted exactly because the STD window gives measurements in decimals" (in Ng's words), which is true.

What is apparent is that Ng knew points could not be plotted beyond the specified x and y values in the view-window, but he did not fully understand why some points could not be plotted exactly on the graphic screen of certain view-windows. In his general remark about how view-windows affected the points plotted, this was what he wrote in his worksheet.

Tetapi penglihatan yang memberikan ukuran tepat mempunyai susunan grid yang sama jarak manakala tetapi penglihatan yang memberikan ukuran tidak tepat mempunyai susunan grid yang berbeza jaraknya.

"View-windows whose grids are equidistant from each other gives exact measurement whereas those whose grids have different distance do not give exact measurement"

It seems Ng thought that only view-windows whose grids were equidistant from each other gave the exact point while those which were not gave otherwise. While this is true in most cases of "square grids", it is not the general case, for example this view-window setting has "square grids" but the point (3,2) cannot not be plotted exactly on the screen:



Ng's answer is also an indication that his learning could be through memorisation. Although Ng saw how the pointer moved a particular distance in different view-windows, he did not really understand how the movement of the pointer was related to the x and y ranges of the view-window. This explains Ng's contradictory answer where at one point he seemed to understand why the point (3,2) could not be plotted on the STD window, but at another point did not understand why the point (8,8) could not be traced in the STD window. Although he saw that the STD window generates coordinates that are not integers, he did not understand why. Therefore, it is reasonable to infer that he could not understand how the tracer worked in relation to view-window setting.


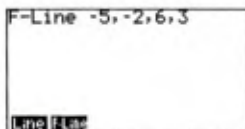
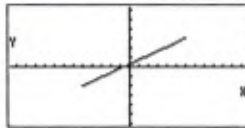
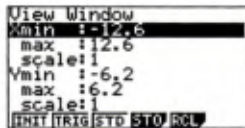
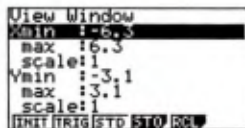
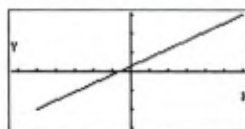
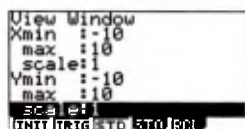
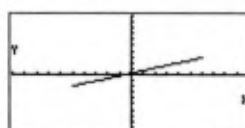
The inappropriate choice for the view-window obscured the task of tracing the point (8,8) and the other points that are integers. The subsequent actions show Ng pressing the cursor to the left a few times, resulting in the y-axis to move to the right (line 11 onwards). These series of actions indicate that Ng has problems with the functioning of the tracer. They also indicate that he was not aware that the x and y values displayed on the screen were getting smaller. The fact that these coordinates display up to nine decimal places each time the tracer moved gives a hint that this view-window is unsuitable for tracing points that are integers - something unnoticed by Ng. This suggests that Ng was not using the graphing calculator with consciousness. The role of the graphing calculator was essentially procedural - as a tool to carry out the procedures of the tasks.

Role of the teacher

Ng's partial understanding of the view-window concept can be connected with how he learned about the view-windows. His tasks execution were all teacher directed or by the contents of the worksheet. There was no evidence of Ng trying to construct view-windows with his own x and y ranges, or to choose his own points in the view-windows. The following excerpts document different aspects of teacher intervention, which significantly influenced how Ng developed his understanding of the view-window concept and how he used the calculator.

Excerpt 5.04 shows a teacher intervention that *was* productive in that it helped Ng to recognise that the view-window settings influenced the appearance of the line.

Excerpt 5.04 Ng's understanding of the view-window in relation to the appearance of the graph

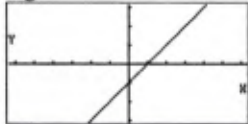
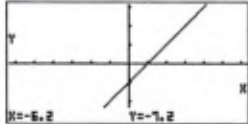
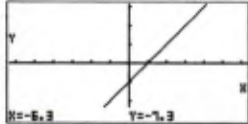

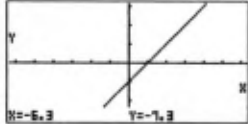

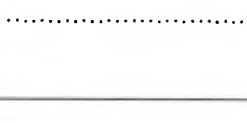


Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Ng, you have not answered... can't you? Okay... to do this, why don't we go....look .. okay we take.... Try to do the F-line. We did the F-line the other day. Click at the F-line	(point to Ng's w/s)	
5		Teacher points to the menu on Ng's cal and Ng presses the F-line command.	
10	T: For F-line, just copy this , -5,-2,6,3 and the calculator will draw it straight away.	Teacher then points to Ng's worksheet at -5,-2,6,3 as Ng operates on the calculator.	
15	T: What is this view-window... INIT and STD?	When the screen shows the display, T asks the view-window that he uses.	
20	Ng: This is view window 5.	Ng shows the view-window 5 setting he used.	
25	T: View-window 5. Change it to the init and standard view window.	(Teacher repeats after Ng) Ng changes to INIT and as he operates on the calculator, Teacher points to the INIT setting (with gesture to stress the length of the line).	
30	T: Now look at the INIT. Now look at the standard....	As Ng is changing the view window and getting the outputs, Teacher asks.	
31	T: What is the difference between INIT and STD? If you observe just now, in INIT the line is long.....	Teacher then moves on.	
32	what is the difference... why is it long and why is it short?		
35	Ng: The scale has changed.		
40	T: Okay, write it here...		

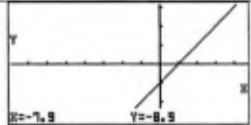
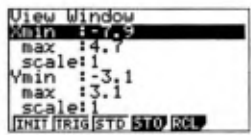

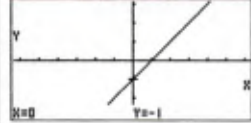
This excerpt seems to indicate that Ng was aware of how the view-window affected the appearance of the graph, specifically the length of the line. The teacher focuses Ng's attention on the relationship between the view-window and the appearance of the line, explicitly pointing out to Ng to look at the *length* of the line (31-32) and asking for Ng's explanation of the differences in the length. It seems plausible that this explicit hint to look at the length (rather than the slope) helped as Ng reasoned with the teacher that the same line in the INIT window appeared shorter in the STD window because the scale

had changed. Ng's explanation of scale here could be said to be in an absolute sense (the distance between two consecutive marks) on the axis. It is not clear whether Ng did understand the meaning of scale used by the calculator. Noticeable too is the teacher's extensive use of the syntax of the calculator with direct instruction on how to execute the tasks.

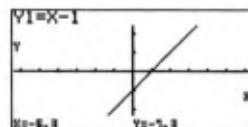
The next excerpt shows how the teacher intervention was not productive in helping him to understand how the tracer functions.

Excerpt 5.05 Ng's difficulty with the tracer

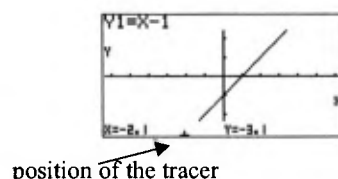
Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T (to whole class): Okay, now you find the coordinate, what is it? Try and trace.... Shift, Trace, okay. Then press the command left/right/up/down of the arrow... you will get the cursor (to mean the tracer) with a positive sign on the line and it will move when you press the arrow. Find what is the intercept that occurs on the y-axis. Try and trace what is the coordinate on the continuous (line) that intercepts the y-axis. (Note: Teacher uses the terminology "arrow" for the cursor)	Students are asked to graph for $y=x-1$ manually and then to graph it using the calculator and trace the values along the line and find the intercept. Teacher gives instructions on how to trace to whole class. As Teacher is instructing, Ng tries but could not find the pointer/tracer, finds his y-axis has moved to the right. First he presses \blacktriangleright , but does not see the tracer.	In RUN Mode Ng's cal: 
5		Then he presses \blacktriangleleft , but still does not see the tracer.	
6		He presses \blacktriangleright again, and still does not see the tracer.	
9		He reverts to pressing \blacktriangleleft again and continues.	
10		When $x=-6.4$, $y=-7.4$, the y-axis moves to the right. Ng continues pressing the cursor back and forth but cannot see the tracer. (-7.5, -7.7,...-7.1,-6.9, -5.5,...-7.1) and finally gives up at $x=-7.9$, $y=-8.9$.	
15			
20			
25			
30			
35			

40	T (to whole class): Use 10... or the INIT, best to use the INIT.	When Teacher mentions this, Ng checks his view window setting and changes them. He plots his graph again, gives up and waits for the Teacher.	
45	T (continues): What is the coordinate where the line cuts the y-axis? Use trace, press SHIFT. (to Ng)		
48	T: You did not press trace just now.	Teacher assumes that Ng did not execute the trace command. Teacher picks and checks Ng's calculator. Teacher returns the calculator to Ng.	
50		Teacher presses on the cursor ► and points on Ng's screen to show him that x and y changes.	
54	T: When you press trace, you see the value of x and y changes.		
59	T: Find the cursor (the pointer), the cursor will move along the continuous line .	Teacher traces along the line in the calculator with his finger and then presses the cursor. Ng then takes over, continues pressing the cursor, the Teacher watches.	
60	T: Press this Until you see it cuts the y-axis.		
63			
65			
67	T: That is where it cuts Now write it down.	When the pointer cuts the y-axis, Teacher points this to Ng.	
70			

The above excerpt sees Ng trying to trace the intercept of the graph of $y=x-1$. The earlier part shows that Ng was not able to follow the teacher's instruction on how to operate the tracer (line 1-35). The teacher's instruction was too fast for Ng who clearly did not understand which cursor key to press. The teacher did not explain this and used the term "tracer", "cursor" and "arrow" interchangeably (e.g., lines 5, 6, 9, 59), which confused Ng. The teacher did not explain that the tracer would not be viewed on the screen if either of the x or y-coordinates on the line fell outside the region of the view-window. In other words if the tracer is activated, to trace the graph of $y=x-1$, the screen will initially appear as follows:



The tracer cannot be viewed as the y coordinate for $x=-6.3$ is outside the INIT window region (whose minimum y-value is -3.1). The tracer will only be viewed starting from $y=-3.1$ (if this value is on the line and in this case it is) as shown below:



Although Ng saw the changing values of x and y as he pressed the cursor, he was not conscious of it until the teacher pointed it out to him directly (line 54). Ng did not realise that the tracer could not be viewed if either of the x or y value on the line is outside the region of the view-window. This is equivalent to saying that the point is outside the region of the viewing window! Ng did not make this connection. He also did not realise that pressing the cursor further to the left for the x -value less than -6.3 resulted in the y -axis being shifted to the right (line 30). The calculator resets the y -axis as the value being traced is outside the range of the view-window. This automatic feature constrained Ng's understanding of the functioning of the tracer. For Ng, the tracer and the points that were being traced on the line were two separate entities. Ng's pressing of the cursor back and forth indicates his confusion about how the cursor worked. More important, it highlights the ineffective role of the teacher in helping Ng. In the intervention, the teacher wrongly assumed that Ng had not pressed the "Trace" button (line 48). Thus the intervention proceeded without identifying why Ng had encountered difficulty. In other words, the teacher did not diagnose Ng's semantic error. Instead the teacher provided explicit procedures and directions for Ng to operate the tracer on the screen (line 59-63). Indeed, the teacher performed the initial act of pressing the cursor and then asked Ng to continue. The intervention steered Ng from further unfruitful floundering as Ng traced the intercept (line 67), but it had not confronted Ng with how the tracer worked in relation to the view-window and the points that lie on the line.

In conclusion, these excerpts emphasise the fact that understanding the view-window is a critical aspect in using the graphing calculator to mediate the idea of continuity. As Ng did not understand that the points that were not traced did not necessarily imply that they were not on the line, the teacher's classroom explanation of the idea of "points that do not satisfy the equation" was too far from Ng's "actual" level of knowledge, making it an

abstract idea for Ng. The teacher's role is critical, both in terms of the language and assumptions that are made about the learners, and the need to be attentive to student semantic errors. The automatic feature in which the calculator resets the y-axis as the value of the (x, y) coordinate being traced is outside the range of the view-window can hamper students' understanding of the working of the tracer. Providing explicit procedures for completing tasks without justification could result in student using the calculator as a "procedural tool", to carry out procedures of the tasks or as a static display tool.

AMIN (LOW ATTAINING BOY)

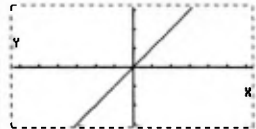
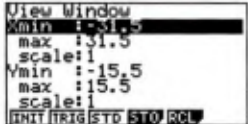
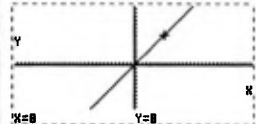
Amin was identified as a low attaining student by his teacher. He consistently performed poorly in his school examination, and this is borne out by his performance in the pre-test which indicates poor algebraic and number skills. Amin saw the points on the line as discrete and countable. In the post-test, he counted all the points plotted on the line and gave the answer that the total number of points on the line as "8". He gave the explicit point $(2\frac{1}{2}, 6)$ as a point between $(2, 5)$ and $(3, 7)$ and did not say how many points there are between them. In his worksheet, he wrote that the tracer could trace every point on the line.

Amin's failure to understand the concept of continuity can be seen as a result of not comprehending the overall functionality of the calculator, and many aspects of the view-window concept, including how the view-window affected the point that was plotted in the view-window. Two issues that are examined in the excerpt are how he seemed unable to understand the tracer's workings, and thus the meaning of the tracing activity, and how teacher intervention was also not productive for Amin.

The view-window and teacher intervention

The following excerpt depicts Amin's difficulty in choosing the appropriate view-window for the task.

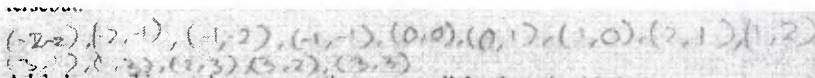
Excerpt 5.06 Amin's difficulty with choosing a view-window

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1		Teacher was explaining how to graph the line $y=x$ in the RUN Mode. Amin peeps at what his neighbour Wan is doing. Teacher is asking to whole class.	
5	T: How is it, is (2.5,2.5) on the line? Is (8,8) on the line?	Amin shows to Fai his screen as he could not find (8,8).	
10	Fai: I am doing it in the STD view-window. You are doing in what?		(This is the screen Amin generates according to what he tells Fai in line 8)
13	Amin: INIT?	Amin is graphing in the INIT view-window.	
15	Fai: Oh there is....	Fai is now operating in this view-window:	
20			
21		Fai reaches for Amin's calculator and changes the view-window for him and activates his tracer. Amin then continues.	
25		Teacher continues asking whole class	(This is the screen that is generated following Fai's action in line 16)
26	T: there is, isn't it? Now (2,1)... Is it on the line?		
29	Am: No	Amin shakes his head .	
30	Am: ..because $x=2$, $y=1$ T asks: Why?	Amin whispers. Teacher ignores the answer.	
34	Wan and Am: It is not the same.		
35	T: Why is it not the same?.... why is it not the same? Becausewhat is the value of y ? $y=1$, $x=2$ If $y=x$ or $x=y$ it must be the same... (2,2), (3,3)... $y=1$, $x=1$... How do we know?... y and x does not satisfy the equation.... Like this, if $y=x$ (10,10) $x=10$, $y=10$.. x and y satisfy the equation (2,1)... is it the same? Does it satisfy or not? It does not satisfy the equation $y=x$. Try and give an example or another point that is on the continuous line $y=x$. Give five points that is on the continuous line $y=x$. T (comments): Someone gave the value 2000.	Teacher does not expect any response. Teacher goes on with long didactical explanation.	

Amin appeared not able to follow the teacher's instructions which could be seen from his actions peeping at what his neighbour Wan was doing. (Many sequences of button

presses are required to draw a line in the RUN mode, starting from pressing “SHIFT”, to “Sketch”, “CIs”, “EXE”, “▷”, “▷”, “GRPH, Y=”, and finally key in the “x”). It is obvious that the teacher assumed that all students had graphed $y=x$ on their calculator using the appropriate view-window (line 5). However, Amin clearly was not following the teacher's classroom instruction as he tried to trace the point (8,8) on the line $y=x$ in the INIT window (line 13). The teacher had earlier reminded students to use a “bigger scale” to “find” the point (8,8) on the line. This implies that Amin did not understand the terminology “bigger scale” that the teacher used. It also shows that Amin was unaware that the INIT window he was working in has the maximum value of 6.3 for x and 3.1 for y and therefore it was impossible to trace the point (8,8). It seems that Amin did not understand that points plotted on the graphic screen were bounded by the x and y view-window values.

The excerpt also shows Amin's peer correcting the view-window and activating the tracer for Amin, pressing the cursor a few times and returned it to Amin to continue (line 21). Therefore it is difficult to gauge whether Amin knew how the tracer worked in relation to the view-window and the line. However, this help did keep Amin on track of the task to trace for (8,8). Amin was also able to decide that (2,1) was not on the line (line 29). The teacher was seen probing further why the values were not the same (line 34 onwards). When Amin did not respond, the teacher continued with long didactical explanation including the notion of x and y does not satisfy the equation (line 40-47). Curiously, the teacher did not use the graphing calculator to support his explanation. The teacher concluded the intervention by asking Amin to furnish five more points that were on the “continuous” line $y=x$. However, the teacher did not wait to see what Amin wrote. It is interesting that Amin gave only positive integers: (5,5), (6,6), (7,7), (8,8) and (20,20). There were no negative numbers or decimals involved. By contrast, Amin did not correct his previous work where he incorrectly listed some of points that lie of the line $y=x$ as shown below.



Handwritten list of points: $(-2,-2), (-2,-1), (-1,-2), (-1,-1), (0,0), (0,1), (1,0), (2,1), (1,2), (3,1), (2,3), (3,2), (3,3)$

Printed list of points below the handwriting:

$(-2,2), (-2,1), (-1,2), (-1,-1), (0,0), (0,1), (1,0), (2,1), (1,2)$
 $(3,1), (1,-3), (2,3), (3,2), (3,3)$

The teacher intervention in the above excerpt was not helpful for Amin. The teacher attempted to build a curriculum script at the beginning of the intervention (line 21-32),

then stopped and continued with didactical explanation. The teacher threw some questions in between his explanation but the fast manner in which he conducted his intervention sent signals that he did not expect any response from Amin. The teacher seemed to carry out a perfunctory type of intervention, that is, the teacher felt obliged to provide the student with the correct answer and explanation regardless of whether student understood or not. So it was not surprising that the teacher did not complete the intervention and examine what Amin wrote, and did not go further to ask Amin how he derived those points.

Role of the calculator

It is unlikely that Amin understood how the tracer worked in relation to the points on the line and the view-window values. This can be seen from his earlier work which shows that Amin did not understand how the plotted points were related to the view-window settings and how the pointer moved on the graphic screen (as defined by the view-window). Examination of his videotape and worksheet indicates that Amin restricted his work to using the view-window settings suggested in the tasks. His exploration was also limited to using the point (3, 2) which was within the regions of all the view-window settings he used. Therefore, most likely, he did not encounter points outside the view-window region that could not be plotted on the screen, explaining why he did not recognise that points outside the view-window could not be plotted.

Examination of his worksheet indicates that he knew that the point (3,2) could not be plotted "exactly" on the STD window, but could not explain why. He also indicated that the point (3,2) could not be plotted exactly on another view-window but could not provide an explanation. One press along the x-axis in the STD moves the pointer a distance of 0.15873015873, and one press along the y-axis a distance of 0.32258064516. For the "other view-window" in the worksheet, one press of the cursor moves 0.0793650793 unit along the x-axis and 0.16129032258 unit along the y-axis. The unit of measurement for each consecutive movement of the pointer in both these view-windows is distinctively difficult to decipher as they are not integers that Amin is most familiar with. Evidently, the teacher did not bring this to Amin's attention, or in classroom teaching explain how the unit of measurement relates to the movement of the pointer when the cursor button is pressed in each of the view-window.

Therefore the graphing calculator for Amin at this stage seemed to be just part of the task. Tracing along the line was merely a mechanical procedure. There was no indication that Amin tried to trace the line of $y=x$ in another view-window. Confining the exploration in only one view-window might have given the misconception that every point could be traced. This must have been what Amin saw on the screen as he traced along the line.

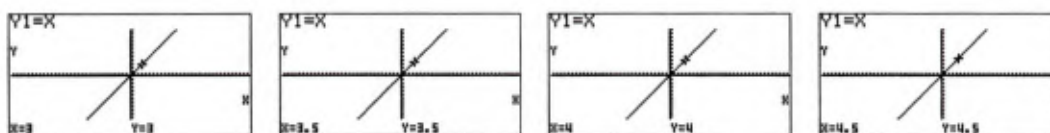


Figure 5.03 Movement of the tracer from one point to the next point

The figures in 5.03 above show a series of tracer movements along the line corresponding to one press on the cursor. The value of the coordinates increases by 0.5 unit each time as the tracer moves upward along the line, giving the impression that these are the only ordered pairs that make up the line. Points between 3 and 3.5 for example, are not traced in this particular view-window and are deemed as non-existent. The series of points traced on the line seem equidistant to each other giving a connotation that these points are discrete.

In conclusion, Amin's learning activity above indicates that in order to appropriate the graphing calculator as a mediational tool for learning the concept of continuity, it is essential to understand the movement of the pointer in different view-windows. Only then can he advance towards understanding how the pointer moves on the line. However, the concept of view-window in relation to the movement of the pointer is evidently not easy for Amin, and careful intervention by the teacher is required. Long didactical explanations, non-expectation to justify one's answer and incomplete intervention experienced by Amin was not productive in assisting him to develop a coherent understanding of the functionality of the calculator, and between the calculator functions and the mathematical meanings embedded in the learning tasks. What also emerges in this case study is that restricting the tracing activity to only one view-window could give rise to the misconception that the points that lie on the line as discrete as the tracer was perceived as jumping from one point to the next point equidistant to each other.

5.4 Conclusion on the concept of continuity in the Structured Learning Model

1. Understanding the concept of continuity requires an understanding of the relationship between the view-window and the trace function. Using the tracer function without an understanding of how it functions in relation to the view-window setting can give rise to the visual illusion of the tracer jumping from one point to the next point.
2. Low and medium attaining students found it confusing when the tracer did not appear on the screen. They were not aware that the tracer starts from the minimum value of x of the view-window which do not necessarily appear on the screen. The tracer starts to appear only when the y -value on the line falls within the y -value of the view-window.
3. The concept of the movement of the pointer on different view-windows is not easy for medium and low attaining students. The confusion lies in the difficulty in recognising the invariant number of presses on the cursor that are required to cover the length of x -axis and y -axis in either direction from the origin regardless of the view-window setting, which results in different unit of measurement as the pointer moves for each press on the cursor in different view-windows.
4. Explicit procedures on how to operate the calculator without connection to the meanings of the tasks predispose towards using the calculator to carry out procedures of the task, resulting in the calculator as a static, display tool.
5. The teacher tended to assume that low and medium attaining students could follow the pace and understand the vocabulary used during the instructions and interactions. This was not the case.
6. Productive interventions involve directing the students' attention to the specific features of the graphic display, enabling them to relate the display to the task. Long didactical explanations, and inconsistent use of terminology confused low and medium attaining students.

7. Semantic errors were not easy for students to rectify themselves, and were also not easy for the teacher to identify when they occurred.

5.5 Analysis of case study students in the Interpretative Learning Model

GRACE (HIGH ATTAINING GIRL)

Grace was identified as a high attaining student, and, as her pre and post-test results indicate, she seems to have benefited from the experience of working with the calculator. In the pre-test she did not express any view about continuity, but in the post-test, Grace wrote that there were many points on the line. She also wrote that there were unlimited points between the two points.

Two issues will be considered to explore Grace's understanding. The first is the close links in her knowledge of the functionality of the TABLE Mode facility, effective teacher intervention, and her ability to make the connections between the various information. The second is the style of teacher intervention that emerges as she explores the activities.

Role of the TABLE Mode

Grace seemed to have built a well-developed notion of the functionality of the calculator in general. She was observed to be working independently with the manual of the calculator by her side and it seems she built the understanding of the functionality of the calculator by herself. It was observed that when working in the TABLE Mode, Grace sought the teacher's help only to explain the meaning of "pitch" in the "Table Range" screen. Grace was specific about what she did not know, suggesting that she was making sense of how the calculator works but continued to work independently. From her conversation with the teacher, Grace chose the range value of x from 1 to 6, so one might expect that she would have generated the following graphs in the G-CON and G-PLT command respectively:

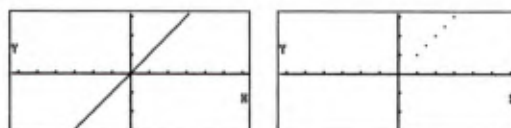
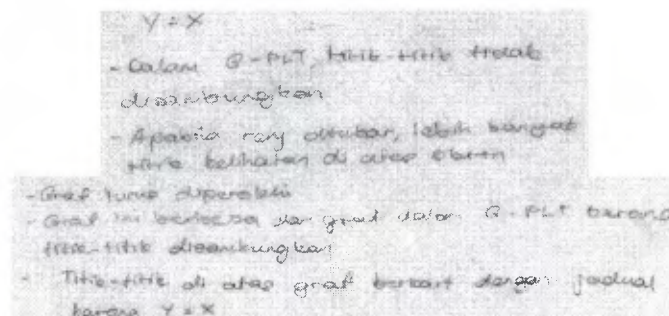


Figure 5.04 Graph of $y=x$ generated in the G-CON and G-PLT with range values of x from 1 to 6

However, the answer she wrote in her worksheet indicates that she experimented with different Table Range values.



"In G-PLT the points are not joined. When the range changes, more points are plotted closer on the screen. The other straight line is different from the G-PLT in that the points are joined. The points on this graph are related to that in the table as $y=x$ "

From what she wrote, Grace saw that many more points are generated when "the range" changes and these points became closer. Here, it seems Grace refers "the range" to include the range values of x , and the pitch values. From her description, the figures 5.05 below show what must have been generated on Grace's calculator screen. They show how the change of pitch value affects the appearance of the graph of $y=x$ drawn in the INIT window using the G-PLT command.

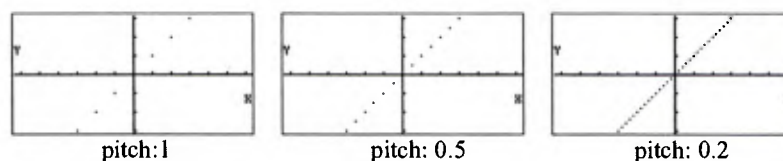


Figure 5.05 Effects of change in pitch value

As the value of pitch decreases, the number of dots increases and becomes closer to each other. Grace's answer points to the fact that she recognised that the G-PLT command is constrained by the range values that are chosen and the graph generated could appear shorter than the graph generated in the G-CON command. This is an indication that Grace had a high level of sophistication with the concept of the view-window. While most students chose a view-window as suggested by the teacher (multiples of the INIT window setting), Grace was observed to choose the following atypical view-window setting:



Figure 5.06 An unusual view-window setting by Grace

In her worksheet, she also wrote that the point (7, 5) could be plotted in the STD window but it will not result in integers. This enabled her to make the connection between the points plotted in different view-windows. It is not surprising that Grace also made the connection that many other points were plotted outside the viewing window in the G-PLT command, if the range values of x were broadened. She saw that the graphs in the G-PLT and the G-CON are the same graphs represented by the equation of the line $y=x$. Grace's answer also indicates that she saw that the points on the graph are related to the table values and the equation. Her understanding of the TABLE Mode enabled her to use it to make sense of the task (e.g., trying different values of pitch, relating the table values to the points on the graph), and her understanding of the concept of view-window, more specifically how the different view-windows affect the points plotted on the screen, enabled her to articulate the various information that came into play. This seemed to suggest that the role of the graphing calculator was a mediational one.

Role of teacher intervention

Grace's success in using the calculator was reflected in the type of interaction that she had with the teacher. This is depicted in the excerpt below where the teacher intervened with Grace when she was trying to derive the equation of the line.

Excerpt 5.07 Teacher intervention with Grace

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Grace, what is the name of the line?	Grace is trying to deduce the equation of the line.	
	T: This is point A, B, C and D	Teacher points to the graph in Grace's worksheet.	
		Teacher gives labels to the points on the graph.	
5	T: What is the coordinate for A?	Teacher points to a coordinate on the graph.	
	Gr: (5,5)	Teacher moves her finger to the next few points and asked Grace for each value of the point.	
	T: (5,5)		
	T: the value?		
	Gr: (1,1)		
10	T: the value?		
	Gr: (0,0)		
	T: the value?		
	Gr: (-3,-3)		
	T: What is your conclusion?		
15	(Emy: The values are the same.)	Emy, a low attaining girl intervenes Note Emy's use of language. She uses "values" to mean coordinates.	
16	T: So what is the name of the equation?		
18	(Emy: same)		
19	Gr: $y=x$		
20	T: When you write the equation,		

22	what must it begin with? (Emy: x) T: We have learnt to name the line before... $y=2x$		
25	(Nurul: aha.... $y=2x$) T: So, what is the name of this line? (Nurul: $y=x$) Gr: $y=x$		

The excerpt shows that Grace was able to follow through the teacher's curriculum script during the intervention. Grace understood the mathematical terminology that the teacher used. When the teacher asked for the coordinates (line 5-10), Grace understood what teacher meant by coordinates and she could also identify all of them correctly. When the teacher asked Grace for the equation (line 16), Grace understood what was meant by equation, and correctly named $y=x$ (line 19), as opposed to Emy who had difficulty in (line 18 and line 22). Grace was able to follow the teacher's intervention because she understood the mathematical language the teacher used. This helped her in following the teacher's line of reasoning, and she understood how the points on the line related to the equation of the line. Noticeably, the intervention progressed from the idea of coordinates to how they relate to the equation of the line. This added another invaluable layer of meaning of continuity for Grace.

In conclusion, Grace's activity shows that the TABLE Mode could confront the idea of continuity provided that students understand the functionality of the TABLE Mode. Her understanding of the TABLE Mode facility enabled her to structure her exploration to include changing the range and pitch values, thereby allowing Grace to use the calculator as a mediational tool to help her frame her thinking about the idea of continuity embedded in the number of points generated on the screen.

What also emerges is that Grace understood the mathematical language that the teacher used in the intervention. This shows the importance of the teacher's curriculum script in assisting Grace to a more advance knowledge.

PRESHA (MEDIUM ATTAINING GIRL)

Presha was regarded as a high attaining student by her teacher. However, her performance in the pre-test and the sometimes unsystematic manner in which she carried

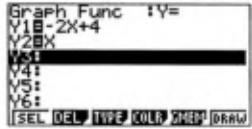
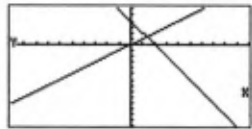
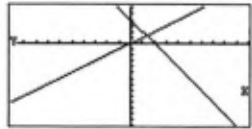
out her tasks did not fit the profile of a high attaining student. Presha did not understand the idea that there are many unidentifiable points on the line and the post-test indicates that she saw the points on the line as discrete. Only points that were plotted were accounted as valid points on the line.


To explore Presha's difficulties in understanding the idea of continuity, the excerpts presented will examine her apparent lack of understanding of the general functionality of the calculator, and the need for precise teacher intervention.

The role of the functionality of the calculator

The excerpt below exemplifies Presha's difficulty in understanding how the calculator works, including which button to push to execute the task that she wanted the calculator to carry out.

Excerpt 5.08 Presha's difficulty with the calculator operations

Line No.		Action	Calculator Display/ Worksheet
1	T: Put in $y=x$ in the Graph Mode.	Teacher instructs to all students. Presha is already in the Graph Mode with the equation $y=-2x+4$. It is suspected the previous user had keyed in and did not erase this equation. She keys in the "x" at Y2.	
5			
10			
15			
17	T: Trace the line.	Teacher instructs to all students to trace the line. Presha manipulates her calculator and it shows these two displays consecutively.	
20			
25			
27			
30	Grace: You need to change the view-window and delete the first graph.	She shows Grace her calculator. Grace explains to her.	

35		Grace then does it for her anyway and Grace also activates her tracer. Grace returns the calculator to her.	
40	Pre: I cannot see the tracer. Grace: Continue pressing the ► key.	She turns to Grace again when she could not see the tracer. Grace tells her to continue pressing the ► key.	

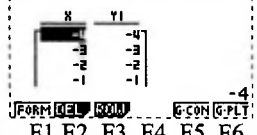
The first graphic display shows that Presha did not know how to erase or “deselect” the existing equation on her text screen. The graph she generated indicates that Presha had not chosen the INIT window. She seemed not to be conscious that she was in the STD window with the x-axis shifted up. This view-window might be from her previous activity. The subsequent graph display (line 17) indicates that Presha had wrongly pressed the following buttons in sequence: “SHIFT Sketch Trace” instead of “SHIFT Trace”. It is highly likely that she pressed “Sketch” because examination of her videotape saw that her previous activity involved execution of this command most of the time. This indicates that Presha had “memorised” some of the operational commands of the calculator without actually understanding what they do. It also indicates her reliance on the teacher's intervention or peer assistance as observed in the task (line 27 onwards). She had little initiative to refer to the manual for operating the functions of the calculator.

The excerpt above shows that Presha did not understand how the calculator operates in general. The next excerpt indicates that operating the TABLE Mode posed a problem to Presha.

Role of the teacher

In the ensuing task that involved tracing the line $y=x$ in the TABLE Mode, Presha exhibited difficulty in understanding how to operate the TABLE Mode. This next rather lengthy excerpt shows the nature of teacher intervention when dealing with Presha's semantic errors.

Excerpt 5.09 Presha's difficulty with the TABLE Mode

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: What mode is this? Pre: TABLE Mode. T: Have you pressed G-Con?	Presha is graphing for $y=x$ in the TABLE Mode.	<p>(This must be the display as this screen is generated after executing the graph $y=x$. Note the empty space that Presha describes in line 9)</p>  <p>(these are the corresponding function keys to execute the command)</p>
5	Pre: No I have not.	Presha starts asking about the menu displayed on screen. Teacher shows and explains.	
7	T: G-Plot? Pre: G-CON..where is it? T: Here. And what is this empty space?		
10			
11	T: Look, F4, there is none. Pre: Ooh..	Presha executes the G-CON command. Presha presses the cursor ◀ a few times instead of ▼.	
13	T: Oo..your graph has gone down...you have also shifted the y-axis.... Where is the y-axis?		
15	Pre: It has disappeared (laughs)..		
18	T: Well, your axis has changed.	Presha tries.	
20	Never mind. Now, how do you get back to the previous screen? Pre: Change the scale? T: Try it.		
24	T: Now you are no longer in the		
25	RUN Mode, you are now in the		
26	TABLE Mode....Now how do you go to the original screen?		
29	Pre: SHIFT F1	Teacher points to the Trace command on the calculator.	
30	T: If SHIFT F1....what is it called? Pre: Trace.		
35	T: Now what does Trace do? Pre: for tracing.... T: Yes, for tracing, now you want to go back to the original screen, how?		
38	T:... You have this button ..EXIT...	Teacher points to the EXIT button.	
40	QUIT..now you want to go back to your original screen,...remember before this there is another screen.		
44	T: What did you press just now?	Presha presses the MENU button.	
45	Why did you press this MENU button? Every button has its function.		
47	Pre: Teacher..., now we need to draw this graph $y=x$ right?		
50	T: Yes. ...what now?	Presha makes gestures of the line $y=x$ on the calculator.	
52	Pre: Draw and trace?		
55	T: So what do you press now? Look...you have to choose G-CON, look at the kind of graph you get and how is this different from G-PLT. Then you trace.		
		Teacher focuses Presha to the task again.	

57	Pre: Now G-CON...	Presha executes the graph.	
59	T: But first...your axis has changed,	Then activates the tracer.	
60	you have to set it back....how do	Presses the cursor a few	
	you do it? Each time you press the	times but it did not appear	
	button, something will change.....	on the screen. Teacher	
		presses the cursor a few	
65	T: This is what happens when you	times but the tracer still	
	simply press the button several	does not appear.	
	times. See the tracer does not	Teacher takes the calculator	
68	appear. Let us look at the set up of	from Presha and resets it for	
	the calculator again.	her.	

The excerpt above indicates Presha's lack of grasp of the general operation of the calculator (e.g., line 7-11) which obscures the significance of the TABLE Mode to her task. What is worthy of note in this excerpt was the teacher's attempt to direct Presha towards aspects of the functionality of the calculator, whilst Presha, was trying to get the teacher to provide explicit procedures so that she could complete the task (e.g., line 47, line 57).

This excerpt also illustrates an interaction that involves a series of continuous conversational exchanges with Presha answering the teacher's question or responding by operating her calculator, albeit wrongly most of the time. The teacher clarified for Presha how the function keys worked (line 8-11) and confronted Presha with how to activate the tracer (line 30-35) but not how the tracer operated, as evidenced from Presha's inability to apply this later in the interaction (line 59). Presha's failure to see the tracer confirms that Presha did not understand how the view-window, the tracer, and the points on the line related to each other.

The excerpt shows a series of semantic errors made by Presha. For example, her confusion on how the cursor buttons functioned. The teacher's remarks (line 13) indicate that Presha had pressed the "◀" cursor successively instead of the "▼" cursor resulting in the y-axis disappearing. The teacher did not ask Presha why this occurred, but instead directed Presha to retrieve her "previous screen" (line 18). When Presha tried to retrieve the view-window, Presha made another error. The teacher realised that Presha had pressed "SHIFT Sketch" instead of "SHIFT V-Window" (line 24). The teacher attempted to guide Presha to rectify the situation herself and asked her how to get back to the earliest screen (line 26), but Presha's wrong answer (line 29) resulted in a diversion

about the tracer. The teacher again tried to get Presha on track to return to the earliest screen (line 35) but when Presha did not respond, the teacher then decided to hint to Presha to use the "EXIT/Quit" command to return to the earliest screen (line 38). Curiously Presha did not carry out the hint that teacher gave, instead she pressed the key the MENU key, thus reverting to the MAIN MENU screen, necessitating her to start the task from the beginning again. It seems Presha chose to start again instead of repairing her error indicating that she did not realise that starting the task again would not change the view-window setting. Although the teacher asked Presha why she chose the MENU key (line 44), the teacher did not expect Presha to answer. The next conversational turn saw Presha trying to circumvent the error as her question to the teacher shows that she wanted the teacher to give her direction in the task (line 47). The interaction continued with the teacher now giving Presha explicit procedures on how to carry out the task (line 52). The semantic error was not resolved. Three things seem to emerge here. First, merely hinting to Presha when she was stuck was not productive if she did not carry out the hint. Second, there was only a limited requirement for Presha to reason about her "wrong actions", which resulted in many other errors to ensue. Third, it shows that the teacher lacked the "pedagogical technology skills" to keep track of the line of argument when a semantic error occurs.

Towards the end of the intervention, the teacher's explanation about the tracer was ambiguous. It was ironic that the teacher "blamed" Presha for this, as the result of her pressing the button recklessly. It seems that teacher herself did not fully understand how the tracer worked as it is not clear why the teacher asked Presha to look at the "set up" (line 68). Presha wrote only two lines in her work sheet: "G Plot = the points are not joined. G Con = Line", describing the graph on the graphic screen. The role of the graphing calculator was at best a display tool.

In conclusion, in order to understand the functionality of the TABLE Mode, an understanding of how the calculator works in general is important, including locating and understanding the various menus. Careful teacher intervention is critical for Presha to develop a coherent understanding of how the calculator works in general. What emerged in Presha's activity was the ineffective nature of teacher intervention which could be characterised as ranging from wrong and ambiguous instructions and inappropriate expectation of Presha to correct her own semantic errors. The teacher's interventions,

for example was simply giving instructions on how to correct a semantic error without explaining why they occur and not correct the mistake. Resolving semantic errors is important in getting student to build an understanding of the functionality of the calculator in general, which requires the teacher's understanding of how the technology works and the pedagogical skills to articulate them to the student.

EMY (LOW ATTAINING GIRL)

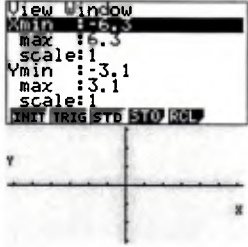
Emy was regarded as a low attaining student by her teacher. This was borne out by the pre-tests results and observations of how she carried out her tasks with many unsystematic explorations. Emy's understanding of continuity was that there were many discrete points on the line. In the post-test, she simply wrote that there are "many" points on the line. It was unclear whether she meant they were uncountable. However, her subsequent incorrect answer where she gave (3,6) as the point between (2,5) and (3,6), perhaps confirms that Emy perceived the points on the line as discrete.

The following extracts focus around the interplay between her grasp of the functionality of the calculator and the teacher interventions.

Mathematical knowledge and teacher intervention

The first excerpt shows Emy's need for key mathematical concepts her difficulties with the mathematical terminology involved.

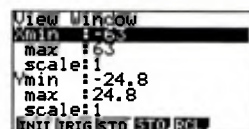
Excerpt 5.10 Emy's difficulty with the mathematical terms

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: You have to take a point that cannot be read...now your screen...	Students are to think of a point that could not be plotted on the screen.	
5	This is X minimum....what is X minimum? What axis is this? Emy: vertical	Teacher points on Emy's screen. Teacher draws the corresponding figure. Teacher points to the horizontal line.	
9	T: the horizontal line what is the name for it?		
10	Emy: x T: x-axis, this one?	Teacher points to the y-axis.	
	Emy: y-axis	Teacher corrects Emy's terminology.	

15 16	T: Xminimum means.. we can read minimum values in the calculator..where is the Xminimum? Emy: How is it?	Teacher asks Emy to point in the graph.	
18 19 20	T: x... the smallest value Emy: here.. (unsure) T: ..the smallest...	Emy points at the positive end . Emy points at the origin. Teacher corrects her. Teacher points are the respective place.	
25	T: This is (0,0), here, the smallest x on the screen is here Emy:aaaa.. T: -6.3 T: What is the maximum?...6.3 Emy: 6.3 (repeats) T: So if you choose ..16 it is not possible	Teacher points at the calculator and to the sketch.	
30	Emy: So Teacher we must take small ones? T: You can change these values. T: Now, y, what is the maximum? Emy: 3.1...	Emy means small numbers. Teacher points on the view-window setting screen of the calculator.	
35	T: minimum?...-3.1 Emy: ooh.... Emy: So if higher values? T: Change the values...look in your manual.	Emy looks at her manual.	

Emy seems confused between vertical and horizontal line and did not remember the term for each axis (line 5-9). Emy did not understand the terminology and the meaning of minimum and maximum (line 16-26). She showed no conception of the number line (line 18-20), including negative numbers. It is not surprising that Emy's lack of mathematical knowledge and the mathematical terminology or vocabulary necessary for the kind of task she undertook posed an obstacle to accomplish the task.

However, what seems to emerge here is the teacher's role in inducting Emy into the use of mathematical language, for example the terminology maximum and minimum and the meaning associated with it. The teacher confronted Emy with the concept of the number line to talk about the position of numbers on the number line. Emy continued her work, and in her worksheet, correctly named the point (18,6) as a point that could not be plotted in the INIT window, reasoning that the x and y value exceeded the x and y value of the INIT window. She was seen choosing the following view-window setting to plot the point (10,8) exactly on the screen.

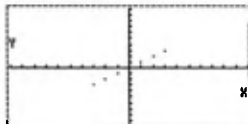
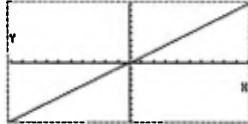
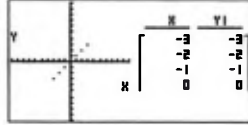
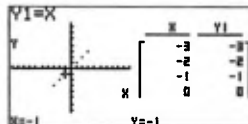


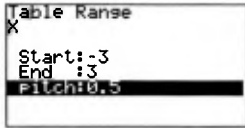

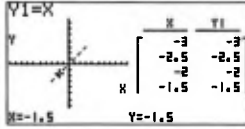
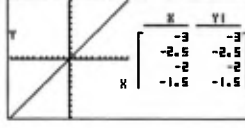
It seems that she multiplied the x-coordinate and y-coordinate with the x value and y-value of the INIT window respectively. Although not very elegant, it was ingenious. It seems with the appropriate teacher intervention, Emy could successfully complete a task.

TABLE Mode and teacher intervention

The following excerpt reveals some aspects of Emy's difficulty with the functionality of the TABLE Mode and how the teacher influenced her understanding of the facility.

Excerpt 5.11 Emy's difficulty with the functionality of the TABLE Mode

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: What is the difference between G-CON and G-PLT?	Emy is exploring the graph of $y=x$ in the G-CON and the G-PLT facility.	
4	T: What is the difference?	Teacher asks whole class.	
5	Emy: G-CON long...the other is short.	Teacher asks Emy. Emy describes what she sees on the screen.	
7	T: Is your equation different?		
8	Emy: It is the same.		(From Emy's response and from evidence of the range of x values below which is between -3 and 3, these are the possible graphic displays).
10	T: How about the Table?		
	Zulfah: G-CON, the line is long and straight, G-PLT it is dotted		
	Emy: aah...aah.. you are right.	Emy confirms her friend's incorrect observation.	
15	T: Go to the range...go to START.. END..PITCH...	Teacher asked her to look at the values.	
19		Teacher leaves and returns a while later. Emy is seen to have split her graphic screen to display the graph and table side by side.	
20		Emy presses the GPLT command.	
25	T: Try to choose GPLT, okay, do you see the points? "kesan" (track) the points.		
	Emy: "kesan" (track) them?	Emy is unsure what to do.	
	T: It means to trace.		
	Emy: Ooh...(proceeds to trace).	Emy knows how to activate the tracer.	
30	T: Look, what is this point?	Teacher points at the point in Emy's calculator.	
	Emy: x..		
	T: x,y, -3,-3.		
	Emy: -3,-3 (Emy repeats)		
	T: Look at the table, what is it?	Teacher points at (-1,-1) on the line.	
35	-1,-1 ...Look ...-1,-1 also in the table.		
36	Emy: Yes.	Emy acknowledges.	

39	T: Yes, now change the pitch...	Emy is quiet.	
40	how?		
41	T: Okay..go to range, yes..say 0.5..ee..okay, choose GPLT..ee..	Emy starts to manipulate her calculator. But not confident. Waits for Teacher's instruction.	
45	Try to trace the point. Look at each time you press, what point did you obtain? Also observe your table.	Teacher guides her along.	
48	Did you see? Now what is plotted on the graph if you choose a different value for pitch?		
49	Emy: The number is the same.		
50			
51	T: What number is the same?	Emy uses the term number instead of coordinate.	
53	Emy: The number...	Teacher points to the x and y values in the table.	
55	T: What do we call that number? What do you mean by this number?.....Coor....(hints)	(Teacher forgets that Emy does not answer the question regarding pitch - the consecutive reading increases by the value of pitch.)	
58	Emy: Coordinates.		
60	T: This is the coordinate, ordered pair, the same as the point that is plotted on your graph.		
61	Emy: The same as that plotted on the graph.		
63	T: Now you want to get a continuous line, a line that is joining (berterusan), what do you press?		
65	Emy: Change the pitch.	Emy repeats and at the same time presses the cursor.	
66	T: Just now when we change the pitch, what changes? (pause)..x and y only, isn't it?		
70	Emy: O..oh..	Emy realises her "error" but still presses her calculator.	
	T (repeats the Q): What if we want a continuous line?		
	Emy: O..ooo		
75	T (asks in another way): This line is dotted, if we want a continuous line, what do we choose?		
76	Emy: GCON		
78	T: Did you see?		

At the beginning of the intervention, it seems that Emy had some understanding of the general operation of the calculator. This can be seen from her ability to generate the graphs using the G-CON and G-PLT command in the TABLE Mode. Emy's screen also showed the graph and table displayed side by side indicating that Emy had changed the "dual screen" command in the SET UP (line 19). However, this understanding is still fragile, since, for example, Emy was not able to locate the "pitch" command (line 39). This is also evident from her description of what she saw on the screen. Emy expressed what she literally saw on the screen (line 4). She saw the graph generated by the G-CON command as continuous and long and the G-PLT command as dotted and short.

The teacher overlooked the fact that Emy chose to generate her graphs in the STD window for the range values for x that she chose between -3 and 3 , and therefore, the graphic display in the “plotted” form appeared shorter. The teacher tried to focus Emy’s attention on the idea of continuity by asking her whether the equations generated by the two different forms were different (line 7). When Emy said it was the same (line 8), the teacher continued with the intervention by asking her to consider the “table” (line 10). The teacher assumed that Emy looked at the table values and was able to relate by herself that the x and y values in the table for both of the graphs were the same. Emy’s affirmation of her peer’s observation that the G-CON command produced a long line and the G-PLT command a short dotted line indicates that she did not. However, the teacher did not give a corrective feedback, instead the teacher hinted to Emy to check out the “range” values (line 15). The teacher asked Emy to look at the settings then left her to explore herself. Again, it seemed the teacher assumed that Emy would be confronted with the range of values for x influenced by the plotted points, and the pitch value influenced by the number of plotted points generated by G-PLT command.

Emy’s poor memory of mathematical language, in this case the term “coordinate” made the teacher intervention difficult (line 49-61). In the excerpt it was apparent that the teacher was constructing a curriculum script to relate the coordinates in the table with the points that lie on the line $y=x$ (line 30-36). The teacher was also trying to get Emy to see that the number of points was determined by the pitch value (line 39-48). Clearly Emy did not know the meaning of pitch and this was exacerbated by her uncertainty in executing the command on the calculator (line 41). Things became more complicated when Emy was vague about the term “coordinate”. The intervention was diverted to the meaning of coordinate (line 51-58), and the teacher subsequently forgot that Emy had not answered the question regarding the pitch. The teacher also assumed that when the graphic screen was generated, and just by pointing to the x , and y values of the table on the screen, Emy had understood the effect of changing the pitch value on the points plotted on the graph (line 53). A script to relate the equation to the points on the line and to the table values was missing.

The teacher continued to ask Emy to generate the continuous line (line 63). Here again it was observed that Emy was confused about the term “continuous”. She wildly gave the answer “pitch” (line 66) which also indicates she still did not know what pitch meant.

The teacher proceeded to tell Emy the incomplete meaning of pitch as “x and y changes”, rather than the precise meaning as increment of the x values.

The teacher had a fixed idea on how to produce a continuous line (line 63-78). Emy was correct in saying that to obtain a continuous line, “change the pitch” (line 66). If the pitch is small enough the points would be very close and a seemingly continuous line would be generated. The teacher was determined for Emy to generate the continuous line using the G-CON command (line 76). It is unclear whether the visual illusion that the G-CON command generated a long line and the G-PLT command generated a short line still persisted. What Emy wrote in her worksheet seems to indicate that she only recognised that the G-PLT command produced “disjointed dots” and the G-CON command produced “line”. She understood in terms of what the two different commands did. She saw two graphical forms generated from the same equation in isolation and did not make the connection between the two. In other words, she did not see the underlying mathematical concept of continuity. The role of the graphing calculator was just as a static tool to reproduce the graph with insufficient mathematical meaning attached to it.

In this excerpt, the teacher interventions were largely ineffective. First, the teacher made the assumption that Emy saw the same thing that she expected Emy to see, that is what an expert would see on the screen of the calculator. Asking Emy to recover from the visual illusion by hinting to her the location of the source, in this case the “Table Range”, but with no suggestion as to how each component in the “Table Range” affects the plotted line, was not useful. The teacher interventions were also incomplete, and the teacher stopped or forgot to continue with several explanations of terminology, about the syntax of the calculator or how to execute certain commands.

In conclusion, the lack of mathematical knowledge including the mathematical language related to the task in low attaining student like Emy could pose enormous difficulties for her to coordinate with the calculator syntax and developing an understanding of the functionality of the calculator and therefore understanding the mathematical meaning embedded in the tasks. When using the TABLE Mode, the lack of understanding of the view-window concept in relation to how the points are plotted on the line could result in a visual illusion in which the line generated with the G-CON command is perceived as long, whereas the line generated with the G-PLT command is perceived as short.

Emy can successfully complete a task with a calculator with careful guidance from the teacher, where the teacher assesses the student's missing knowledge pieces including the mathematical terminology and ensures that the intervention is complete. On the other hand, unspecific teacher intervention, strict adherence to a curriculum script and incomplete intervention do not support low attaining student in developing his/her own understanding or meaning making about the functionality of the calculator and how it relates to the mathematical activity.

5.6 Conclusion on the concept of continuity concept in the Interpretative Learning Model

1. The TABLE Mode facility can support the understanding of the idea of continuity but students need to understand the relationship between the view-window and the points plotted on the line.
2. Using the TABLE Mode facility could lead to a visual illusion to perceiving the line generated with the G-CON command as long and the line generated with the G-PLT command as short.
3. Teacher intervention plays a critical role in structuring medium and low attaining students' mediational use of the calculator, which requires technological pedagogical competence on the part of the teacher in dealing with students' semantic errors with the calculator.
4. The teacher tended to adhere to curriculum scripts in the interventions, which middle and low attaining students found difficult to follow because of the missing knowledge pieces, including the mathematical language.
5. Low and medium attaining students did not systematically explore different values of pitch or range values of x in when using the TABLE Mode facility.

CHAPTER 6

CASE STUDY 2: THE CONCEPT OF GRADIENT

The background to the gradient concept is given in section 6.1, which also presents the performance of the three selected students in each learning model on the related test items in the post-test. Section 6.2 describes the expected learning outcomes in relation to the gradient concept using the graphing calculator. It begins with a brief outline of the mathematical tasks on the concept of gradient in each learning model, and follows with an explanation on how the graphing calculator is thought to assist in understanding the concept of gradient. Section 6.3 describes how the case study students in the StructLM appropriated the concept of gradient with the graphing calculator, with a conclusion of their development of the concept in section 6.4. Finally section 6.5 discusses how the case study students in the InterLM appropriated the concept of gradient with the graphing calculator, with a conclusion on their development of the concept in section 6.6.

6.1 Background

In this study, gradient is considered for the case of a line in Cartesian coordinates. There are several ways of thinking about gradient. Below are a few ways of thinking about gradient that are central to this study for the purposes of looking at which of these ideas pose greatest difficulty for students:

- (i) Looking at a line as an object with the attributes of gradient and the direction, implying that parallel lines have the same gradient. The greater the magnitude of the gradient, the steeper is the line. A horizontal line has zero gradient. The gradient is positive for a line which slopes from bottom left to right as x increases, and the gradient is negative for a line which slopes from top left to right as x increases.
- (ii) Attributing “slope-related graphical” properties to equations that have differing values of “ m ”. The idea of this property is illustrated in figure 6.01 below.



Figure 6.01 Graphs of equations with different values of “m”

- (iii) Thinking of the gradient as the ratio of the vertical change (increase in y) to the horizontal change (increase in x). If the line is drawn on the graph paper, the numerical gradient can be determined by completing the right-angle triangle method with negative sign assigned to it if the line is in the negative direction. If the line passes through any two points, say $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the line, the gradient can therefore be computed as $(y_2 - y_1)/(x_2 - x_1)$ (see figure 6.02). The order of the coordinates is important to determine the direction of the line. $(y_2 - y_1)$ and $(x_2 - x_1)$ are directed line segments and the ratio thus indicates the gradient and the direction of the line.

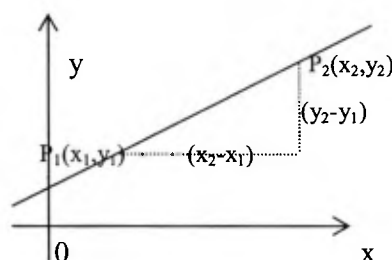


Figure 6.02 Gradient as the ratio of $(y_2 - y_1)$ to $(x_2 - x_1)$

- (iv) Thinking of the gradient of a straight line is the same at all points of the line. This means the same gradient is obtained if any two different pairs of points on the line are chosen.

The ideas of gradient in (i) and (ii) above are sometimes referred as the global or qualitative idea of the gradient involving attention to the entire graph, whereas (iii) and (iv) are referred as the local idea of the gradient involving only certain segments or parts of the graph (Moschkovich et al., 1993). It follows that to understand the concept of gradient of a line holistically, the students have to recognise the following relationships:

- (i) “m” denotes the gradient of the line in the general equation $y = mx + c$;
- (ii) “m” in the equation is the ratio of the increase in y to increase in x ;

- (iii) “m” in the equation is the same numerical gradient of the line that could be computed by using the right-angle triangle method or using the point-slope formula.

Clearly, the gradient concept is a “high density” concept where many other concepts are interrelated, and arguably may present difficulty in students’ learning. Barr (1980, 1981) noted that when gradient is presented in the decimal form, students have difficulty in considering it as a ratio, suggesting that students could have difficulty in representing the numerical gradient geometrically (or graphically). Underlying the gradient concept is the notion of Cartesian Connection²⁰. Moschkovich et al. (1993) assert that the key to interpreting the ratio $(y_2 - y_1)/(x_2 - x_1)$ for “m” graphically lies in the connection between global and local conceptions of gradient. Since the local idea of gradient and the geometric representation for the ratio “m” would be very difficult in the graphing calculator environment, this suggests that hand-graphing in the learning of the gradient concept have to be considered in students’ activities.

The following issues were examined in the post-test: whether students

- (i) recognise the connection between the varying values of “m” and the orientation and the gradient of the line (Question 16),
 - (ii) can assign the same numerical gradient to two lines parallel to each other (Question 10(c)(i))
 - (iii) can determine the gradient of the line drawn on the grid paper (Question 10(a)),
 - (iv) can identify the value of the gradient from the equation (Question 12),
 - (v) recognise that the gradient of a straight line is the same at all points of the line, and can use the idea of the gradient to decide whether two lines that looked seemingly parallel are indeed parallel (Question 9).
- (i) and (ii) can be considered as the global idea of the gradient, (iii) as the local idea of the gradient, and (v) involved coordinating and relating both local and global aspects.

It should be pointed out that only 3% of the CSMS (Kerslake, 1981) children of the same age group (14 years old) were successful in Question 9 indicating its difficulty. Tables

²⁰ Refer section 2.2.3

6.01 and 6.02 below depict the responses of the students in the StructLM and the InterLM respectively in the post-test.

Table 6.01 Students' performance on gradient in the StructLM

Name of student	Concepts/ tasks			
	connection between the varying values of "m" and the orientation and the gradient of the line (Question 16)	determine the gradient of the line drawn on the grid paper Question10(a)) and can assign same numerical gradient to two lines parallel to each other (Question 10(c)(i))	identify the value of the gradient from the equation $2y=5x-8$ (Question12)	recognise that the gradient is the same at all points on the line, and use this idea on to recognise parallel lines (Question 9)
Wan (High Attaining Boy)	Correctly matched all equations with lines of differing gradients and directions. Reason given was by looking at the gradient values of the equations.	Correctly computed the gradient of the line using point-slope formula as $-\frac{1}{2}$ and used this gradient value for the other line parallel to it.	Correctly gave the gradient of the line as 2.5.	Correctly gave the values "3" and "6" for the second triangle whose length is twice the first. Correctly answered the two lines are not parallel. Showed the gradients are not the same: $3=6/2$the other is $1\frac{1}{4}$.
Ng (Medium Attaining Boy)	Correctly matched all equations with lines of differing gradients and directions. Reason given was "the bigger the value of the gradient, the higher is the gradient".	Used the point-slope formula to compute the gradient but left out the negative sign in the final answer ($\frac{1}{2}$). Used this "incorrect" value for the gradient of the line parallel to it.	Wrongly gave the gradient of the line as 5.	Ignored the numerical value "3" as the gradient of the equation which was given as $y=3x-5$. Also ignored the value "3" given in the first diagram. Scribbled several values: 3, $\frac{6}{2}$, $\frac{5}{2}$ and $1\frac{1}{4}$
Amin (Low Attaining Boy)	Matched the equations of lines with negative gradients in the reverse order where equations of lines with higher magnitude are assigned to the lines that are less steep. Did not explain his answer.	Correctly calculated the gradient of the line as $-\frac{1}{2}$ using the point-slope formula. Did not use this value but repeated using the point slope formula to determine the gradient of the other parallel line, computed it wrongly and accepted this wrong value.	Wrongly gave the answer as -5 . The working shows: $2y=5x-8$ $y=5x-8-2$ $y=-5x$	Did not recognise the gradient from the equation $y=3x-5$. Shows confusion by giving absurd values of "4" and "7" respectively. Incorrectly answered the two lines are parallel with no reason given.

Table 6.02 Students' performance on gradient in the InterLM

Name of student	Concept/ tasks			
	connection between the varying values of "m" and the orientation and the gradient of the line (Question 16)	determine the gradient of the line drawn on the grid paper Question 10(a)) and can assign same numerical gradient to two lines parallel to each other (Question 10(c)(i))	identify the value of the gradient from the equation $2y=5x-8$ (Question 12)	recognise that the gradient is the same at all points on the line, and use this idea on to recognise parallel lines (Question 9)
Grace (High Attaining Girl)	Correctly matched all equations with lines of differing gradients and directions. Reason given is lines are steeper if the magnitude of the gradient is bigger.	Correctly computed the gradient of the line using point-slope formula as $-\frac{1}{2}$ but repeated using the point-slope formula to compute the gradient of the other line parallel to it.	Correctly gave the gradient of the line as $\frac{5}{2}$.	Correctly gave the values "3" and "6" for the second triangle whose length is twice the first. Correctly answered the two lines are not parallel. Reason given is the two lines have different gradients.
Pre (Medium Attaining Girl)	Correctly matched all equations with lines of differing gradients and directions with an absurd reason "when the value of x increases, it gets closer to the origin".	Correctly computed the gradient of the line using point-slope formula as $-\frac{1}{2}$ and used this value to determine the gradient of line parallel to it.	Correctly gave the gradient of the line as $\frac{5}{2}$.	Gave the length of the first line as "3" and the second line as "5.5" by wrongly using the visual clue, did not use the clue from the given equation and from the first diagram. Incorrectly answered the two lines are parallel and did not give any reason.
Emy (Low Attaining Girl)	Correctly matched all equations with lines of differing gradients and directions but no reason was given.	Correctly computed the gradient of the line using the right-angle triangle method and left the answer as $-\frac{2}{4}$ and used this value to determine the gradient of line parallel to it.	Did not answer the question.	Gave length of the first line as "4" and the second line as "6" by inappropriately using the right-angle triangle method. Correctly gave the two lines are not parallel but no reason was given. Many scribbles on the paper: $\frac{3}{1}$, $\frac{2}{1}$, $\frac{16}{8}$, 2.75 and evidence of using the Pythagoras theorem to calculate the length of PQ.

The results indicate that only the high attaining students had a deep understanding of the concept of gradient. Other students showed a fragmented understanding of the concept. Most students seemed to privilege the point-slope formula to calculate the gradient of the line. Only the high attaining students recognised that the gradient of a straight line is the same at all points and they were able to extrapolate it to lines that have the same gradient are parallel (Question 9).

It must be pointed out that other questions in the post-test namely Question 11, Question 14, Question 15 and Question 17 also gauged in part students' understanding of gradient. As these questions are intertwined with the straight-line concept, they will be discussed under the straight line case study but where relevant, references will be made to student's performance in these questions in this section for purpose of cross-examining their level of understanding.

6.2 Expected learning outcomes with the graphing calculator on gradient

The tasks were designed to assist students in conceptualising a global idea of the gradient concept through exploiting the facility of the graphing calculator (the GRAPH Mode, DYNA Mode, and TABLE Mode; see below for detailed explanation). Hand graphing activities were included to develop the local idea of gradient alongside the global idea of gradient. It was also to consider the aspect of geometric representation for the ratio for "m" that would be difficult in the graphing calculator environment, that is interpreting the numerical "m" in a given equation of a line in terms of its graphical representation.

In the StrucLM, the tasks involved students drawing several graphs of $y=mx$ with varying values of "m" manually and with the calculator in the GRAPH Mode as well as in the DYNA Mode. The idea of gradient was also developed in a mathematical activity that compared the characteristic of two parallel lines. Included was graphing several parallel lines and non-parallel lines with the calculator. Other tasks included determining the gradient of a line by completing the "right-angle triangle" or applying the point-slope formula, $(y_2-y_1)/(x_2-x_1)$.

In the InterLM, the task involved students reproducing the STARBURST picture (Magidson, 1992) using the GRAPH Mode. They were encouraged to explore in the

DYNA Mode for graphs of $y=mx$. The other task involved investigating families of graphs of $y=mx+c$ with different values of “m” and “c” with their graphing calculator in any mode they choose. Students also hand graphed $y=x$ and $y=2x$ and investigated how to determine the gradients from the graph they had drawn. The task also involved trying to derive the point-slope formula and applying it to determine the gradient of the line.

The GRAPH Mode of the graphing calculator enables student to see immediately the graph being generated. It was expected that this immediacy would enable student to recognise the connection between the varying values of “m” and the orientation, and the slope of the line. It was expected that by generating graphs by using equations with the same numerical value for m, students would see that the lines are parallel.

The DYNA Mode of the graphing calculator allows student to see dynamically what happens when the coefficients or parameters of an equation change. It was expected that seeing the graphs dynamically displayed one after another as the value of the coefficient changes allows student to see (i) the line as an object that have gradient and orientation, and (ii) slope as a parameter, and develop the correlation between the values of “m” and the orientation of lines of slope “m”.

It was hoped that the TABLE Mode would confront students with how the ordered pair (x, y) on the line related to the table values, and to the x and y in the equation of the line. (The TABLE Mode, as explicated in the preceding section, allows the generation of the table of values for the graph.)

6.3 Analysis of case study students in the Structured Learning Model

WAN (HIGH ATTAINING BOY)

Wan’s responses to the related questions in the post-test and the way he wrote down his observations in his worksheet indicate that he seemed to grasp both the global idea and the local idea of gradient, and was able to make the connection between them. He correctly computed the gradient of the line by selecting a segment of the line and applying the point-slope formula, which indicates a local understanding of the gradient concept. By assigning the same value of gradient to another line that is parallel to the

other line, it indicates a global understanding of the gradient concept. From both these correct answers and the reason he gave on how he matched the equations to lines of varied direction and slope in Question 16, it is reasonable to infer that he not only recognised the straight line as an object with the slope-related graphical property, he also recognised this slope-related graphical property is related to the equations that have differing “m”. By correctly giving the vertical length of “3” and “6” of another segment of the same line whose horizontal length remains the same and is twice respectively to the first triangle, it indicates that Wan recognised that the gradient of a straight line was the same at all points of the line. This answer and the correct answer he gave that the two seemingly parallel lines are not parallel by showing that their numerical gradients are not the same indicates that Wan could associate between the local and global idea of the gradient.

This case study will focus on the relationship between Wan’s understanding of the calculator’s functionality, his own mathematical knowledge and teacher intervention that seemed to support Wan’s coming to understand the different ideas of gradient and making the connection across them.

Role of the calculator

Two examples are illustrated here to show how Wan’s understanding of the different aspects of the functionality of the graphing calculator helped him to structure his thinking about gradient. In the first example, Wan is engaged in a task involving exploration of the properties of the graphs of $y=x+1$, $y=x$ and $y=x-1$ in the TABLE Mode. After generating the graphs in the INIT window, Wan deliberately chose a view-window that displayed all the three lines seemingly as one as shown below.



Figure 6.03 Graphs of $y=x+1$, $y=x$ and $y=x-1$ in two different view-windows

Wan seemed to have an expectation that if all the three lines were parallel to each other, and conjectured that a view-window whose x and y ranges were large would produce a

line that will be so close to each other that appear as one line. He used his knowledge of the view-window concept to test his conjecture. The graphing calculator acted as a supporting tool to extend and structure Wan's thinking about parallel lines. It seems to have enabled him to build an intuition that something in the "x" rather the constants (+1, and -1) that was related to the "steepness" of the line.

The second example depicted in excerpt 6.01 shows how Wan's understanding of the functionality of the DYNA Mode and ability to make sense of the graphic display influenced the way he used the graphing calculator. It also shows how the graphing calculator played a role in further developing his mathematical disposition.

Excerpt 6.01 Wan's understanding of the functionality of the DYNA Mode

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1		Wan is already operating his calculator when Teacher is still explaining how to control the "speed" of the dynamic graphs.	
5	Wan: What is this thing hah?	Wan's screen shows the graph "moving". Wan and Amin compares their display.	
9	Amin: It is moving, isn't it?		
10		Wan puts two calculators side by side, one with the text display before the command to generate the graph is executed and the other with the graphic display.	Graphing Calculator 1 Dynamic Func+Y= Y1=MX Y2: Y3: Y4: Y5: Y6: [SEL] [DEL] [TYPE] [VAR] [BIN] [RCL]
15			Graphing Calculator 2 Y1=MX Y M=2 X
19	Wan: It is actually...-1 multiply by x,	He looks at each screen as he presses the EXE button.	
20	-2 multiply by x, 1 multiply by x..., any number multiply by x.....		

The excerpt shows Wan's rapid response to the task. He seems to have had an expectation of how the menu that appeared at the bottom of the text screen worked - that they worked similarly to the other modes that he had used before.



Wan also used the calculator astutely and thoughtfully as illustrated in his action of placing two calculators side by side (line 10). It seems to indicate he was trying to relate the “M” in the text screen and the “M” in the graphic screen. In other words, it seems Wan was trying to verify what “M” signified and how it affected the graphs each time he pressed the EXE button to execute the graph (line 18). One may conjecture that Wan used the graphing calculator as a mediational tool to articulate his thinking about the relationship between “M” in the equation and the appearance of the graph.

Wan’s understanding of the functionality of the calculator in the DYNA Mode helped him discern which aspects to ignore and which to focus on in the graphic screen. The conversation between Wan and Amin when comparing their output display indicates that Wan knew different lines were generated depending on the number in front of x (line 19) whereas Amin’s focus was on the movement of the lines (line 9). The DYNA Mode facility seemed to enable Wan to act on the line as an object that changes in direction and gradient. Worthy of note was how Wan developed both the local and global idea of the gradient.

The following figure below is an extract of Wan’s sketch of the graphs of $y=\frac{1}{2}x$, $y=x$, $y=-x$, $y=2x$, $y=4x$ on the grid paper. The videotape shows Wan constructing the graphs of on the grid paper with ease. They were all correct, indicating Wan’s algebraic competency (substitution) and at the same time clear understanding how the coordinates were related to the equation of the line.

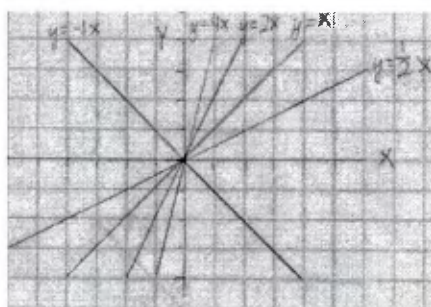


Figure 6.04 Wan’s sketch of graphs with different gradients

The correct graphs also illustrate that Wan saw the connection between the geometric (diagrammatic) representation of gradient and the numerical gradient in the equation. He was also seen graphing $y=2x$, $y=\frac{1}{2}x$, $y=-\frac{1}{2}x$ using the GRAPH Mode of his calculator to verify his construction, which also indicates his understanding of how the GRAPH Mode


functions. Wan's attempt at different constructional activities related to the idea of gradient such as the above, might have added another layer of meaning to the global aspect of the gradient and how it relates to the numerical gradient in the equation.

Role of the teacher

The following two excerpts reveal Wan's mathematical language and algebraic competency and how they influenced his interaction with the teacher.

Excerpt 6.02 The influence of Wan's mathematical language

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Try and see the difference between the two lines. The question asks what is the difference...ya?	Teacher directs question to whole class and students are keying in the equations $y=x$ and $y=x-1$ into their calculator. Teacher also intervenes Ng.	
5	How is this line different from the one we have done?		
	So let us look...okay try to describe it here. They both have the same gradients....What is the difference, first and second between $y=x$ and $y=x-1$? There are two or three....		
10	We have done both yesterday, we have drawn it on the graph paper.		
14	Wan: The position of the line.		
15	T: The position of the line, what else? The position of the line, describe it.. there are more...	Teacher asks to a group of students near Wan as well	
20	Wan: The value of y ... (pause) the value of the coordinates		
21			
23	T: The value of the coordinate, what else?		
25	Wan: It does not pass through the centre, the origin.		
30	T: What else?aaa describe it. Now if we look at these line ... we have come across it in Form 2 and Form 1... parallel lines, you have encountered them before, haven't you.... That is their similarity... parallel. The difference between these parallel lines that is $y=x-1$ and $y=x$ well what is it.. try and look carefully.	Teacher directs Wan to check the intercept on the calculator.	
32			
35			
36			
40	Wan: The equations are different		

43	T: What is important is the point of intercept. Check the intercept. Try and describe the point of intercept.		
45	What did you get?		
	Amin: The intercept is not the same.		
50	Fai: The line $y=x-1$ does not pass through the origin.		
	T: aaaa.. okay, you tell....	Teacher points to Wan's work sheet.	
55	Wan: The other one passes the origin.	Wan mumbles the response to his neighbour.	
60	T: What is the other one Wan? Your friend says that one does pass through the origin ... the other one means ...		
62	Wan: The other one intercepts at different place on the y-axis, isn't it?		
65	T: Write it in your worksheet. The important difference is the place of intercept, the location of the intercept, describe it.... What is the first one, the second one...		
70			
75	T: There are two, one is the difference and another is the similarity. What is the difference? It means.....	Teacher walks away. Wan is seen graphing: Wan writes in his work sheet that the graph of $y=x-1$ does not pass through the origin and the positions of both graphs are different. He also writes that both graphs have a gradient at 45° and are parallel.	
80			

This excerpt shows students graphing the lines $y=x$ and $y=x-1$ in their graphing calculator where they immediately saw two parallel lines being displayed. No connection to the equations was explicated. The teacher used the graphing calculator as an instructional tool to mediate the idea of gradient intuitively. Wan was observed to be participating actively. The excerpt illustrates a series of exchanges between Wan and the teacher, which developed from the position of the line (line 14) to the value of the coordinates (line 21-23), the different forms of equations (line 40), the origin (line 55), and ultimately to the idea of the intercept (line 62). Wan was able to follow through the teacher's curriculum script. The smooth flow of the curriculum script was made possible

by Wan's knowledge of the mathematical language the teacher used during the intervention for example, coordinate (line 21), parallel (line 32-36), intercept (line 43) and origin (line 59). In this intervention, it is reasonable to infer that Wan began to think of the meaning of parallel lines in terms of having the same slope. In his worksheet, he included this description of the two graphs as ".....they both have a gradient at 45^0" (line 67). Not only does this indicate Wan's precise observation of the properties of these lines, it also suggests that Wan started to construe that something in the "x" in the equation to have the "parallel effect" on the line. The graphing calculator also seems to be structuring Wan's thinking about the slope of the line with the help of the teacher.

The second excerpt shows how Wan's algebraic competency enabled him to follow through a conceptual connection which the teacher attempted during classroom teaching.

Excerpt 6.03 The influence of Wan's algebraic competency

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: What are the gradients of the lines $y=x+2$ and $y=-x+2$.	Teacher asks whole class.	
4	Wan: 1 and -1.		
5	T: What is the same and what is different between the two lines?		
9	Wan: The gradients are different		
10	and the intercept is the same.		

Wan recognised the equivalence between $1x$ and x and $-1x$ and $-x$ (line 4). He also understood the symbolic representation of "m" and of "c" in the general equation $y=mx+c$ (line 9). Wan understood the teacher's question indicating that he knew what features of the graph to attend to.

It seemed that the teacher's classroom teaching provided Wan with the critical source to make the conceptual connection between the different ideas of the gradient. In an episode where the teacher was explicating the idea that the gradient was the same at any point on the graph of a straight line using the two point slope formula $m=(y_2-y_1)/(x_2-x_1)$, Wan pointed out that the y-coordinate for the second point for the "other gradient" of the

line that the teacher gave was wrong. Wan recognised that the “other gradient” of the same line should be the same.

Wan’s learning activities show that his ability to integrate the different ways of thinking about gradient enabled him towards a full understanding of the gradient concept. His active involvement shown by contributing to the teacher’s explication guided him towards bridging the diverse forms of the gradient concept.

In conclusion, Wan’s coming to understand the different ideas of gradient and making the connection across the different ideas was based on his ability to connect the local and global ideas, which could be attributed to his sound grasp of the functionality of the graphing calculator and the related mathematical knowledge. His understanding of the overall functionality of the calculator, specifically the functionality of the DYNA Mode and the GRAPH Mode, together with his capability to make sense of the graphic display enabled him to use the calculator to mediate the global conception of gradient. Both the DYNA Mode and the GRAPH Mode enabled him to make the connection between the slope-related graphical property of the line to equations that have varying values of “ m ”.

Wan’s algebraic competency and computational skill enabled him to follow teacher’s classroom instruction, which provided him with a critical source to fill the gaps between the different notions of gradients that he was developing. His mathematical competency also enabled him to make sense of the hand graphing activities which accompanied the graphing calculator activities. Wan’s activity highlights the fact that the hand graphing which accompanied the graphing calculator activities, was necessary to have a holistic understanding of the gradient concept, and to make the connection between the geometric and the numerical gradient in the equation.

NG (MEDIUM ATTAINING BOY)

The post-test shows that Ng could match the equations of lines of differing gradients with differing directions but he could not recognise that the numerical gradient he computed was wrong for a line in the negative direction. His post-test also shows that Ng clearly did not understand that gradients on any points on the line were the same. The scribbles on his post-test paper indicates that Ng was expecting that the two

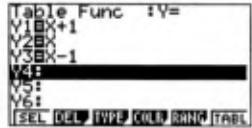
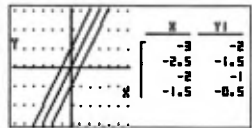
seemingly parallel lines were indeed parallel, and was confused when he worked out that their gradients were not the same. Ng also incorrectly deduced that the gradient in the equation $2y=5x-8$ was “5” indicating that Ng did not transpose the equation in the form of $y=mx+c$.

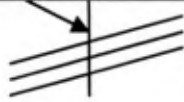
Many factors seemed to contribute to Ng’s fragmented understanding on the gradient concept. In the following excerpts, two aspects will be considered: his lack of understanding of the functionality of the calculator in general and more specifically his inability to make sense of the graphic display, and ineffective teacher intervention.

Role of the calculator

The following excerpt reveals the manner in which Ng used the graphing calculator, and in particular, the way in which his usage prevented him from making the connection between what was displayed on the graphic screen and the “m” of the equation.

Excerpt 6.04 The graphing calculator as a display tool

Line No.	Dialogue	Action	Display on Calculator/ Worksheet
1	T: What are the similarities that you get?	Teacher asks whole class to add another line $y=x+1$ to the existing ones ($y=x$ and $y=x-1$). Ng was seen graphing in the calculator. Teacher approaches Ng.	
4	Ng: The gradient.		
5	T: The steepness....		
7	Ng: The gradient.		
9	T: The gradient is.....		
10	Ng: The gradient is the same.	Teacher repeats. Ng is silent.	
13	T: The gradient is the same. What is the other one?		
15	Ng and Wan: Parallel		
20	T: It is parallel. You have identified two, what is the other one? Another one.		
22	Wan: continuous line.		
	T:...continuous, that is the gradient.	Teacher gestures with his hand like “cutting” or “axing”. Ng looks at his graphic display.	
	What is it? What else does it have?		
	Do you remember it has this?		
	Wan: the intercept.		
	T: The intercept at the y-axis, The intercept at the y-axis do not look at		

25	the x-axis...They all intercept at y.	Teacher illustrates the diagram on the blackboard.	
30		Ng writes in his work sheet: (i) They have the same gradient. (ii) They are parallel lines. (iii) All intercept at the y-axis.	

One focus of the above activity was the slope attribute of the line with lines of the same slope being parallel to each other. The other focus was on the intercept of the line. The teacher did not point out how the numerical gradient is related with the equation and where it is signified in the equation. The teacher focussed Ng on the global idea of slope (line 4, 9, 13) and the meaning of intercept (line 22). This could also be seen from how Ng correctly sketched the lines of $y=x+1$, $y=x+2$ and $y=x-1$ without an understanding that the coefficient “1” in front of “x” represented the gradient of the line. One could say that the graphing calculator was simply used as a display tool. Ng was seen drawing these graphs by looking at the display of these lines. The following graph replication illustrates the situation.

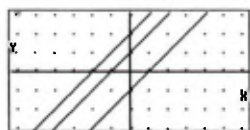


Figure 6.05 Graphing calculator graphs

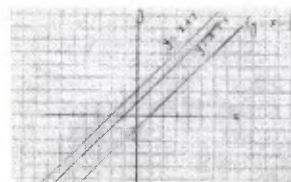


Figure 6.06 Ng's sketch

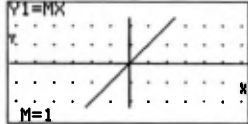
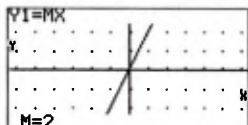
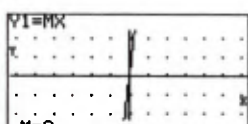
Ng's sketch of the graphs were all correct not because he understood how the “m” in the equation was related to the slope of the line, but because the grids of the calculator provided Ng the guide to draw the graphs. The graphs were also easy to sketch because some of the points of the graphs were exactly on the grids and so identifiable. It seemed the visual display of the graphing calculator enabled Ng to interpret the teacher's explication of the gradient in terms of “steepness” of the line (line 4) and lines with the same gradient are parallel (line 7-13). However, it could be said that Ng's use of the graphing calculator only as a display tool prevented him from seeing beyond what was pointed out to him by the teacher. This was evident from the equations he listed in his worksheet. He listed the equations $y=x+3$, $y=x-3$ and $y=x+2$ as belonging to a similar family, excluding a family of lines such as $y=2x+3$, $y=2x-3$, and $y=2x+3$. This indicates


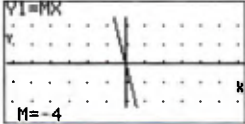
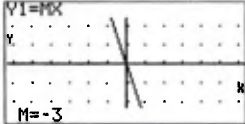
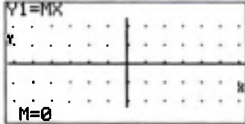
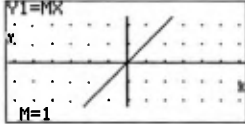
that he only developed a “static” notion of the global gradient and did not develop an intuition about how the equations were related to the graphs displayed. Notable in the interaction, Ng was silent after a few turns (from line 13 onwards). The teacher’s incomplete explanation which did not include the connection between the equations and the slope property of the line could have resulted in Ng misconceiving the “x” in the equation as the gradient.

Role of the teacher

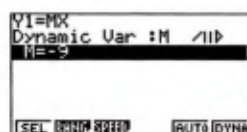
In the next excerpt Ng’s lack of understanding of the general operational syntax of the calculator affected his execution of the DYNA Mode and what he saw on the screen. It also shows how the teacher’s specific directive on which aspect of the graph to focus assisted him to make the connection between the “m” and the orientation of the line.

Excerpt 6.05 Ng’s difficulty with the DYNA Mode

Line No.		Action	Calculator Display/ Worksheet
1		Teacher instructs students to explore for the graphs of $Y=Mx$ in the DYNA Mode. Teacher explicates to students how to operate the DYNA Mode but does not explain about the range values for M.	
5			
10		Ng generates the graphs. He is pressing the EXE button. The graphs displayed were all in the positive direction for $M=1$ to $M=9$.	
15			
16		He keeps on and on.	
20			
25	T: Look at the direction of the graph.....when M is positive and	Teacher instructs to whole class. Ng looks at Wan.	
27	when M is negative.	He seems confused why he does not obtain the negative values for M.	
30			

31	T: Go to the range menu and key in the following values of m from 4 to -4 in the RANG menu.	Teacher intervenes Ng. Teacher then tells Ng to observe the screen.	
35	T:..... $4x$, $-3x$, $-2x$, $-x$, $0x$, x All these graphs pass through the origin.	As Ng presses the EXE key, Teacher verbalises-as each graph appears on the screen. Teacher also points out that these graphs pass through the origin.	
40			
45			
50			
55	T : Graphs of mx are straight lines whose gradients could be positive or negative with m positive or negative respectively.	Ng listens intently as Teacher then explains to whole class.	

In the above excerpt Ng was not able to change the range for “M” values (line 27) indicating that he did not know where to locate the menu to change the “M” values. The menu is not located on the text screen. It has to be generated from the “VAR” menu on the text screen (on the “VAR” screen, the “RANG” menu is displayed where the range of values for “M” can be specified). In the excerpt above, the way Ng generated the dynamic graph one after another by pressing the “EXE” button in turn (line 10-16) shows that Ng had changed the “SPEED” of the dynamic graphs. This means he had encountered the “VAR” screen before as shown below.



It also implies that Ng was not aware of the “RANG” menu located at the bottom of the “VAR” screen that contained commands to change the value of “M”. In other words, he did not appropriate the meaning of the “VAR” screen that clearly showed the value that “M” starts with. When the teacher intervened and asked Ng to go to the “range menu”

(line 31), Ng's actions indicated that he had pressed the EXIT button, found himself in the "VAR" screen and pressed the key for the RANG menu which, he had earlier overlooked. By pressing the EXIT button, it shows Ng knew how to get out of the graphic screen he was currently in. Had Ng pressed the EXIT button before the teacher intervened, he would be confronted with the "VAR" screen. But Ng did not. This suggests that Ng lacked the confidence to try for himself with calculator commands that he was not instructed to manipulate. It also suggests that he had blindly memorised many of the steps of the calculator operations without an understanding how they worked.

However, the teacher intervention that directed Ng to change the values in the RANG menu (line 31) did keep Ng on track enabling him to see the effects of "M" values that are negative. This intervention was very specific. The teacher made the conceptual connection between the changing values of "M" and "m" in the equation and while the teacher was verbalising, the calculator displayed the related graph (line 35-55). Notably, the teacher used the calculator as an instructional aid to support the explication of the connection between the "direction-related" graphical property of the line and the "m" in the equation of the line $y=mx$. However, there was no mention of the gradient. Ng did not make the connection of "m" in the equation to the slope-related graphical property. The absence of the teacher's explanation of this idea of the slope could have precluded Ng from making the connection. This could be observed from his drawing of the graphs of $y=2x$, $y=\frac{1}{2}x$, $y=-1x$ and $y=4x$ on the grid paper. They were not all correct. Graphs of $y=4x$ and $y=\frac{1}{2}x$ were labelled in the reverse order and incorrectly drawn as shown in the figure below.

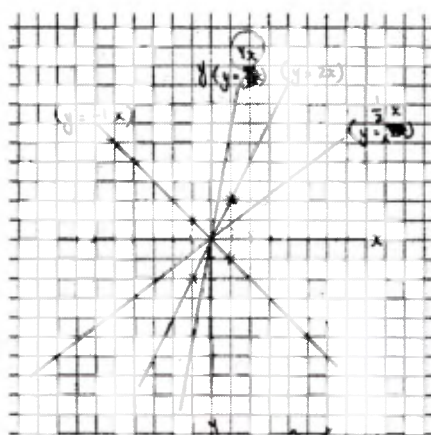


Figure 6.07 Ng's hand draw graphs of $y=mx$

However, the teacher drew to Ng's attention the mistake and asked him to check the graphs with his calculator. Ng executed the graphs in his calculator in the GRAPH Mode and then relabelled the graphs of $y=4x$ and $y=\frac{1}{2}x$ but did not correct the wrong coordinates he had chosen to draw on these graphs. Here again is evidence that Ng used the calculator according to teacher's instruction and it was not surprising that he did not see beyond what he was asked or discover for himself features of graphs that were not explicated to him. The DYNA Mode had not assisted Ng to associate "M" with the slope of the graph. It was the graphic display in the GRAPH Mode displaying all the graphs on the same screen that afforded Ng to see to the extent that the "m" in the equation of the form $y=mx$ affected the slope of the line where the greater the gradient, the steeper is the line.

In conclusion Ng's lack of understanding of the functionality of the calculator in general affected the manner in which he used the graphing calculator, mostly to carry out the procedures of the task. It also prevented him from understanding how the DYNA Mode worked as there are various menu keys contained in the DYNA Mode text screen, which resulted in him to missing the connection between the "m" in the equation $y=mx$ and the slope-related graphical property of the line.

Teacher intervention seems to be an important aspect of Ng's making sense of what was displayed on the graphic screen and the mathematical ideas embedded in the display. Ng not only depended on the teacher's instruction of what to observe on the screen but he also relied on the teacher's exposition of the operational commands of the calculator that were related to the activity if they were different (new) from that he had encountered in his previous activities. However, the inconsistent way in which the teacher intervened did not help Ng to make sense of the graphic display and the mathematical ideas underlying his activities. The teacher sometimes provided the instruction as to which command key to press without ensuring Ng knew why the command was executed although the teacher explicated what was generated on the screen. Vice versa, sometimes the calculator command was explained without relating it to the graphic display it produced.

AMIN (LOW ATTAINING BOY)

Amin's understanding of the gradient was obscure and at times ambiguous as indicated in his performance in the post-test. In Question 16, Amin matched the equations of the lines of $y=mx$ that have negative gradients in the reverse order. Equations of lines with higher magnitude are assigned to the lines that are less steep as shown in the extract below.

16. Padankan persamaan-persamaan yang diberi dengan graf-graf di rajah berikut:

$y=5x$ A

$y=3x$ B

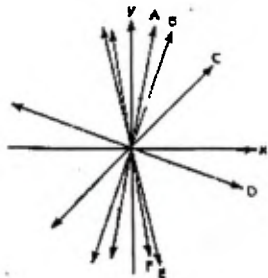
$y=-4.6x$ E

$y=-\frac{1}{3}x$ F

$y=1.1x$ C

$y=-6x$ D

Terangkan jawapan anda.



Although Amin seems to recognise the positive and the negative sign of the coefficient denotes the direction of the graph, he confuses between the magnitude of the coefficient of x that designates the slope attribute and the sign that designates the direction attribute. Amin generalised the idea of “the greater the coefficient, the steeper is the line” for the lines in the positive direction to the lines in the negative direction.

Amin's response to Question 9 and 12 in the post-test indicates that he did not recognise the gradient of the line from the equation. Although he correctly calculated the gradient of the first line, he continued to calculate the gradient for the second line that is parallel to it. However, he correctly drew the straight line parallel to the line $y=2x-5$ in Question 17! His incorrect answers in Question 9 also indicate that he did not grasp the idea that the gradient is the same at any point on the line.

First, it would be relevant to look at Amin's level of algebraic competency. He had very poor algebraic skills. The pre-test shows that he attempted only one of the eight questions that required him to transpose the equation by making y the subject of the equations presented. In attempting to identify the gradient for the equation $2y=5x-8$ in Question 12, in his post-test, he gave the incorrect answer as “-5”. The following extract shows how he attempted to solve the problem.

$$\begin{aligned}
 2y &= 5x - 8 \\
 y &= 5x - 8 - 2 \\
 y &= -5x
 \end{aligned}$$

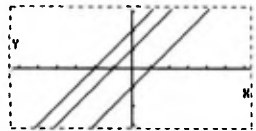
It clearly shows that Amin had difficulty in his algebra on the one hand, but on the other hand, it shows that he was aware that he had to transpose the equation into the form $y=mx+c$.

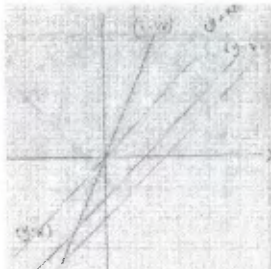
Amin's fragmented understanding of the gradient concept could be tracked through the nature of teacher intervention and the difficulty he had in interpreting the graphic display of the calculator.

Role of the teacher

The following excerpt shows how the teacher's ineffective intervention hampered Amin from making the connection between the numerical gradient in the equation and the slope-related graphical property of the line.

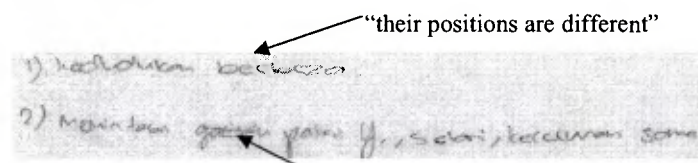
Excerpt 6.06 Teacher's ineffective intervention

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Where is the gradient in these equations?	Amin is graphing for the equations $y=x+1$, $y=x+2$ and $y=x-1$ Teacher instructed. Amin is quiet.	 <p>(This is probably the display as these were the graphs instructed by the teacher and the teacher's proceeding questions in line 14)</p>
5	T: Look at the three equations. What is the same in these equations and what is different?	Amin remains quiet.	
6	Amin: Only x is the same.		
7	T: plus 1, plus 2 and negative 1...where are they in your graph?		
10	T: Can you trace them? T: Try...where is trace?	Teacher points to the SHIFT Trace button on Amin's calculator. Amin remains passive.	
15	T: $x=0$, what is $x+1$...what is y ?... $x=1$, what is y ? T: eem, can't you? Try to write it down...where is it that says $y=x+1$...where?	Teacher waits for Amin to manipulate his calculator.	
20	T: Determine which is $x+1$ first....you have learnt to trace just now.	Teacher gestures with his hand, points at Amin's calculator and leaves without waiting to see what	

25		Amin does. Amin does not carry out Teacher's suggestions.	
30		Teacher later asks students to sketch these graphs and Amin was seen looking at his calculator and sketching his graphs from it.	 <p>Amin's sketch graphs</p>

The graphing calculator allowed Amin to see that the graphs of $y=x+1$, $y=x+2$ and $y=x-1$ are parallel. But Amin had no conception of how the equation could inform him of the gradient of the line (line 6). Those equations with gradient “1” present an ambiguous meaning as the value of “x” in these equations is “invariant”. Amin’s answer indicates that he incorrectly deduced that “x” is the gradient (line 6). In his subsequent work with perpendicular lines discussed in the next section Amin declared that there is no “m” in the equations $y=x+2$ and $y=-x+2$. The teacher ignored Amin's misconception about the numerical gradient signified in the equation, and gave no corrective feedback, but instead moved to the other aspects of the graph (line 7). This is followed by a series of questions with no expectation of Amin to respond. Amin remained passive throughout the interaction. Notably in the intervention, the teacher only asked in general where the “+1, +2 and -2” were located in the graphical space (line 7). As the task was to induct students to develop an intuition of the line as an object with some properties the teacher made no attempt to explicate the conceptual connection between the numerical gradient in the equation and the slope-related graphical properties of the graph. However, as shall be seen later, Amin’s misconception that “x” is the gradient persisted and hindered him from making the connection between the numerical gradient in the equation and the slope-related graphical property. The intervention ended with the teacher instructing Amin to trace the graphs but the teacher did not wait to see how Amin carried out the task. The excerpt shows that Amin did not even pursue the tracing task and the intervention seems incomplete.

Amin's drawing of the graphs (line 30) shows incomplete sketches, with only the graphs of $y=x$, and $y=x-1$. The other parallel line was unlabelled, and a graph of $y=2x$ presumably was sketched in an activity after this. These sketches evidently were produced from what he saw in his graphing calculator as the following was what he wrote in his worksheet about their differences and similarities respectively.

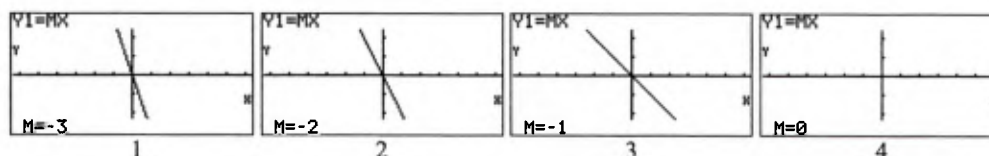


“they intercept at the y-axis, parallel and have the same gradient”

This description fits the global aspects of the line. In other words, he saw the graphs as objects with some attributes including the slope attribute. Amin was able to deduce that these graphs have the same gradient, but they had no connection to the numerical gradient in the equation. This perhaps explains why he could draw a line parallel to a given line as in Question 17, but was not able to deduce that the numerical gradient obtained from the point-slope formula was the same numerical gradient for any other line parallel to it. Amin went on to give three other hypothetical equations whose graphs will be parallel to these graphs: $y=x-2$, $y=x+3$ and $y=x+4$ with the misconception that “x” attributes the slope to the lines.

Role of the calculator

An area of genuine confusion for Amin was the interpretation of the graphic display. Figure 6.8 shows Amin was observed working in the DYNA Mode, indicating that he understood how to enable the DYNA Mode. However, the dynamic display was another issue, and although Amin remarked to Wan that he saw the lines were moving, Amin was unclear about the meaning of the screen display (see excerpt 6.1, line 9). The following series of graphic display depicts what Amin saw on his graphic screen.



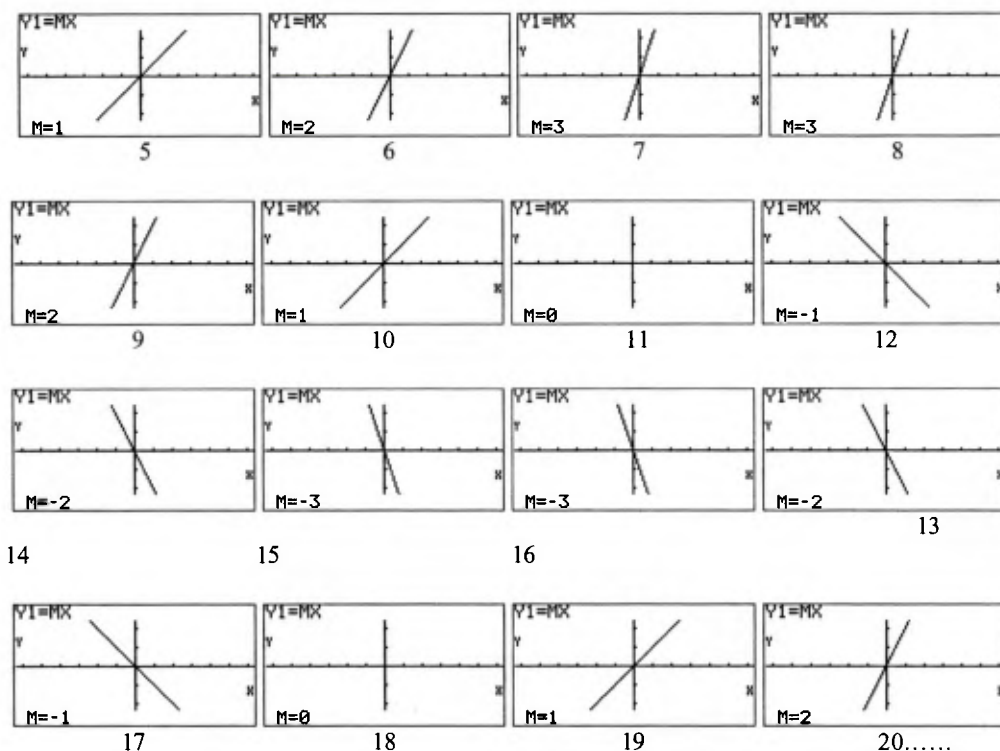
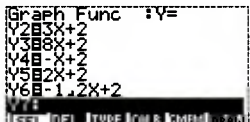
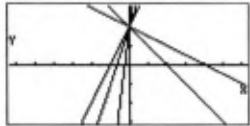


Figure 6.08 Graphs of $y=Mx$ generated in the DYNA Mode for range of values of “M” between -3 and 3

Note that the graphs that were being generated dynamically in Figure 6.08 apply for the values of “M” between -3 and 3 . It seems the dynamic generation of each graph one after another gave Amin the visual illusion that the DYNA Mode set the lines in motion with the direction changing. Thus, instead of focussing on the change in slope or the “steepness” and the direction of the line as the value of “M” changes, the attention was on the movement of the line. Amin’s peer, Wan commented that what was happening was due to the change of coefficients in front of “x” (also from excerpt 6.01, line 19). This might have made Amin aware of the effect of the coefficients on the orientation of the line but it is doubtful whether he discriminated which sign corresponds to which direction of the line. This further shows that Amin’s misconception of “x” being the gradient of the line hindered him from thinking of how the coefficient in front of “x” influenced other characteristics of the line besides its orientation, specifically its slope.

The following excerpt shows how Amin’s difficulty in interpreting the graphic display affected how he used the graphing calculator and his knowledge construction.

Excerpt 6.07 Knowledge compartmentalisation arising from difficulty in making sense of graphic display

Line No.	Action	Calculator Display/ Work Sheet
1	Amin keys in the equations and generates the graphs as requested by the Teacher.	 <p>(From the display below this is the text screen)</p> 
5		
10	When the graphic display is produced, he waits for further instruction.	

This excerpt shows Amin graphing $y=2x+2$, $y=3x+2$, $y=8x+2$, $y=-x+2$, $y=2x+2$ and $y=-\frac{1}{2}x+2$, following the teacher's instruction. In his worksheet, Amin concluded that these graphs have the "same y", describing literally what he noticed on the screen - "y" as the intercept value. His description indicates both his use of the graphing calculator as a display tool and his inability to make sense of the graphic display. That is, he fixed on one aspect of the graph only, and excluded other aspects that were not apparent to him, or when not accentuated explicitly. Notably here was his inattention to the gradient. The graphs with the same intercept presented a different context for him. It also clear that Amin did not make the connection between the text screen which he keyed the equations and how the equations were related to the graphs, indicating he did not make the connection between the slope-related graphical property of the line and the numerical gradient of the equation of the line. Given the difficulty Amin has with his algebra and computation, it is not surprising that Amin did not understand that the gradient of the line was the same at any point on the line from the teacher's classroom explication.

In conclusion, Amin's fragmented understanding can be tracked to his lack of the necessary mathematical knowledge related to the task, his difficulty in making sense of the graphic display, and ineffective teacher intervention. Although he knew how to generate the dynamic graphs in the DYNA Mode, his inability to make sense of the graphs that were generated prevented him from understanding the mathematical ideas underlying the button pushes that produced the graphic display. Amin's activities show that the dynamic generation of graphs in the DYNA Mode could give rise to visual illusion of the graphs as moving as "M" changed. Amin's difficulty in making sense of

the graphic display resulted in concentrating on only certain features of the graph that was displayed on the screen. His lack of the related mathematical knowledge also influenced the way he structured his learning, which was disorganised, resulting in many uncompleted tasks, evident from his worksheets where there were many blank spaces of unanswered questions or unwritten observations.

What emerges from Amin's learning was the importance of the teacher intervention to direct his attention when focussing on the graphic screen in order to make sense of the graphic display. The teacher's ineffective interventions, which among others included incomplete explanation and a lack of corrective feedback, prevented him from making the connection between the numerical gradient in the equation and the slope-related graphical property of the line. Amin's activities also highlights the fact that using equations such as $y=x$, $y=x+1$, and $y=x-1$ could give the misconception that the invariant "x" is the entity that governs the gradient of the line.

6.4 Conclusion on the concept of gradient in the Structured Learning Model

1. The complexity of the gradient concept involving both the local and global aspect means the role of the calculator is very significant. The display on the calculator without the numbers means hand-graphing activities to support the global aspect of the gradient is equally important as the hand-graphing activities support the local aspect of the gradient. An understanding of why the graphs displayed on the calculator screen and those reproduced on the graph or grid paper are the same or different is important to help middle and low attaining students make the connection between the two displays.
2. Interpreting the graphic display is an important aspect to avoid attention on certain features of the graph only, which could result in compartmentalization of knowledge construction.
3. Dynamic images need careful interpretation. The dynamic graphs generated in the DYNA Mode can give rise to visual illusions that the lines are moving or turning as the value of "M" changes. Students can also miss the connection

between the changing values of “M” and the steepness and orientation of the lines.

4. Medium and low attaining students need careful intervention by the teacher to direct them on specific features of the graphic display and how they are connected to the task and equations that produce them.
5. Teacher interventions and classroom instructions provided the high attaining student with a critical source in making the conceptual connection between the different ideas of the gradient concept that he was developing.
6. The high attaining student benefited from the teacher’s curriculum script as he understood the mathematical language and the mathematical concept that the teacher presented.
7. Operating the DYNA Mode requires an understanding of the general functionality of the calculator which includes an understanding how the various menus are located in the various screens.
8. The GRAPH Mode of the calculator supports the global conception of the gradient, in particular parallel lines have the same gradient but students do not necessarily refer this gradient as the numerical gradient in the equation.

6.5 Analysis of case study students in the Interpretative Learning Model

GRACE (HIGH ATTAINING GIRL)

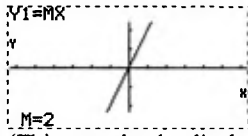
Grace answered correctly all the questions related to the gradient concept in the post-test. Her working in her worksheet indicates that she understood both the local and the global concept of gradient. She was also able to make the connection between slope-related graphical properties and equations that have different values of “m”. She also understood that the gradient is the same at any point on the straight line.

Two key issues are examined to illuminate how Grace developed both the local and global aspect of gradient and made the connection between them, namely the role of the graphing calculator and the role of hand graphing.

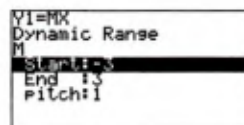
Role of the calculator

The following excerpt illustrates how Grace's understanding of the functionality of the DYNA Mode enabled her to develop an intuitive knowledge of the line as an object with the "slope" attribute. It also shows a supporting role played by the teacher.

Excerpt 6.08 Grace's understanding of the DYNA Mode

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: $y=2x$, how does the line $y=2x$ looks like?	Grace is exploring in the DYNA Mode for graphs of $y=Mx$ when Teacher intervenes.	 <p>(This must be the display as evidenced from the conversation).</p>
3	Gr: $2x$? $M=2$?	Grace seeks confirmation and displays the graph.	
5	T: Yes. What does it mean...change in M ?	Teacher confirms. Teacher points to the M on Grace's screen. Teacher tries to advance Grace's conception of gradient by questioning, followed with a hint.	
7sla...(kecon..)	Grace picks the hint and completes the word.	
10		Teacher inducts Grace to the terminology.	
11	Gr: slant (kecondongan)	Grace repeats and nods her head.	
13	T: M is the slant or gradient.		
15	Gr: Gradient.		

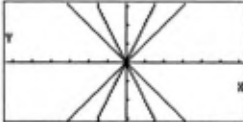
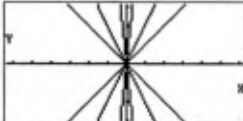
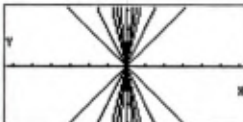
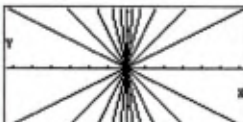
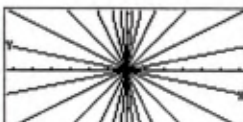

It could be construed here that Grace was using the calculator as a mediational tool as she was aware that " M " relates to the coefficient in $y=2x$ (line 3). The dynamic display where the appearance of the line changed as " M " changed in value facilitated Grace to think intuitively of " M " as the "thing" that governs the "steepness" of the line. This is evident from Grace's ability to complete the teacher's phrase (line 7). This intuition, however, was built upon Grace's ability to discern what was displayed on the graphic screen as opposed to merely seeing the graphs as moving, which was a consequence of her knowledge of the functionality of the DYNA Mode. Grace was conscious that each time she pressed the "EXE" button, she was changing the value of " M ", and this " M " was related to the range of values she had previously defined to generate the graph:

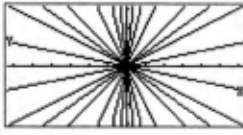


The teacher's confirmatory feedback helped Grace to establish what "M" signified and to see it as the entity that affected the slope of the line. The teacher also inducted Grace to use the terminology "gradient" (line 13).

Grace's understanding of number concepts and algebraic knowledge helped her to structure her tasks. The next excerpt shows how the graphing calculator was used to support her thinking about the gradient, and to interpret the teacher's hint.

Excerpt 6.09 The influence of Grace's mathematical knowledge

Line No.		Action	Display on Calculator/ Worksheet
1	T: Try and graph $y=x$, $y=-x$, $y=2x$, $y=-2x$, ...	Grace graphs the lines suggested by the Teacher.	
5		She then graphs for $y=9x$ and $y=-9x$.	
6			
10		She repeats with $y=4x$ and $y=-4x$, and then with $y=6x$ and $y=-6x$ generating the graphs after each turn.	
12			
15			
17		She later includes decimals: $y=0.5x$, $y=-0.5x$.	
20		She then includes $y=0.2x$, $y=-0.2x$. She explains what she did to Chan. She also passes her calculator for other students to see.	
25			
30			
31	Gr: Look, this is x , $-x$, $2x$, $-2x$, $4x$, $-4x$, ...you have to include decimals too.	She deletes $y=6x$ and $y=-6x$ and replaces with $y=0.7x$ and $y=-0.7x$. When she obtains a horizontal line display, she immediately knows she has made a mistake and corrects it to	
35			

40		$-0.7x$. She continues her work with $y=0.4x$, $y=-0.4x$, $y=0.8x$, $y=-0.8x$. She adjusts the input equations until she obtains the Starburst picture.	
----	--	---	---

The teacher's instruction to the whole class to key in the graphs of $y=x$, $y=-x$, $y=2x$ and $y=-2x$ in the GRAPH Mode provided Grace with the hint that $y=-x$ and $y=-2x$ were reflections of the graphs of $y=x$ and $y=-2x$ respectively (line 1). The graphic display indirectly provided the visual clue as Grace was seen subsequently keying in the equations $y=9x$ and $y=-9x$ and generating the graphs (line 6).

Grace was seen repeating a few more equations, and the graphic display showed that they were gaps under the line $y=x$ and $y=-x$ (line 12). She tried for values less than 1 indicating that Grace knew that the coefficients for $y=x$ and $y=-x$ were 1 and -1 respectively (line 17). She tried several decimal values and at one point when she saw a horizontal line being generated, she knew that she had not keyed in the "x" in the equation (line 31), which she subsequently deleted. Grace's actions show how the calculator acted as a mediator to support her conceptualisation of the gradient concept. The calculator enabled Grace to generate the graphs easily and quickly enabling her to observe how the parameter in front of x affected the orientation and gradient of the graph. Grace's understanding of decimal also enabled her to refine her STARBURST picture. The GRAPH Mode facility enabled Grace to concretise her intuition of the "M" in the equation, which was also facilitated by her algebraic knowledge of equivalence, negative numbers and decimals.

It must be pointed out too that Grace was observed graphing several lines of $y=mx+c$ that have the same "m" in the GRAPH Mode. Given her ability to interpret the graphic display, this must have helped her to develop an intuition towards seeing how the same "m" affected the slope of graphs that were drawn at different positions in the graphical space. It is reasonable to infer that the global conception supported by the graphing calculator enabled Grace to extend her knowledge of the gradient of a line passing through the origin ($y=mx$) to the gradient of a line that did not pass through the origin ($y=mx+c$).

In conclusion, Grace's understanding of the gradient concept could be attributed to her understanding of the functionality of the calculator in general, and the related mathematical knowledge including algebraic equivalence, decimals and negative numbers. Her knowledge of the functionality of the DYNA and GRAPH Modes, and her ability to make sense of the graphic display enabled her to use the graphing calculator as a tool to support her thinking. The DYNA Mode enabled Grace to build an intuitive insight of the straight line with "attributes". The GRAPH Mode assisted Grace to concretise the intuition by establishing the gradient "m" of the line as the entity that governed the slope graphical property of the line. Her firm grasp of negative numbers and decimals enabled Grace to extend her exploration with the graphing calculator in the GRAPH Mode and structure her thinking about how the "m" affected the orientation and the "steepness" of the line. Teacher intervention, both whole class and individual played a role in inducting Grace into new terminology, gradient and its meaning of "steepness".

PRESHA (MEDIUM ATTAINING GIRL)

Presha's understanding of the gradient concept was fragmented, as her post-test results indicate that although she could deduce the gradient of the line from the given equation $2y=5x-8$ correctly, she was not able to recognise the gradient for the line $y=3x-5$ in Question 9. The right-angled triangles drawn in the figure seemed to distract her attention, indicating the fragility of her conceptions of gradient. Presha did not understand that the gradient is the same at all points on the line although she could successfully calculate the gradient of lines drawn on grid paper using the right-angle triangle or the point-slope formula.

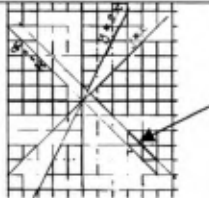
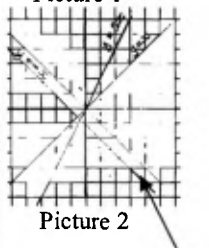
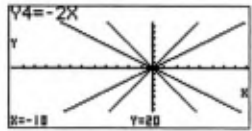
While Presha seems to understand that the parameter "m" governs the slope of the line (with the greater magnitude having a steeper slope and the negative value denotes the line with a downhill slope), her answers on questions involving parallel lines were ambiguous. She did not recognise that graphs with different gradients in Question 9 are not parallel but she was able to draw the parallel line passing through the origin and parallel to the given line in Question 17, indicating a global conception for gradient.

A key focus in examining Presha's fragmented conception of the gradient concept will be the teacher intervention - how it shaped Presha's understanding of the functionality of the calculator and how it impinged on her development of the gradient concept.

Role of the teacher

This excerpt shows several aspects of ineffective teacher intervention and how Presha used the calculator in her activities.

Excerpt 6.10 The graphing calculator as a display tool

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Presha, try to draw $y=-2x$ on the blackboard.	Pre checks the graph in her calculator and then goes to the draw on the blackboard incorrectly! She draws a line parallel to $y=-x$.	 <p>Picture 1</p>
5	Pre: Wait...		
	T: Do you want to correct your graph?	Pre labels the graph as $y=-2x$!	 <p>Picture 2</p>
10	T: Your graph is wrong.	Teacher tells explicitly. Pre proceeds to redraw another line at the bottom!	
15			
16	T: Look here..this is $y=x$. ..one	Teacher points to the graph $y=x$.	
	T: $y=2x$..What is the number here?	Teacher points to the graph $y=2x$	
20	Pre: 2		
22	T: If it is $3x$, where is the line? Is it here? Right?	Teacher gestures the line $y=3x$ on the graph. Pre nods.	
25	T: Now look here...-1..this is $y=-1x$, where is $-2x$?	Pre is silent.	
26			
27	T: Go and look in your calculator..differentiate which is x , $-x$, $2x$ and $-2x$...Use Trace.	Pre returns to her table, graphs the lines and seen tracing the line $y=-2x$.	
28			
30			
32	Pre: Grace, what are these lines?	Pre asks Grace while she is tracing.	
34	Gr:any numbers of x ...	Grace points at the graphs.	
35			
36	Pre: How do we go back ?	Pre asks how to return to the text screen. Grace shows the button to press.	
37	Gr: Press here.		
39	Pre: How to get the ones here?	Pre points to the region.	
40	Gr: Choose negative decimals...		

	T: Presha, let me see....	After a while, Teacher returns to examine Presha's Starburst.	
--	---------------------------	---	--

This excerpt suggests that the DYNA Mode used in her preceding activity only led Presha to think about the line as an object with the direction attribute, but not the slope. Presha incorrectly drew the line $y=-2x$ as parallel to the graph $y=-x$ on the black board, despite having generated the graphs (picture 1) in her calculator. Presha was also not aware that the line passed through the origin. During the earlier part of the lesson, when the teacher asked the whole class what they observed as the value of “M” changes, Presha responded that she saw the line “rise and fall”. It was probably only later when the teacher pointed out to the whole class to observe the changes in the value of “M” when each graph was generated, that Presha came to realise it affect on the direction of the line. Apparently, Presha did not see what she was expected to see on the screen. Presha was also observed to try to produce the STARBURST picture in the DYNA Mode although she saw only one line being generated at a time as the “dynamic variable” changed. All these suggest that Presha’s operation of the calculator procedurally resulted in her using the calculator as a display tool to provide visual display, with no understanding of how the graphs were being generated. It also resulted in Presha fixing on certain aspects of the graph, in this case the direction of the graph, thereby missing the point that all graphs $y=Mx$ pass through the origin.

It is apparent in the above excerpt that the teacher was constructing a curriculum script to direct Presha to the numbers in front of x and the influence they had on the appearance of the (line 16-26). The teacher was also quick to give the answer (line 22-25), allowing Presha little opportunity to suggest the answer herself. When Presha could not correct the graph of $y=-2x$ and remained silent, the teacher was aware that Presha did not understand and decided to stop the intervention (line 28). However, the teacher did not give Presha the correct answer but suggested to Presha to continue her work with the calculator (line 28-30). It seems the teacher was unsure on how explicit her explanation should be, and at the same time had a desire for Presha to discover for herself the concept of the gradient. What is clear is that this kind of intervention, that was not specific on what features of the graph to look at, was not helpful in Presha’s learning, as is evident

from Presha's confusion on what lines were being generated on her screen (line 32). It resulted in Presha seeing the graphs as static displays with no connection to the equation.

Notably, Presha's knowledge of the general functionality of the calculator at this stage was still lacking, as observed from the display where the y-axis has shifted to one side of the screen (line 28), and her uncertainty on how to return to the text screen (line 36). The later part of the excerpt shows Presha asking questions to Grace, which could be taken as indication that Presha was gradually trying to make sense of the graphical output. It is suspected that Presha began to associate the coefficient of x with the slope of the line from Grace's answer to Presha's questions (line 32-34, 39-40). Grace's explicit direction on which button to press on the calculator (line 37) maintained Presha's engagement in the task.

Examination of her work sheet indicates that Presha did not complete her work involving the STARBURST picture. It shows that allowing Presha to explore with the graphing calculator to discover the gradient concept herself consumed a lot of time but examination of her subsequent work noted that Presha explored parallel lines in the GRAPH Mode of her calculator. She concluded that the lines $y=2x+0$, $y=2x+3$, $y=2x-3$ and $y=2x+4$ were parallel because their gradients were the same and the graphs of $y=3x+2$, $y=4x+2$ and $y=-4x+2$ were not parallel as their gradients were not the same. It seems to indicate that the graphing calculator did enable Presha to see how the slope of the line was related to the numerical gradient in the equation of the line. This is discussed in greater detail in the next section.

It must also be pointed out that Presha missed the session on hand graphing of the graphs of $y=x$ and $y=2x$, and the investigation on how to determine the gradient of each graph by finding the ratio of the difference in y to the difference in x for each line. Therefore, it is most probable that the lack of hand graphing did have an impact on Presha not understanding that gradients at any point on the line are the same.

In conclusion, Presha's partial understanding of the gradient concept typically evolved from her lack of understanding of the overall functionality of the calculator, her inability to make sense of the graphic display, both aggravated by the ineffective teacher intervention. Her lack of knowledge of the functionality of the DYNA Mode and her

inability to make sense of the graphic display predisposed her to seeing the graphs as “rising and falling” instead of correctly seeing them as a line whose gradient and orientation changes as a result of the changing values of “ m ”.

Presha’s activities show that teacher intervention was necessary in her concept development, in particular towards helping her to make sense of the graphic display, and guiding her to understand the functionality of the graphing calculator. Ineffective teacher intervention led her to many syntactic and semantic errors. With the help of her peer, the graphing calculator in the GRAPH Mode did enable Presha to relate the coefficient of x with the slope of the line. Presha’s activity also highlights the teacher’s lack of knowledge of the kinds of difficulties that students have when working with the calculator in certain modes, and how to deal with students’ syntactic and semantic errors.

EMY (LOW ATTAINING GIRL)

It is encouraging to note that Emy was able to answer some of the questions involving the gradient concept in the post-test. By correctly assigning the same numerical gradient to two parallel lines and correctly matching all equations with lines of different gradients and direction, they indicate her understanding of the global idea of gradient. Her answer to Question 9 indicates that she could calculate the gradient of the line using the right-angle triangle method. Emy also recognised the need to assign the negative sign if the line has a downhill slope from left to right.

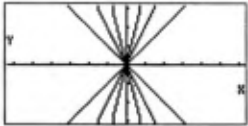
However, in Question 9 Emy did not recognise that the gradient of the line was “3” from the equation. The triangles that were drawn on the line seemed to have more impact on Emy’s attention. Her working shows that she was somewhat confused and clearly she did not understand that the gradient is the same at all points of the line, but she was able to recognise another line is not parallel to a given one if the gradients are not the same.

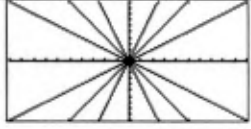
Emy’s case study will centre around the manner teacher intervention impinged on her development of graphical concepts, which interfaced with Emy’s mathematical knowledge and her knowledge of the functionality of the calculator.

Role of the calculator

An observation of Emy working in the DYNA Mode indicates that Emy did not have problems with the operational syntax of this Mode. It seems that the teacher's explicit classroom instruction on how to work in the DYNA Mode, which included questioning whether students had successfully executed each command after each instruction, enabled Emy to work in the DYNA Mode. However, Emy was observed having difficulty in making sense of the dynamic graph being generated on the screen. When the teacher asked the whole class why the line changed and what happened each time the EXE button was pressed, Emy described that the "number" had changed. Although the teacher later explained to the students that when $M=-3$, the equation $Y=Mx$ became $Y=-3x$, it seems Emy did not make the connection between the "M" and the appearance of the line on the graphic screen, as the teacher made no reference to how the "M" was related to the graphic display. This was evident in the activity to produce the STARBURST picture using the GRAPH Mode, as shown in the following excerpt.

Excerpt 6.11 The role of the GRAPH Mode

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	Emy: Mine is not good.mine is not the same. T: These are on STD	After exploring the DYNA Mode, Emy is trying to reproduce the Starburst using the GRAPH Mode. Emy and Nurul ask for intervention. Teacher points to Nurul's calculator. (Emy is using the INIT view-window).	 <p>Emy's display in the INIT window</p>
5	Nurul: Mine is the same with Zulfah. T: Yes, both are on the STD. Put in the equations...just now until Y4, now do Y5 (to mean the 4 th and the 5 th equation)	Teacher has earlier suggested to key in: $y=x$, $y=-x$, $y=2x$, $y=-2x$	
10	Zulfah: What equations do we put in? T: Whatever.		
15	Emy: Do we key in $4x$ and $-4x$? T: Yes, until 20	Teacher quickly interjects.	
20	T: Record all the numbers that you have done... what are you doing there? Why α , α , α ,... we do not want that.	Teacher instructing Kath and Zulfah at the same time.	
22	Emy: I cannot....	There are gaps in Emy's picture, despite generating in the STD window.	

25	T: Work with fraction, or points... do what you like, just put a number.	Teacher gives suggestions to work with fractions or decimals.	
	T: You did not get?	Teacher instructs students to make adjustments to the input equations after observing the display.	
29	T: Look at the star... which direction does it go... then you adjust.....	Teacher looks at Nurul's screen and asks her to choose points (to mean decimals)	
30		Emy is not sure what to do and tries to attract Teacher's attention.	
	T: That is why Nurul, haa... choose points...		
	Is that a star?		
35			
36	Emy: Draw eee?		
	T: You can adjust the cursor.		
40	Emy: Huush....	Emy exclaims and Teacher looks at her screen.	
	Zulfah: Wow! It didn't turn out.	The whole group of girls laugh. Teacher also laughs and looks at Emy's screen.	
45	T: Try again.	Teacher turns to look at Emy's screen again.	
	Nurul: What else do we do?		
	T: We want to produce a star		
50			
51	T: Look at the changes....change the value of M, aa... Observe what you change.	Teacher instructs Emy to change the value of M.	
	Zulfah: This sits in the middle.		
55	T: I know... but what did you change? What did you change now, hah... Zulfah?	Emy is quiet.	
	Zulfah: The number...		
	T: What number?		
60	Zulfah: The number.		
	T: What did you replace the number with?		
	Zulfah: positive..		
	T: Yes... but look, what is the equation? What is y equal to?		
65	Zulfah: x	Emy remains quiet.	
67	T: $y=Mx$... the basic equation		
	Now you change... the value...		
	Nurul, what is it Nurul? Which value did we change.... Observe what happens when "M" changes.	Emy remains quiet	
70			

Emy did not understand how the view-window influenced the appearance of the line (line 1). Her display was generated in the INIT window, whereas her peers generated theirs in the STD window. She still could not understand why there were gaps in her display despite graphing $y=x$, $y=-x$, $y=2x$, $y=-2x$, $y=4x$, and $y=-4x$ in the STD window (line 22). This could be interpreted as Emy using the graphing calculator to carry out procedures of the tasks. Her question to the teacher whether to key in $y=4x$ and $y=-4x$

(line 15) indicates that she was unsure which types of equations to key in. This uncertainty suggests that Emy did not make the connection between the goal of her activity and the role of the graphing calculator in her activity. Emy's use of the graphing calculator as a tool to carry out procedures of the activity was also reflected in her uncertainty on what to do (line 36).

Emy's response that she did know how the gaps were generated in her picture (line 22) indicates that she did not understand how "M" influenced the slope of the line from her previous activity with the DYNA Mode, although she could successfully generate the dynamic graphs. The teacher did not focus specifically on what to observe on the screen. The teacher did suggest to the students to work with decimals (line 25), but did not make clear what happened to the slope of the line when the parameters were in decimals. The teacher provided further general hints to "look at the changes... as M changes" (line 51) and to "observe what happens when the "M" (in the equation $y=Mx$) changes" (line 67) without specifically pointing out how the "M" affected the slope of the lines and how it was related to the "M" in the equation. However, the teacher hinted that "M" influenced the orientation of the (line 29). As shall be seen later, this direct hint, focussing a specific aspect of the graph to look at, enabled Emy to make the connection between "M" and the orientation of the line.

Role of the teacher

This excerpt shows Emy's confusion about the graphic display and how the teacher's intervention impinged on Emy's interpretation of it.

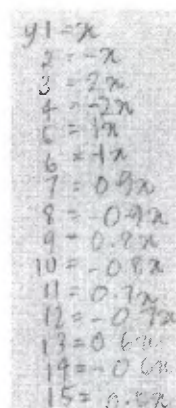
Excerpt 6.12 Emy's misconceptions

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Why do use decimals? Emy: I want to make it smaller....	Emy is drawing her starburst and keying in the equations. Teacher asks why Emy is using the decimals with the "Starburst" task.	
5	T: What are you trying to make smaller?		
6	Emy: I want to make the line smaller...shorter T: If you want the line to be smaller,	(The lines between $y=x$ and	

10	you change the view-window....	$y=-x$ looks shorter).	
	Emy: Well, I want to change the cursor.		
15	T: No.... what do you want to change... what is this...this line is more steep compared to this one...	Teacher points to the lines in the Starburst figure.	
16	...Here this is $y=x$		
	T: (then asks Emy) If $y=x$, which is it?		
20	Emy: This (points to the diagonal)		
	T: If $y=3x$? Where approximately would it be?	Emy counted and chooses the third line	
	T: If $y=4x$?	Emy chooses the fourth line.	
24	T: If $10x$?		
25	Emy: Gosh!	(Emy exclaims.. !) Counts up to tenth line.	
26	T: Does it follow the order?		
	Emy: (nods)		
28	T: Afterwards have a look at it again.. Okay, now ...if it is below here?	Teacher points to the section below the line $y=x$.	
30			
32	Emy: Negative.		
34	T: Where is it negative?	Emy gives the wrong response.	
35	Emy: Here, on the left side	Emy looks at Teacher.	
		Emy points to the left of the Starburst.	
	T: When you do this, try and draw each display one by one to note down what happens...if you do like this, you will not see.	Teacher gestures Emy to draw the graphs generated on the work sheet.	
40			

The teacher asked Emy why she was using decimals to reproduce the STARBURST picture. Emy replied that the decimals made the lines shorter! (line 2 and line 6). This is partially true if one observes the STARBURST picture when it is drawn on a “square window” as in the students’ worksheet (in line 4). Lines below $y=x$ or $y=-x$ did appear to be shorter. This same situation could also be observed when the graphs are generated in the STD window. Emy also had the misconception that the coefficients in front of the equations of the STARBURST picture were in linear order (line 20-24). Emy misinterpreted the first line was $y=x$ (the diagonal), the second line $y=2x$, the third line $y=3x$ and so forth. The teacher pointed out the difference in the “steepness” between the lines (line 15 to 16) to confront Emy with the fact that the number in front of x affected the “steepness” of the line, with the larger number denoting the line being more steep. However, Emy’s misconception prevented her from appropriating the teacher’s explicit hint. The teacher then confronted Emy’s misconception (26) but did not provide any explanation of how the coefficient was related to the ratio of increase in y to the increase in x . Instead, the teacher told Emy to check later with her calculator (line 28). The

teacher then tried to get Emy to focus on the lines whose gradients were in decimals (line 29). Emy's incorrect response (line 32) indicated two things. Firstly, she did not see that decimal numbers such as 0.9, and 0.8 were smaller than 1 in this context of the task (in the pre-test she could sequence the numbers involving decimals in the correct order). Secondly, she did not associate the negative sign in front of x governed the orientation of the line. The teacher then tried to get Emy to correct her error by re-questioning (line 34). Emy's response indicates that she was confused. It is not clear whether she meant "negative" to mean the left side of the graphing space of the y -axis, or the $y=x$ line segment that was located in the third quadrant. The teacher ended the intervention without raising with Emy the fact that lines "below" $y=x$ would have gradients that were decimals which were less than 1. She asked Emy to hand draw the graphs one by one and provided the general hint "to note down what happens". However, this general hint did not help Emy to focus on how the coefficients affected the slope of the line, as is evident from what she wrote in her worksheet as shown below.



A photograph of a handwritten list of 15 linear equations on a piece of paper. The equations are written in a cursive, somewhat messy hand. They are as follows:

- 1 $y = x$
- 2 $y = -x$
- 3 $y = 2x$
- 4 $y = -2x$
- 5 $y = 1x$
- 6 $y = -1x$
- 7 $y = 0.9x$
- 8 $y = -0.9x$
- 9 $y = 0.8x$
- 10 $y = -0.8x$
- 11 $y = 0.7x$
- 12 $y = -0.7x$
- 13 $y = 0.6x$
- 14 $y = -0.6x$
- 15 $y = 0.5x$

These equations were most likely the ones that she chose to key into her calculator, and would have produced the following "imperfect" STARBURST picture as shown in the figure below. This also indicates that the role of the graphing calculator was simply to carry out the mathematical procedures.

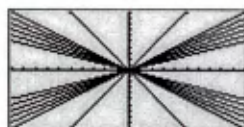


Figure 6. 09 Emy's imperfect STARBURST picture in the STD window

The extract of Emy's work above also highlights her poor algebraic competence. She did not recognise the x and $-x$ are equivalent to $1x$ and $-1x$ respectively. However, what

emerges from the intervention is that the teacher's explicit hint went unheeded by Emy, indicating that explicit hints were ineffective if misconceptions related to the task are not corrected. Getting Emy to resolve her own misconception was not easy for her.

In conclusion, Emy's disjointed understanding of the gradient concept is in part contributed to by her inability to make sense of the graphic display, her lack of mathematical knowledge, and ineffective teacher intervention. Her inability to make sense of the graphic display resulted in her seeing certain aspects on the graphic display only. It also resulted in visual illusions: she saw that the graphs of $y=mx$ got shorter as "m" got smaller. Her lack of mathematical knowledge, including negative numbers, decimals, algebraic manipulation, and substitution limited the scope of her investigations and sometimes prevented her from making sense of the activity or from carrying out the tasks. It also made it difficult for her to build the units of graphical knowledge, and at the same time, to establish the relationships between the units of graphical knowledge that were developing. Not only did the lack of mathematical knowledge become a source of missing connections, it resulted in compartmentalised learning.

Teacher intervention was an important aspect of Emy's concept development. Careful intervention by the teacher enabled Emy to hand draw graphs successfully, and this helped Emy to proceed with her task of examining the relationship between the "m" in the equation and the numerical ratio obtained from the geometric representation of the slope. Making explicit which features were to be observed on the screen was important to enable low attaining students like Emy to make sense of the graph display as they need to see what they were expected to see. The teacher's specific hints, like asking Emy to look at the direction of the line as "m" changes, assisted Emy in focussing on the essential features of the graphic screen. General hints, like asking Emy to look at the changes on the graph as "m" changes, or to work with decimals without specifically directing what to look on the screen were not productive in helping her to make sense of the graphic display.

Emy's activity showed that it took time for her to internalise the different ways of thinking about gradient - making the connection between the action on the calculator, the display it produced, and the mathematical concepts underlying in each of them, was not easy.

6.6 Conclusion on the concept of gradient in the Interpretative Learning Model

1. Understanding the gradient concept requires an understanding of the number line concept and being able to discern that negative sign denotes a negative gradient. The low attaining student confused smaller numbers with lines having smaller gradients or being less steep.
2. The DYNA Mode could facilitate an intuitive understanding of the line as an object with the slope and orientation attributes but medium and low attaining students required the intervention of the teacher to focus on both these features and to relate the display to the equation that generates it.
3. Medium and low attaining students need more specific type of intervention in interpreting the graphic display, whether it is in the GRAPH Mode or DYNA Mode. Making explicit which features of the graph should be attended to is necessary to enable them to distinguish between the effects of the sign in front of the parameter and the magnitude of the parameter itself.
4. Recognising and addressing students' syntactic and semantic errors, which were prevalent in medium and low attaining students is not easy. Providing explicit procedures can result in students using the calculator purely as a display tool.
5. The lack of mathematical knowledge, specifically in decimals and negative numbers in the low attaining student, made it difficult to systematise her investigation of varying the values of "m". It also resulted in visual illusions that the decimals produce a shorter line.
6. The calculator syntax for range can be a source of confusion. The term range used in the TABLE Mode and the DYNA Mode but refers to different meanings. In the TABLE Mode, the range refers to the changes of the value of the variable x whereas in the DYNA Mode, the range refers to the changes of values of the "dynamic coefficient" under consideration.

CHAPTER 7

CASE STUDY 3: THE CONCEPT OF LINEAR GRAPHS

The background to the linear graph concept is given in section 7.1. The performance of the students in each learning model in the related test item in the post-test are summarised in tables 7.01 and 7.02. Section 7.2 presents the expected learning outcomes using the graphing calculator for the linear graph concept. It begins with a brief rationale of the tasks and then outlines the mathematical tasks on the linear graph concept in each learning model. This is followed by an explanation on how the graphing calculator is thought to assist in understanding the linear graph concept. Section 7.3 describes how the case study students in the StructLM dealt with the linear graph concept, with a conclusion on their development of the concept in section 7.4. Similarly section 7.5 presents the analysis of the case study students in the InterLM, with a conclusion on their development of the concept in section 7.6

7.1 Background

Understanding linear graphs requires being able to make the Cartesian Connection (Moschkovich et al.1993). That is, being able to see the connection between the equation of the graph, the table of values and the graph itself. It also involves being able to think of the linear equations of the form $y=mx+b$ as members of a two-parameter family in which the parameters m and b

- (i) are independent,
- (ii) determine the position of a line graphically, and
- (iii) determine each position the graph of a line in particular ways as they are varied.

Students' partial understanding of the linear graph concept has been attributed to overemphasis on point-wise or local interpretation of the graph instead of a global interpretation in which the line is seen as an object (Leinhardt et al., 1990), that can be manipulated as whole (Sfard, 1991; Moschkovich et al, 1993). Another view is the tendency for students to concentrate on only one aspect of the information available to them and exclude considering the other related information (Bell et al., 1987).

The following issues were examined in the post-test: whether students

- (i) can determine the position of a graph graphically (Question 17),
- (ii) can distinguish which of the parameter in the equation is the intercept and which is the gradient (Question 11, Question 12),
- (iii) can deduce the equation of a line that is parallel to another line (Question 10(c)(ii),
- (iv) recognise the connection between the equation and the graph (Question 14),
- (v) determine the equation of a line with a negative gradient (Question 15).

Question No. 17 was from the 1990 National Assessment of Educational Progress (source: Moschkovich et al., 1993, p. 70). It was reported that only 32% of high school seniors (average 17 years old) drew the new parallel line on the graph and only 16% answered both parts correctly. Tables 7.01 and 7.02 below depict the responses of the students in the StructLM and the InterLM respectively.

Only the high attaining students correctly answered all the questions but all the case study students were able to draw a line parallel to the given line $y=2x-5$ that goes through the origin. There were variable answers from low and medium attaining students, which shall be discussed in each of the detailed analysis.

Table 7.01 Students' performance on linear graphs in the StuctLM

Name of student	Concepts/ tasks				
	Determine the position of a graph graphically (Question 17)	Distinguish between the gradient and intercept from the equation (Questions 11(b), 12(ii))	Recognise the connection between equation and the graph (Question 14)	Determine the equation of a line with a negative gradient (Question 15)	Determine the equation of a line parallel to another line (Question 10(c)(ii))
Wan (High Attaining Boy)	Correctly drew the line parallel to $y=2x-5$ and passing through the origin and correctly named the it as $y=2x$.	Correctly named $2x+3y-5=0$ and $2x-3y+5=0$ have the same intercept with workings shown. Correctly identified -4 as the intercept of the line $2y=5x-8$	Correctly identified $2y=x+4$ as the equation of the given graph with workings shown.	Correctly named $y=-2x+6$ as the equation with workings shown.	Correctly deduced $y=-\frac{1}{2}x-1$ as the equation of the second line, used the gradient value of the first line and substitution to determine "c".
Ng (Medium Attaining Boy)	Correctly drew the line parallel to $y=2x-5$ and passing through the origin but the equation of the line is given as $y=4x$.	Incorrectly named $2x+3y-5=0$ and $4x+6y-5=0$ have the same intercept giving the absurd reason "they both have the same total number of coordinates". Incorrectly gave -8 as the intercept of the line $2y=5x-8$	Incorrectly identified $y=2x+2$ as the equation of the given line with the reason "2" in the equation has the same value as the place where the given graph intercepts.	Incorrectly wrote $y=3x+3$ as the equation of the line with faint scribbles surrounding the question.	Used the incorrect value of gradient obtained from the previous calculation: $\frac{1}{2}$. Gave the equation of the line $y=\frac{1}{2}x-1$, with correct intercept value.
Amin (Low Attaining Boy)	Correctly drew the line parallel to $y=2x-5$ and passing through the origin with evidence of the completing the right angle triangle but the equation is not given.	Incorrectly named $2x+3y+5=0$ and $2x-3y+5=0$ have the same intercept, did not give any reason. Did not name the intercept value of the line $2y=5x-8$, working shown: $2y=5x-8$ $y=5x-8-2$ $y=-5x$	Incorrectly chose $y=2x+2$ as the equation of the given line, no reason was given.	Correctly gave the equation $y=-2x+6$ as the equation of the given line and used the point-slope formula to determine the gradient again $\frac{0-6}{3-0} = -\frac{6}{3} = -2$	Used the point-slope formula to determine the gradient and computed the wrong value: $\frac{5}{6}$. Gave the equation of the line as $y=\frac{5}{6}x-1$, the intercept is correct.

Table 7.02 Students' performance on linear graphs in the InterLM

Name of student	Concepts/ tasks				
	Determine the position of a graph graphically (Question 17)	Distinguish between the gradient and intercept from the equation (Questions 11(b), 12(ii))	Recognise the connection between equation and the graph (Question 14)	Determine the equation of a line with a negative gradient (Question 15)	Determine the equation of a line parallel to another line (Question 10(c)(ii))
Grace (High Attaining Girl)	Correctly drew the line parallel to $y=2x-5$ and passing through the origin and correctly named the it as $y=2x$.	Correctly named $2x+3y-5=0$ and $2x-3y+5=0$ have the same intercept with workings shown. Correctly identified -4 as the intercept of the line $2y=5x-8$	Correctly identified $2y=x+4$ as the equation of the given graph but no working is shown.	Correctly named $y=-2x+6$ as the equation, used the point-slope formula to determine the gradient and substitution to determine "c".	Correctly derived the equation of the line as $y=-2x$, used the point-slope formula to determine the gradient and substitution to determine "c".
Presha (Medium Attaining Girl)	Correctly drew the line parallel to $y=2x-5$ and passing through the origin but did not name the line, used "geometric equivalence" by counting the squares to obtain the same steepness.	Did not answer Question 11. Correctly identified -4 as the intercept of the line $2y=5x-8$ but did not show any working.	Correctly identified $2y=x+4$ as the equation of the given graph but no working is shown.	Correctly named $y=-2x+6$ as the equation with working shown. Determined the gradient using the right-angle triangle: $6/3 \dots -2$, the intercept read from the graph and then substituted the values in $y=mx+c$ $y=-2x+6$	Incorrectly gave the equation of the line as $y=-2x+2$. Both the gradient and intercept values are wrong.
Emy (Low Attaining Girl)	Correctly drew the line parallel to $y=2x-5$ and passing through the origin and correctly named the it as $y=2x$. Evidence of using the right-angle triangle method to determine the gradient.	Did not attempt Question 11. Also did not attempt Question 12.	Did not choose any of the given equations but wrote down (correctly) the equation as $y = \frac{x}{2} + 2$ and used the right-angle triangle to compute the gradient.	Correctly named the equation of the line as $y=-2x+6$, and used the right-angle triangle method to determine the gradient.	Correctly derived the equation of the line as $y = -\frac{x}{2} + (-1)$ without simplifying it. Used the right-angle triangle to determine the gradient.

7.2 Expected learning outcomes with the graphing calculator on linear graphs

The tasks that were designed in both models were such that the students would use all the graphing facilities they had encountered in their previous activities. The graphing calculator activities were accompanied by the hand graphing activities so that students also were inducted to graphical conventions and symbolism (seen by many researches to be important aspect of graphing). Some “standard text book” exercises on linear graph problems (for example, determining the equation of the line that passes through any two given points and through a given point with a known gradient) were included to see how much students made use of the graphing calculator in their solutions.

In the StrucLM the scope of the tasks ranged from looking at families of straight line graphs that had the same gradient (“ m ” is held constant and the intercept c varied) and those having different gradients (“ m ” varied and the c held constant). Graphs were investigated with the graphing calculator by using the GRAPH Mode, the DYNA Mode and the TABLE Mode.

The tasks also included hand graphing of the equations $y=3x+9$ and $3x+5y=30$. A few tasks involved determining the equation of the straight line of given graphs. Other tasks included determining the straight-line equation given the gradient and passing through two points. Some activities involved graphs that are perpendicular to each other. Several pairs of equations of perpendicular lines were given and investigation of the influence that the view-windows have on the appearance of the lines was included. Students were encouraged to use the “Dual Screen” facility and the “zoom” facility in their investigations.

The tasks in the InterLM were more open. Students were asked to investigate several graphs of $y=mx+c$ by varying values of “ m ” and “ c ” by using the GRAPH Mode, or TABLE Mode, or DYNA Mode as they felt appropriate. They also did some hand graphing of linear graphs. In the exploration of graphs that were parallel to or perpendicular to each other, students chose their own equations and were expected to deduce which parameters characterise the lines as such. Their activities also confronted them to use the “zoom” facility and the view-window facility of the graphing calculator.

The GRAPH Mode or TABLE Mode enables the graph of an algebraic expression to be generated easily and quickly with one button push. It gives the user the option to modify the algebraic expression and observe immediately the effect of changing a parameter. It was expected that by using the GRAPH Mode facility in their activities, it would shift students' attention to view the line as an object that could be manipulated as a whole. For example, students may shift attention from the y-intercept (local) to vertical translation (global). Students have to transpose the equation to the form $y=mx+c$ before keying it into the calculator which, was expected to confront them with the general form of the straight line. The TABLE Mode of the graphing calculator requires student to move readily from the table of ordered pairs for the algebraic expression and the graph, thus using this facility was expected to expose the relationship between the ordered pairs and the line itself. It was hoped that "Dual Screen" facility in the SET UP (which displays the graph and its numeric table of values side by side) would confront students to see the link between them.

The DYNA Mode of the calculator lets the user to see what happens to a graph when the parameters are changed: one parameter can be varied while holding the others constant. Students needed to specify the values of the parameter that was to be varied, which was expected to confront them which parameter they were manipulating with and therefore the characteristic of the graph that was involved. In addition when the graph is dynamically generated, each graphical output will display the respective value of the parameter again, linking students to the features of the graph that are affected by the parameter change.

Students in this study were expected to view the same line in different view-windows which was intended to confront them the visual illusion generated by the choice of scales as described by some researchers (see section 2.3.2.2). Likewise the "zoom" facility allows a graph or a selected region of a graph on the graphic display to be enlarged and reduced enabling the student to see the graph from closer or further away.

7.3 Analysis of case study students in the Structured Learning Model

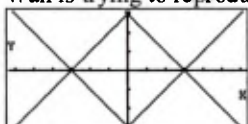
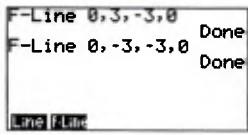
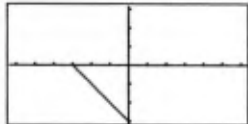
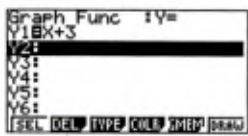
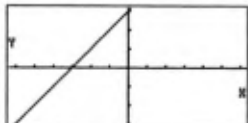
WAN (HIGH ATTAINING BOY)

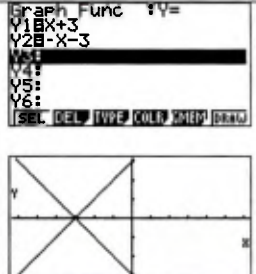
As the table 7.01 shows, Wan was successful in answering all the questions in the post-test. The question is what makes him successful? Two issues will be examined to explore his approach.

Role of the calculator

In an episode where Wan used the calculator as a resource with his calculation, he was seen using the calculator to obtain the table of values to hand graph the line $y=4x+9$. The following excerpt shows how Wan's understanding of the overall functionality of the calculator enabled him to use it to support his thinking by providing him with instant feedback of the conjectures he made.

Excerpt 7.01 Using the graphing calculator to support conjectures

Line No.	Action	Calculator Display/Worksheet
1	Wan is trying to reproduce this figure below:	
5		
7	He uses the RUN Mode to draw the lines by inputting the coordinates joining the line.	
10		
12	He checks the output display.	
15		
17	He changes his mind and decides to use the GRAPH Mode instead by keying each equation and displaying the output in turn until he successfully reproduces the required figure.	
20		
25		

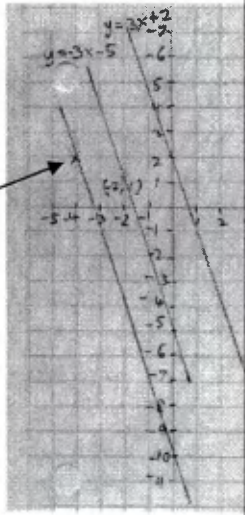
28	Wan keys in the expressions: $x+3$ and $-x+3$	
30		

Wan started the task of drawing a rhombus by using the F-line facility which he had used in earlier tasks to draw line segments (line 7). This shows that Wan reflected and related what he had previously learned (drawing of lines) to new situations or tasks. The output display showed only part of the line segment (line 12). Wan then realised that the adjacent pair of lines were perpendicular to each other and used his knowledge of the gradient and intercept concept to test the first equation as this method would produce the whole figure instead of just the rhombus (line 17). He then applied his understanding of the perpendicular concept to test out the next line (line 28). It is clear Wan was using the graphing calculator to provide him with feedback to test his conjectures.

Role of the teacher

The following excerpt shows how Wan's mathematical competency and use of calculator enabled him to benefit from the teacher's curriculum script.

Excerpt 7.02 Wan's ability to follow the curriculum script

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: If it passes through the point $(-4,2)$...where is the point $(-4,2)$? T: Okay, what is equation of this line now?	Wan is trying to find the equation of the line that is parallel to $y=-3x+2$ and $y=-3x-5$ and passes through the point $(-4,2)$. He had drawn the graphs of $y=-3x+2$ and $y=-3x-5$. Wan marks on the graph correctly. Wan is silent.	
5	T: What is the general equation of the straight line?		
7	Wan: $y=-3x...$		
8	T: plus c		
10	T: What is the general equation of the straight line? Y is equal to.....		
11	Y is equal to m....	Wan writes down $y=mx+c$	<p>Wan's graphs</p>
13	T: What is the gradient of the line		

14	that is parallel to $y=-3x+2$?		
15	Wan: -3		
16	T: So $y=-3x$ plus what?	Wan writes down $y=-3x+c$	
17	T: Which point does it pass through?		
	Wan: (-4,2)		
20	T: Can you obtain the equation now? What can you get from here?	Teacher refers to (-4,2)	
21	Wan: x and y	Wan substitutes the values as Teacher watches:	
24		$Y=-3x+c$	
25		$2=-3(-4)+c$	
		$-c=12-2$	
		$-c=10$	
		$c=10$	
28	T: What is the equation of this line now?	Wan writes: $y=-3x+(-10)$	
30	T: Simplify it.	Wan writes: $y=-3x-10$	
32		He also graphs the line.	

The above transcript shows the teacher intervention leading Wan to solve the problem in determining the equation of the line parallel to another given line. The teacher initially asked Wan to point out where the point (-4,2) is on the graph (line 1). Wan knew that the parallel line would have the same gradient (line 7) but could not continue. The teacher completed the Wan's statement "plus c" (line 8) but when Wan did not respond further, the teacher began to structure a solution template and proceeded to ask for the general equation of the line (line 5). When Wan did not reply, the teacher provided with hint "y is equal to....?" and added "y is equal to m...." (line 11). Wan got the hint and wrote down $y=mx+c$. The teacher then focussed Wan to the gradient, and then back to the equation "So $y=-3x$ plus what?" (line 16). The teacher then guided Wan that the point (-4,2) is equivalent to (x,y) (line 17-22). The excerpt shows the teacher was crafting the overall problem into sub-problems that Wan could manage. From the transcript, it is apparent that the interaction pattern involved a series of conversational exchanges between Wan and the teacher rather than the teacher being directive. The curriculum script was maintained as Wan also understood what the teacher asked, for example, gradient (line 13), parallel (line 14). This clearly enabled more information to be generated in each turn with the teacher providing hints (line 11, line 16, line 21) until Wan was able to relate that (x, y) are the points that lies on the line $y=-3x+c$ and subsequently solve the problem.

Wan's ability to draw the graphs $y=-3x+2$ and $y=-3x-5$ manually indicates several things. He knew that the negative sign denotes the "negative direction" of the graph, and the meaning of the coefficients. More importantly, Wan was able to translate the symbolic or numerical gradient in the equation geometrically. That is Wan also related the numerical gradient in the equation in terms of the ratio of increase in y to increase in x , and so was able to visualise that "3" can be written and represented as the ratio "3/1" or "6/2" etc. He also had a conceptual understanding of the mathematical procedures (line 24-32).

Wan was also observed participating actively in the teacher's classroom explanation by contributing to the explanation and answering the questions that the teacher intermittently posed. In one episode, Wan was feeding answers as the teacher was simplifying the equation, indicating he could follow the teacher's explanation. This is also an indication that Wan understood the terminology that the teacher used and had the level of competence in algebra and computation necessary to follow through the teacher's explanation.

In conclusion, Wan shows a deep understanding of the straight-line concept, successfully making the Cartesian Connection. Three major factors that seem to facilitate his learning were his understanding of the functionality of the calculator, the related mathematical knowledge (including the gradient concept), and the ability to follow teacher's classroom explanations. His understanding of the overall functionality including the functionality of the different modes enabled him to utilise the graphing calculator in many different ways, for example sometimes as a resource tool to help him with computations in his hand graphing work, and sometimes as a supporting tool to support his thinking about the various aspects of the straight-line concept. It was clear that he could associate the table values in the calculator to the table values that he had to generate himself to construct the graph manually, indicating Wan was clear on to what extent the calculator could assist him.

Wan's understanding of related mathematical knowledge including the mathematical terminology enabled him to follow the teacher's classroom instructions and explanation. His understanding of the gradient concept in the previous learning activities (including how the gradient in the equation relates to the slope-related graphical properties) led to an expanding knowledge base. Wan's understanding that any point that lies on the line

$y=x$ must satisfy the equation, derived in his earlier work, facilitated him towards understanding that the x and y in the equation $y=mx+c$ again represent any point (x,y) on the line that satisfies the equation.

What emerges from Wan's activity is that the teacher's occasional conceptual connection during classroom teaching provided Wan a critical source to integrate all the related concepts. His understanding of the mathematical language that the teacher used and the related mathematical knowledge presented by the teacher enabled him to contribute actively and to follow the teacher's explanation and knowledge presented, and to make use of this knowledge in his activity with the calculator. Wan's mathematical knowledge also enabled him to benefit from the teacher's use of curriculum scripts.

NG (MEDIUM ATTAINING BOY)

Ng's understanding of the straight line concept could be described as partial. He correctly drew the line parallel to $y=2x-5$ for Question 17 indicating that he understood the meaning of origin and that two lines that are parallel have the same gradient. The equation for this line was given, however, as $y=4x$! Paradoxically, he assigned the same gradient to the second line that was parallel to the first line in Question 10 (although his calculation for the gradient is wrong). And yet in Question 11, Ng was not able to distinguish which pair of lines are parallel to each other when the equations of the lines are given in the form $ax+by+c=0$ instead of $y=mx+c$. He incorrectly picked $2x+3y-5=0$ and $3x+2y+5=0$ as being parallel to each other giving the seemingly absurd reason that they both have the same total number of x and y coordinates! The coefficient in front of x and y connotes a different meaning. Ng treated straight lines represented in the form $ax+by+c=0$ as separate entities from those in the form $y=mx+c$.

An examination of Ng's derivation of equations of the straight lines in Question 10, indicates that Ng seemed to recognise that " c " in the general equation as the y -intercept from the diagram. In Question 14, he incorrectly chose the equation $y=2x+2$ as the diagram shows that the line cuts the y -axis at " 2 ". This response, however, also indicates that Ng had compartmentalised his conception of gradient and intercept, applying each when it suited the situation or when the evidence for one was more profound (from the

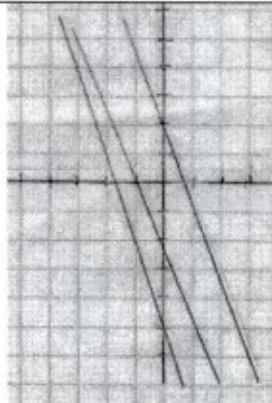
diagram), while ignoring the other aspect if the facts presented were less apparent and doubtful to him.

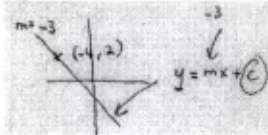
The following extracts will try to illuminate the different aspects of the role of the teacher and the role of the calculator that contributed to Ng's fragmented development of the linear graph concept.

Role of the teacher

This excerpt shows the nature of the teacher intervention, which adhered to a curriculum script and was directed towards a correct solution. It also illustrates how Ng's lack of mathematical knowledge (including the terminology) precluded him from following the teacher intervention.

Excerpt 7.03 Ng's inability to follow the curriculum script

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: What is the general equation of the straight line....the straight line, what is its general equation?	Ng was trying to find the equation of the line that is parallel to $y=-3x+2$ and $y=-3x-5$ and passes through the point $(-4,2)$.	 <p>Ng's hand draw graph</p>
4	Ng: Like this?	Ng shows his sketch of the graphs of $y=-3x+2$, $y=-3x+5$ and the required graph.	
5	T: No...the general equation. My question is what is the common equation for the straight line.	Teacher asks again.	
10	Ng: $y=mx$		
12	T: No...the general equation...do you see there?...that is the general equation for the straight line.	Teacher points to the equation $y=mx+c$ written on the black board. Ng nods his head.	
15	T: Now if the line that is parallel to this ($y=-3x+2$), what is the gradient?	Ng is quiet.	
20	T: If here (points to $y=-3x+2$), what is its gradient? Ng: -3	Teacher repeats question.	
25	T: -3...this line...can you determine what is the equation of this line? T: This line is parallel to this line ($y=-3x+2$). So what is the gradient of this line? It is parallel to this line	Ng shakes his head.	

28	($y=-3x-5$). What is the gradient? Ng: also -3		
30	T: Okay, good, it must be -3 too. Now can you determine where is the intercept? It passes through here ($-$ 4,2). Okay, can you determine the equation of the line now?	Teacher affirms Ng's response.	
33		Ng keeps quiet.	
35			
36	T: It means you have a line whose gradient is -3 ... m is -3 . Now what do you have to determine? The inter.... (T "rounds" the "c" in the equation $y=mx+c$) Ng: intercept.	Teacher sketches and explains.	
40			
45	T: Can we determine the intercept if we know the line passes through this point ($-4,2$)? T: Try ..try to calculate...how? Ng: 14	Ng calculates.	
47		Teacher inspects Ng's calculation and finds mistake.	
50	T: If the intercept is 14...so this becomes $-3x+14$...But your intercept....check your intercept again...c is equal to.... Where did you get the "2" from? "2" is equal to what? How do you obtain this. Can you show me? Ng: y....	Ng is seen writing (presumably): $Y=mx+c$ $2=-4(-3)+c$ $2=-12+c$ $2+12=c$ $c=14$ Ng acknowledges his computation error and corrects it. Ng nods.	
60	T: -4 multiply by -3 ?		
65	T: Now what is the equation... $y=$ Ng: $y=-3x-10$ T: Do you understand? T: If I give you another one, a line that is parallel and passes through ($-$ 1,-4), can you determine the equation. Ng: Yes.		

First it would be in point to mention that the teacher had already hinted at the value of the intercept during classroom instruction. Ng's wrong calculation (line 47) adds support that the teacher's classroom instruction had very little effect on Ng's understanding. This was probably due in large part to Ng's misunderstanding of terminology, as evident from the transcript.

In the intervention depicted above, the teacher followed a curriculum script to determine the equation of a line parallel to a given line. At the beginning of the intervention, Ng did not understand that the term “general equation of the straight line” that the teacher requested refers to the equation $y=mx+c$ (line 4). Ng instead showed his drawing of the graphs of $y=-3x+2$ and $y=-3x-5$ on the grid paper. Curiously, the teacher did not ask Ng to identify the graphs he drew. The gradient is graphically inaccurate as the ratio of increase in y to increase in x is not equal to “3”, although the direction of the line is correct. This indicates that Ng did not understand how to represent the gradient “3” geometrically and that he did not understand the gradient concept in the ratio form. It also shows that Ng had not checked these graphs with his calculator. In addition, it also shows that Ng did not relate the points (x, y) on the line to the equation of the line. After three turns, the teacher decided to tell Ng what he meant by general equation (line 12).

The teacher then focussed Ng to the gradient of the line. It seems Ng understood that the coefficient represented the gradient (line 20 and line 28). The interaction was disrupted when Ng could not relate the point $(4,2)$ with the general equation $y=mx+c$ (line 33). The teacher gave additional hints by giving a pictorial representation of the problem (line 36). It seems this pictorial representation of the problem enabled Ng to proceed to solve the problem despite getting the calculation wrong, but it is doubtful whether Ng really understood the concept that any point on the line passes through the line only if the x and y values satisfy the equation of the line, as the teacher did not relate Ng’s final answer to his drawing of the graphs. There was a strong element of getting the answer correct towards the end of the intervention, rather than trying to get Ng to reason why the intercept “ $c=14$ ” was inappropriate. There was a mixture of scaffolding, direct telling and focus on correct answer in the teacher’s intervention. It is reasonable to infer that the way the teacher intervened predisposed Ng to solving the task according to a set of procedures.

Role of the calculator

The two examples that follow show how Ng’s inability to make sense of the graphic display prevented him from seeing the straight line as a two-parameter family. The first comes from observations of how he carried out his investigation on perpendicular lines.

After graphing the two pairs of perpendicular lines in his graphing calculator, this was what he drew on his graph paper.

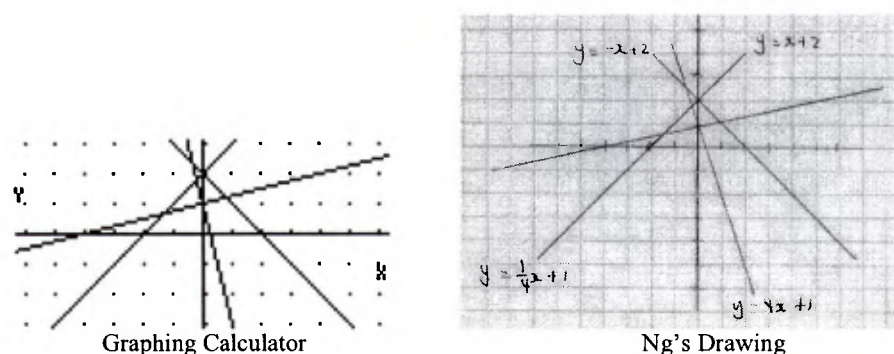


Figure 7.01 Replication of the graphic display

Notably the graph of $y = -4x + 1$ was erroneously drawn revealing that Ng used the graphing calculator to “copy” the graphs. The graph $y = -4x + 1$ was more difficult to replicate as it does not pass through identifiable “grids” on the graphing calculator. It also shows that Ng did not understand gradient in the equation as the ratio of increase in y to increase in x . Ng carelessly labelled the equations $y = \frac{1}{4}x + 1$ and $y = x + 2$ on the same graph. There was no evidence that Ng attempted other pairs of lines. This also characterised Ng’s activity in general, following the teacher’s instruction or instructions contained in the worksheet. The conclusion that Ng wrote, shown below, clearly indicates that Ng had not grasped the concept of perpendicular lines.

Kecenderungan dalam persamaan kedua-dua garis itu adalah berbeza manakala titik pintasananya adalah sama.

“The gradients in both the lines are different while the point of intercept is the same.”

This shows that although Ng successfully operated the calculator to produce the lines, he was unconscious of what he entered into his graphing calculator. To him the equations all generated straight lines with certain attributes without understanding why. The calculator afforded Ng to see that equations with different numerical gradients produced lines with different gradients and intersected each other. But Ng’s inability to make sense of the task resulted in him fixing his attention to the points of intersection and prevented him from attending to the more important feature of the graphs, which are their gradients.

In the second example, Ng was observed to operate in the TABLE Mode. He was able to generate table values for the graphs of $y=x$ and $y=x-1$. He moved the cursor across the columns to see the values displayed as shown below.

Y1=X			
X	Y1	Y2	
-3	-3	-4	
-2.5	-2.5	-3.5	
-2	-2	-3	
-1.5	-1.5	-2.5	
FORM DEL ROW ECHO G-PLY			

Y2=X-1			
X	Y1	Y2	
-3	-3	-4	
-2.5	-2.5	-3.5	
-2	-2	-3	
-1.5	-1.5	-2.5	
FORM DEL ROW ECHO G-PLY			

However, there was no evidence that Ng used these table values to draw the graphs. It seems Ng only saw that the table values in each equation were different without making the connection of how the x and y values were related to the equation and points on the line. Here it could be construed that Ng used the graphing calculator simply as a display tool.

One would expect that the requirement to key in the equation in the form $y=mx+c$ in the graphing calculator would confront Ng with the need to transform equations making y as the subject and establish the equivalence between the equations of the form $y=mx+c$ and other forms. For Ng, it did not. Two things seem to emerge here. First, it shows that Ng used the graphing calculator, as a tool to carry the procedures of the task. This approach made it difficult for him to make sense of the graphic display. To him, the equations all generated straight lines with certain attributes and features, for example differing steepness or intercepts, without really understanding why. Second, it also indicates that, the teacher's classroom teaching that emphasised the need to transform the equation in terms of x , for example in $3x+5y=30$, had little significance in Ng's understanding. Examination of his worksheet indicated that Ng ignored the portion of activity that required him to transform the equation and to subsequently hand graph the line.

In conclusion, Ng's fragmented understanding, in general is contributed to by his lack of understanding of related mathematical concepts, his inability to make sense of the graphic display, and ineffective teacher intervention. His lack of competency in algebraic manipulation made it difficult for him to transpose the given equations into the form of $y=mx+c$. His lack of understanding of some of the mathematical terminology that the teacher used made it difficult for him to follow the teacher intervention. The inability to make sense of the graphic display predisposed him to use the graphing calculator as a tool to carry out procedures of the task as directed in the worksheet or by the teacher. It also resulted in him confining his attention only to certain aspects of the graphic display

and confusion as to what aspects of the graphic display to attend to. For example, in the task that involved ranging the numerical gradient and keeping the intercept constant Ng focussed on the point of intersection instead. Therefore, for Ng, the graphing calculator afforded the viewing of the straight line as an object with attributes but with no understanding as a two-parameter family.

Teacher intervention was an important aspect of Ng's learning to make the connection between the many gaps in his knowledge. However, the teacher interventions that adhered to a curriculum script and emphasised correction solutions were not productive to help Ng develop a coherent conceptualisation of the linear graph.

AMIN (LOW ATTAINING BOY)

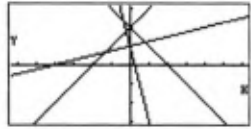
The post-test shows that Amin had fragmented understanding of the linear graph concept. In his answer to Question 10, the equation for the second line is wrong because of the incorrect gradient value which Amin incorrectly computed using the point-slope formula. Amin recognised that the intercept in the graph is the same as "c" in the equation. (Also confirmed by the way he answered Question 15 although he obtained the wrong answer as a result of not following the order of the coordinates when he computed the gradient using the point-slope formula). His workings for Question 12 shows that Amin could not transpose the equation $2y=5x-8$ in the form of $y=mx+c$. So it is not surprising that that Question 14 was also wrong. In Question 17, Amin drew the parallel line correctly passing through the origin. The equation was not given, however.

The following extracts will try to illuminate the various aspects of teacher intervention and Amin's usage of the calculator which shaped his development of the concept of linear graphs.

Role of the teacher

The teacher's intervention had an influence on the way Amin solved problems of this type as illustrated in the lengthy excerpt below. This excerpt also shows Amin's lack of mathematical knowledge and difficulty in making sense of the graphic display.

Excerpt 7.04 Amin's lack of mathematical knowledge

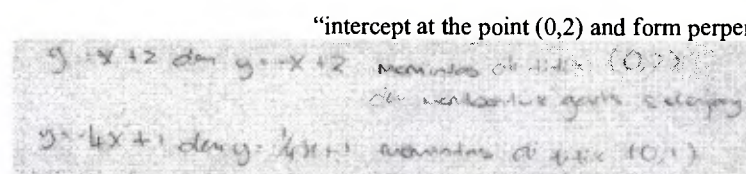
Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Is there m in these equations?	Students are to investigate the properties of the perpendicular lines Teacher writes on the board: $y=x+2$, $y=-x+2$ and asked whole class. Teacher points to the equations.	
3	Am: No, there isn't.		
5	T: That is impossible. Wan: There is. T: What is m here?	Teacher points to the place in front of x in the equation $y=x+2$	
10	Wan: 1 T: 1 T: How about here? Wan (and a few other students): -1	Teacher points to the place in front of x in the equation $y=-x+2$ T points to the 2 in $y=x+2$ and $y=-x+2$.	
15	T: What is this? Wan: c T: What is the same and what is different? Wan: the intercept.		
20	T: the intercept.. is it the same or different? Wan: the same. T: the intercept is the same, what is different? What is m?		
24	Wan: the gradient.		
25	T: Now describe it, what is the same and what is different. Try and sketch the graphs. T: Where is the intercept?	Am's calculator display has other graphs ($y=\frac{1}{4}x+1$ and $y=-4x+1$) besides $x+2$ and $-x+2$. Am points at the intercept "2" in his calculator.	
30	Am: Here T: Okay, it is there. Now, can I say this is $y=-x+2$? It also intercepts at 2	Teacher points to the graph $y=x+2$.	
35	Am: this....(mumbles) Wan: Say it louder. Am: Aaaa...		
40	T: How do we know which one is $y=x+2$ and $y=-x+2$, which one?		
41	Am: This is $x+2$because this is positive (points to the 1 st quadrant) and this is negative (points to the second quadrant).	Am points at the respective lines in the calculator.	
45	T: What is positive and what is negative? Am: The values here are positive and the values here are negative.	Am is quiet. Am points to the 1 st quadrant and the second quadrant respectively.	
50	T: What value is positive and what value is negative?		

53	Am: eem.... X and y T: I do not see the value of y is negative here.		
55	T: When we speak of straight line, we look at its intercept and...what is the other one that we look for? Am: Gradient.	Teacher points at the y-intercept.	
60	T: What is the general equation for straight line. Write down the general equation for ...the general one for the straight line. T (rephrase): If you see a straight line, what is its general equation.	Am is quiet.	
65	T: It has been written down many times.. What is the general equation? T: Do you understand my question?	Am is still quiet.	
70	What is the general equation? T: What is y equal to? T: Now look at the black board, what is it?	Am is still quiet. Am is still quiet.	
75	Am: $y=mx+c$ T: $y=mx+c$...What is m ? Am: gradient T: What is c? Am: the intercept		
80			
81	T: Now when you look at the equation $y=x+2$, what is the gradient? Am: One		
85	T: What is the intercept? Am: 2 T: Here, what is the gradient?		
	Am: -1	Teacher points to the equation $y=-x+2$	
90	T: The intercept? Am: 2 T: Now if you look here, what is the equation for this graph?		
95	T: Just now you said that it is $x+2$...Why did you say this is $y=x+2$ and this is $y=-x+2$? T: Can I say this is $-x+2$?	Teacher points to the graph of $y=x+2$ in the calculator. Am is quiet. Teacher points to the respective graphs again. Am is quiet.	
100			
101	T: What is the gradient for this graph? T: When you look at a line, you look out whether the gradient is positive or negative....right?	Teacher points to the line $y=x+2$. Am is still quiet. Am is still quiet.	
105	T: So is the gradient here positive or negative?	Am nods his head.	
	T: Where is its direction?	Teacher points to the line $y=x+2$. Am is still quiet.	
110	Am: Positive. T: How do you know that it is		

115	positive? T: How do you know that the gradient is positive? Am: The direction is positive.	Am is quiet.	
120	T: Where does it slants? Slants to the right or to the left? Am: To the right. T: Slants to the right...now if you look at this equation, what is the gradient? Is the gradient positive or negative? Am: Positive.		
125	T: This one? Am: negative. T: If in this sketch, which one has negative gradient?	Am points to the correct graph in the calculator.	
130	T: Why do you say it is negative?		
131	Am: It slants to the left.		

The above excerpt involves Amin's investigation on the concept of perpendicular lines. Amin was not able to assign the given equations to the graphs displayed. The teacher laboriously tried to structure a curriculum script to point out to Amin the two important features that characterise the graph of straight-line, that is, the gradient and the intercept. He also tried to address Amin's misconception of the sign assigned in front of the variable x (line 45 and line 50), confronted it (line 53), and subsequently tried to confront Amin the relationship between the sign in front of the variable x and the direction of the line. The curriculum script was disrupted a few times when Amin misconstrued the sign in front of x as the gradient of the line (line 41), did not understand the term "general equation of the line" (line 60), and the connection between the gradient and the direction of the line (line 110). The many missing mathematical knowledge pieces made it impossible to maintain the script flow and ended without Amin making the connection between the numerical gradients of two lines that are perpendicular to each other. This was also evident in what he wrote in his worksheet where his focus was on the intercept of the lines.

"intercept at the point (0,2) and form perpendicular lines"



"intercept at the point (0,1)"

The first answer was more likely expressing that the graphic screen displayed lines of $y=x+2$ and $y=-x+2$ that obviously looked perpendicular rather than the connection that the two lines have “opposite” gradients. Interestingly the extract shows that Amin made the connection between the constant “c” in the equation and the y-intercept and recognise that the y-intercept as (0,c). His attention on the intercept resulted in him overlooking at the value of the gradients.

What is also evident from the transcript is that a low attaining student like Amin had little opportunity to participate in the interaction during the teacher's classroom instruction depicted in the earlier part of the excerpt (line1-24). It was observed that the question the teacher was asking and the knowledge that the teacher was presenting was beyond Amin's level of comprehension. For example, Amin did not recognise the equivalence between x and $1x$ and between $-x$ and $-1x$ (line 3).

It was expected that Amin made the connection between the numerical gradient and the direction of the line from the painstakingly way the teacher focussed Amin the direction of the graph and the sign assigned to the gradient of the equation of the straight line (line 75, line 81, line 101-131). Yet, when given in a different context, Amin was not able to apply what he had previously learned. It is not unreasonable to infer that the above intervention was concerned with the solution of the task, resulting in Amin only memorising the procedures of how to solve similar problems as evident in the way Amin solved Question 10 in his post-test. It was a repetition of what he did in the learning activity. According to his worksheet, Amin first determined the gradient of the line using the point-slope formula to determine the gradient rather than the right-angle triangle method which is easier, as shown in the following extract taken from his worksheet.

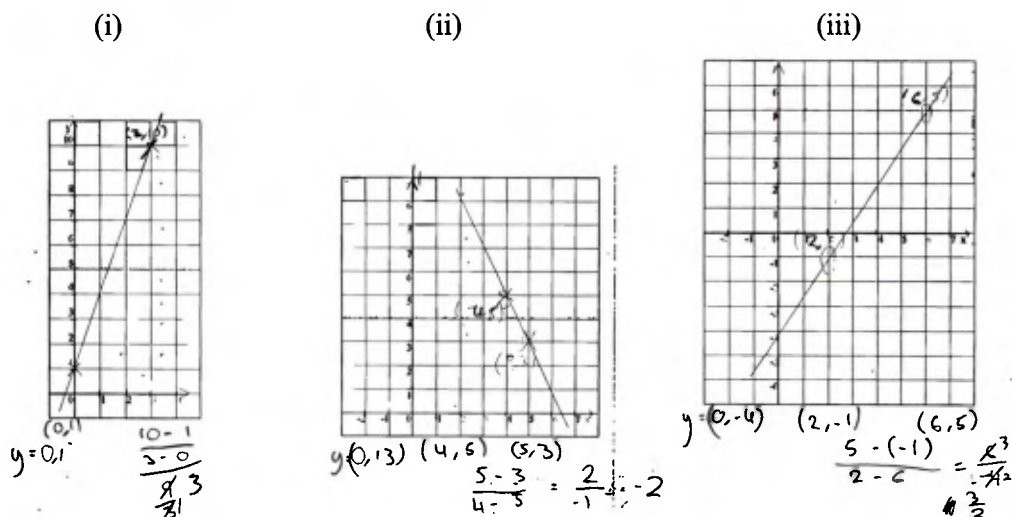


Figure 7.02 Amin's work on determining the gradient

In the third graph, his wrong computation results in the wrong sign for the gradient. He scribbled out the negative sign and disregarded it, indicating that he was aware that the gradient of this line had to be positive. He later successfully obtained the equations of these straight lines by applying the general equation of the straight line as shown below.



$$\begin{aligned}
 1) & y = 3x + 1 \\
 2) & y = -2x + 13 \\
 3) & y = \frac{1}{2}x + (-4) \\
 & = \frac{1}{2}x - 4
 \end{aligned}$$

Role of the calculator

The following excerpt shows Amin's usage of the calculator including his lack of consciousness on what is being keyed in the text screen.

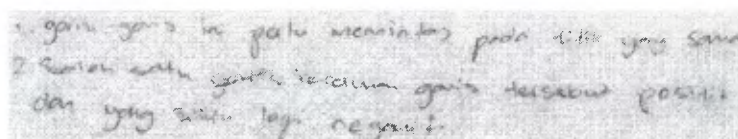
Excerpt 7.05 Amin's lack of consciousness on what is keyed in

Line No.	Action		Calculator Display/ Worksheet
1	T: Test with your calculator whether the two lines are perpendicular: $y = \frac{1}{3}x + 5$ and $y = -3x + 5$	Amin is seen to key in: $y = \frac{1}{3}x + 5$ and $y = 3x + 5$ and generates the graph. Amin moves the x-axis down a few times.	
2			
5			
6			
10			

12	Amin: Why is this?	He asked Wan and showed Wan his calculator.	
15			
18		(Wan corrected him)? Amin finally gets it correct. He has graphed in $y=-3x+5$.	

The above excerpt shows that Amin had shifted the x-axis down a few times to enable him to view a bigger portion of the graphs. From the reaction he showed (line 12), he was confused why his graphs did not appear to be perpendicular. It seems Amin did not realise that both the equations he keyed in have positive gradients (line 2). This indicates that Amin was not aware of what he had keyed in, indicating his usage of the calculator according to the teacher's instruction and as a display tool. Amin finally got it correct (line 18). It is not clear whether his peer Wan helped him but it seems the calculator did confront him that two lines that were perpendicular to each other had opposite signs to the numerical gradients. This was what Amin wrote about perpendicular lines in his work sheet.

"these lines have to intercept at the same point"



"one of the gradient of the line is positive and the other is negative"

His remark that the "lines have to intercept at the same point" indicates the strong visual impact of the graphing calculator. He finally realised that one of the numerical gradients had to be positive and the other negative for two lines to be perpendicular to each other. Although it could be described that Amin's use of the graphing calculator was restricted to a display tool, it is nevertheless a powerful tool to aid Amin's learning of straight line. It would have been very difficult for Amin to hand graph the two lines above.

In conclusion, Amin's fragmented understanding by and large evolved from the interplay of three factors: his lack of mathematical knowledge, his inability to make sense of the graphic display, and the ineffective nature of the teacher intervention. His

lack of understanding of the concept of equivalence, for example, posed difficulty for him to make sense of tasks involving graphic display. His difficulty in making sense of the graphic display, focussing only on certain features, subsequently prevented him from seeing lines as members of a two-parameter family. These two problems prevented him from following the teacher's intervention, thus exacerbating his difficulty. What emerges from Amin's activities is that teacher interventions characterised by a curriculum script and strong emphasis towards procedures to the solution resulted in rote learning (Hiebert and Lefevre, 1986).

7.4 Conclusion on the concept of linear graphs in the Structured Learning Model

1. Medium and low attaining students found it problematic to interpret the graphic display which resulted in them focussing on certain aspects of the graph only, in particular either exclusively on the intercept or on the gradient. The high attaining student had developed awareness on to what extent the calculator could assist in the task he undertook and had built an expectation of the display graphs.
2. The lack of related mathematical knowledge has several implications in low and medium attaining students' grasp of the linear graph concept. The lack of knowledge in algebra in particular transposing of equations and solving of equations pose problems in graphing of graphs not in the form of $y=mx+c$ and finding the "unknowns" in the equations respectively. The lack of graphical language also hampered them from following the teacher's curriculum script and therefore following the conceptual connection between that the teacher attempted.
3. Teacher's classroom teaching had very little significance in medium and low attaining students' knowledge construction. However, for the high attaining student, classroom teaching provided the critical source to integrate all the related concepts underlying the learning activities.
4. Teacher intervention that follows a strict adherence to curriculum scripts, emphasis on correct solutions and lack of opportunity for student participation encourage rote learning in that students solve the tasks according to a set of

procedures evident from the way students determine the equations of straight lines by mechanically applying the general equation of the straight-line and using the point-slope formula to determine the gradient.

5. There is very little evidence of the teacher guiding students to give a comprehensive and coherent reason for their answers in the worksheets. Prevalent were incomplete interventions that were unproductive in enabling medium and low attaining students to construct a coherent conceptualisation of the linear graph concept.
6. The ability to translate the symbolic or numerical gradient in the equation geometrically facilitated the development of a coherent understanding of the Cartesian Connection, indicating a need for students to be assisted to represent the gradient graphically.
7. There need to be a careful use of equations in the tasks as equations like $y=-x+2$ and $y=x+2$ can cause confusion for low and medium attaining student who do not recognise the equivalence between $-1x$ and $-x$.
8. The display of graphs by the calculator provided a strong visual impact on students in that it helped to concretise that equations in the form of $y=mx+c$ is a straight-line.

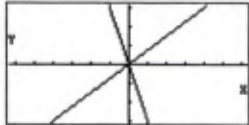
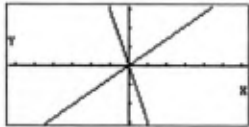
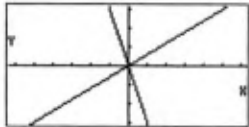
7.5 Analysis of case study students in the Interpretative Learning Model


Table 7.02 shows that Grace answered correctly all the related questions in the post-test. She correctly drew the line parallel to $y=2x-5$ and correctly deduced the equation for this line. Grace was able to distinguish the two equations that have the same intercept and knew how to derive the equation of straight-line in many different situations, indicating a high level of conception of the linear graph concept. The following extracts will try to explore Grace's development of the linear graph concept.

Role of the calculator

The following excerpt demonstrates how the graphing calculator supported and extended Grace's mathematical disposition. Underlying the calculator usage was her mathematical knowledge in fractions and decimals, which also enabled her to follow through the conceptual connection that the teacher was attempting to present.

Excerpt 7.06 Grace's knowledge in decimals and fractions

Line No.	Dialogue	Action	Calculator Display/Worksheet
1	T: Grace?	After asking students to find the line perpendicular to $y=2x$, Teacher asks whole class and points to Grace:	
3	Gr: negative 0.5		
4	T: negative half		
5	Gr: negative half x	Teacher then asks whole class to find the equation of the line perpendicular to $y=4x$. Students explore and after a while Teacher explains on the board:	
8	Gr: It is $-0.25x$ right? (asks Chan)	$y_1=2x$ $y=4x$ $y_2=-\frac{1}{2}x$ $y=-0.25x=-\frac{1}{4}x$ $0.5=\frac{1}{2}$ $-0.5=-\frac{1}{2}$	
10			
13			
15		Teacher then ask students to explore for $y=-3x$. Grace explores with her calculator and tilting the calculator each time after the graphs are displayed. She keys in the following pair of equations and executes the graph after each turn:	
20			
25		1. $y=-3x, y=0.75x$	
27		2. $y=-3x, y=0.7x$	
30		3. $y=-3x, y=0.6x$	
31		4. $y=-3x, y=0.65x$	
35			
40	T: Why not try with $\frac{1}{3}$?	At this juncture Teacher hints to whole class when most students used	

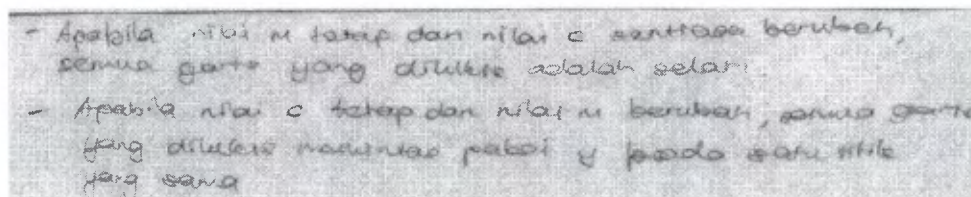
43	T: If $y=\frac{1}{4}x$, what is the line perpendicular to it?	decimals. Teacher begins to craft a curriculum script to guide students by induction to generalise the relationship between two perpendicular lines. Grace responds to Teacher's questions.	
45	Gr: -8		
48	T: $y=7x$?		
50	Gr: $-\frac{1}{7}x$		
55		Teacher explicates to students of the equivalence between the decimals and fractions and reverts to what she has written on the board before. Teacher then explicates the relationship $m_1 \times m_2 = -1$, proceeds with example and reverts to questioning and answering the question for $y=\frac{1}{4}x$, and $y=7x$ again.	
60			

The excerpt shows Grace taking an active part in the classroom instruction in which students were trying to find the relationship between two perpendicular lines. Grace attempted to respond to the questions the teacher put forward (line 1-4, line 43-50), indicating that she was following through the teacher's instruction. Grace successfully obtained the equation of the line perpendicular to $y=4x$ (line 8). When the teacher instructed students to find the line perpendicular to $y=-3x$, Grace began by experimenting with $y=0.75x$ (line 25). The sequence of equations she chose showed that she was executing a systematic exploration by estimating the gradient of the line perpendicular to $y=-3x$ (line 27-31). The graphing calculator afforded her a tool to test her conjectures. Her mathematical knowledge of decimals enabled her to structure her exploration.

Grace continued to work with decimals despite the teacher hinting to students to work with the fraction form instead of the decimal form (line 3, and line 10-13). This shows that intervention that only gave general hints was not useful for even a high attaining student like Grace to recognise what was incorrect in her solution steps, that is, the use of decimals in the equations. Note that towards the later part, the teacher pinpointed the location of the "error" of the solution by directly hinting to try with $\frac{1}{3}$ (line 39). The teacher then contrived a curriculum script to push students to reason inductively the relationship between two perpendicular lines. This proved successful for Grace, which was evident from her responses to the teacher's questions (line 43-48). Her unsuccessful attempt at her investigation might have been due to her erroneous methods and made her

more insightful to the explanation that the teacher presented towards the end of the task. However, what was clear was that Grace's knowledge of decimals and fractions enabled her to structure her exploration and use the calculator to support her conjectures. The teacher's classroom instruction provided Grace the source to move her to a higher level of conception. The graphing calculator afforded her to transform her overall approach to the tasks into a process of extending her mathematical disposition.

This is also reflected in what she wrote in her worksheet in her exploration of graphs of $y=mx+c$ with the calculator:



- “ - When the value of m is constant and the value of c varies,
all the lines drawn are parallel.
- When c is constant and the value of m varies,
all the lines that are drawn intercepts the y axis at the same point.”

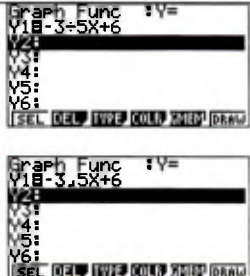
The conclusion that Grace articulated at the end of her investigation shows that she understood the graphs of $y=mx+c$ as a two-parameter family. More importantly, it shows her ability to structure her investigation and make use of the calculator to mediate the linear graph as a two-parameter family.

Role of the teacher

The following excerpt shows a teacher intervention that kept Grace on track of the task.

Excerpt 7.07 Teacher's corrective feedback on syntactic error

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Okay, you obtain graphs with asymptotes..... Why?	As soon as Teacher finished showing how to make y the subject of the equation $3x+5y=30$, Grace keys in the equation into her calculator. She is surprised by this display and asks Teacher. Grace is quiet.	
4	T: Go back to your equation...	Grace reverts to the text screen and Teacher detects	
5			

7 10	T: If it is fraction....how do you key in ? Look at the manual....here 3 divided by 8....	Grace's mistake. Teacher directs Grace to look at calculator's syntax for keying in fraction. Grace keys in the equation and obtains her graph.	
-------------	---	---	---

The excerpt depicts Grace keying in the equation into her graphing calculator immediately after the teacher had demonstrated how to make y the subject of the equation, before she proceeded to hand graph the line as required by the task. It seems she wanted to inspect the graph before she constructed it. When the graphic display was not a straight line, she was surprised and asked the teacher (line 1). This indicates Grace had an expectation of the graphical output of the equation. The teacher directed Grace to look at the equation that she had keyed in (line 4), however Grace was not able to detect the syntactic error she had made (keying in $-3 \div 5x$).

When Grace did not respond (line 3), the teacher gave an explicit corrective feedback by telling Grace how to key in the calculator syntax for fraction (line 7). This corrective feedback (line 4-10) by way of direct telling did not address Grace's missing knowledge (reciprocal equation generates reciprocal graph). However, the teacher's action kept track of Grace's task goal that was to ascertain what the graph of $y = -\frac{3}{5}x + 6$ looked like. The excerpt also indicates that getting students to repair a syntactic error is not easy, even for high attaining students like Grace. It also shows the value in the explicit corrective feedback that the teacher gave to Grace to recover from her syntactic error.

In conclusion, Grace's conceptualisation of the linear graph concept could be attributed to her ability to make sense of the graphic display and her rich conceptual knowledge base related to the concept. In one activity Grace used the graphing calculator to compare the graph of $y=mx$ and $y=10x+20$. Her understanding of the gradient concept where "m" in the equation as the entity that governed the slope-related graphical property and her ability to interpret the graphic display provided her the structure to relate it to graphs of another family that have the intercept, contributing to her appropriation of the linear equation of the form $y=mx+c$ as members of a two-parameter family. Her mathematical knowledge in fractions and decimals also enabled Grace to

systematise her exploration with the graphing calculator, and follow the conceptual connection that teacher made during classroom instruction. What also emerges in Grace's learning activities was that her ability to make sense of the graphic display predisposed her to have an expectation of the graph, recognizing errors when they occur.

PRESHA (MEDIUM ATTAINING GIRL)

Presha's performance in the post-test indicates that her understanding of the straight-line concept is disjointed. She was able to answer Question 12 correctly but it is unclear why she did not attempt Question 11. Presha had no difficulty with algebraic manipulation as shown in the pre-test. She answered Question 14 correctly, confirming that she recognised the equivalence of $y=\frac{1}{2}x+2$ and $2y=x+4$. She correctly drew the line passing through the origin and parallel to $y=2x-5$ in Question 17 but did not name the equation for the new line. For Question 10, she determined the gradient of the first line using the point slope formula although it is easier to use right triangle method which she applied when solving Question 15. She determined the equation of the first line correctly but the equation of the second line parallel to it was wrong. Both the intercept and gradient values were wrong, indicating Presha's confusion with the gradient and intercept concepts.

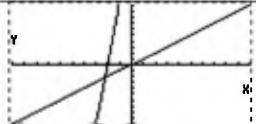


The following extracts will try to illuminate several aspects of Presha's difficulty with the learning tasks focussing on her calculator usage and teacher intervention.

Role of the calculator

The following excerpt shows Presha's inability to make sense of the graphic display, which precluded her from developing the concept of intercept. It also shows the teacher's incomplete intervention.

Excerpt 7.08 Presha's inability to make sense of the graphic display

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: What is that display about?	Presha is graphing the graph of $y=x$ and $y=10x+20$ with	

4 5	<p>Pre: This is the graph of $y=10x+20$... here $y=x$</p> <p>T: Yes...but your graph is not complete, so what must you do?</p> <p>T: why do you do like this?</p>	<p>her calculator.</p> <p>Presha points to the respective graph.</p>	 <p>(This must be the display as the following display sees the STD view window being generated when Presha activated the view window.)</p> 
10	Pre: ..because the view-window.	Pre's view window shows the STD view window.	
12	<p>T: Okay..... can you use INIT?</p> <p>Pre: No</p>		
15	<p>T: Why?</p> <p>Pre: The graph..the scale is different.</p>		
20	T: The scale is different...but why can't you?		
22	Pre:... because it does not give the value.		
25	<p>T: Yes, what next? Look at this value.</p> <p>T: Here...what is the value?</p> <p>Pre: 20</p> <p>T: Here:45</p> <p>Pre:...It is not large enough.</p> <p>T: Yes, it is not large enough, so what value do you need?</p> <p>Pre: Much bigger.</p> <p>T: Okay</p>	Teacher points to the values in the work sheet.	
29 30			
35		<p>Pre proceeds with this view window and generates her graph.</p> <p>Teacher moves away without inspecting.</p>	

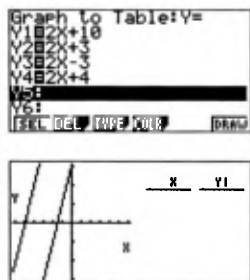
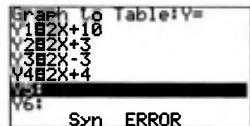

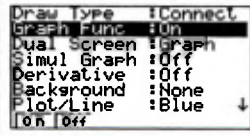
Here it is apparent that Presha did not know what features of the graph she ought to choose to make the comparison, whether it is the slope, and/or the intercept, and/or the origin of the graphs of $y=x$ and $y=10x+20$. She chose an inappropriate view-window that did not show the intercept of the other graph of $y=10x+20$ (line 4). The teacher did not rectify Presha's vague use of the word "value" to indicate "intercept" (line 22). The teacher's focus of the intervention was to get Presha to resolve the error of the incorrect choice of view-window (line 12). The teacher pursued this extensively to the point of asking her an invalid question that had nothing to do with the goal of the task (line 29). The value "45" was not the intercept of the graph, it was the value of y when $x=2.5$. It resulted in Presha choosing a view-window to include this value! Astonishingly, the teacher did not wait to see Presha's choice of the values of the view-window. The teacher did not complete the intervention to get Presha to focus on the intercept. Presha's

finding in this task as written in her work sheet was restricted to: "the graph of $y=x$ passes (0,0) and the graph of $y=10x+20$ does not pass through (0,0)". There was no mention of the intercept or the slope, although the graphic display would have displayed both features. Clearly, Presha missed these two features. It could be said that the role of the calculator essentially was just to execute the tasks and to display the graphs.

Role of the teacher

The following excerpt shows many aspects of unproductive teacher intervention including not correcting Presha's syntactic error.

Excerpt 7.09 Intervention that did not resolve Presha's difficulty and syntactic error

Line No.	Dialogue	Action	Calculator Display/ Worksheet
		Pre is exploring graphs of $y=mx+c$. Pre is keying in the first set of equations suggested by the Teacher.	
1	Pre: SYN Error...why?		
2	T: Key in again....what did you do in?	After generating the second graph, the Syn ERROR appears on the screen.	
5	Pre: GRAPH	Pre verbalise the screen she is working in.	
	T: Graph...Graph to Table?	Teacher advises to change to Graph.	
	Pre: Graph to Table Y equal to...		
10	T: Okay, no need Graph to Table, go to Graph only		
12	Emy: Yes.....	Teacher points out to Emy's mistake.	
	T: Graph only...		
	Pre: I have entered Graph.		
15	T: Emy, not Graph to Table ..G to T		
	T: Okay now compare the value....	Pre is still confused.	
	Graph Function... Look at the difference, the m is the same.		
	Some girls: Still Syn Error!		
19	Pre: Teacher, Graph Function.... on or off?	Teacher moves away.	
20	Nurul: Do we change the view window?		

This excerpt sees Presha wrestling to coordinate the functionality of the calculator and the goal of the task. Presha used the examples of each family of graphs suggested by the teacher in her exploration of families of straight-line graphs. Presha chose an inappropriate graphic screen that showed the graph and table side by side to draw the graphs. Presha did not know that when the “Syn ERROR” appears on the screen, a syntax error had been committed (line1). As this error message appeared after the second graph was generated, the error would be in the notational syntax in “Y3” which Presha failed to detect. Her failure to detect the “Syn ERROR” sign when it appeared on the screen also indicates Presha’s lack of awareness of what was generated of screen.

The teacher’s initial reaction was to ask Presha to key in the equations again (line 2). The teacher then decided to give hints so that Presha (and the students) corrected the mistake themselves by asking what was the text screen the equations were keyed in (line 2, second half of the sentence). The teacher did not realise that this hint conveyed a connotation that the error was related to the modes of the calculator that Presha (and the students) were working with. Presha was working in the TABLE Mode and using the dual screen facility. When the teacher instructed to change to “graph” (line 12), she meant operating in the GRAPH Mode but for Presha who was already operating in the TABLE Mode, she interpreted it as changing the dual screen command in the SET UP to “Graph” (line 13). When the teacher directed to compare the values (line 15) in the “Graph Function”, what she meant was asking Presha (and students) to look at the text screen of the GRAPH Mode to view the equations and observe the values “2” in them. However, Presha thought the teacher was referring to the “Graph Function” command in the SET UP (line 19). Clearly the teacher and Presha were referring to different screens of the calculator but both screens had the same syntax that were mentioned as shown below in the figure below.

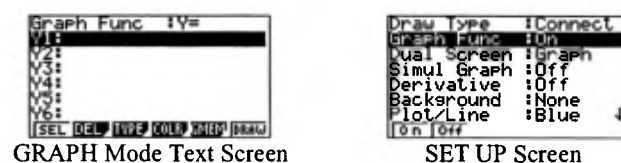


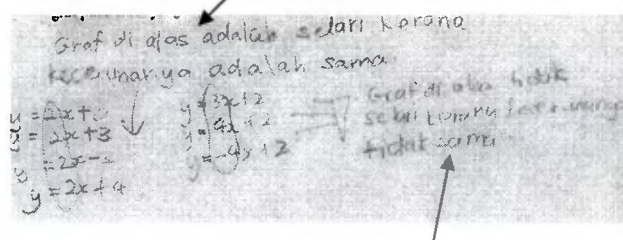
Figure 7.03 “Graph Func” syntax on both screens

Several things emerge here. First, the teacher tended not to diagnose Presha's error and did not ensure that she recovered from her error. Second, the teacher lacked some

awareness of the commands in the calculator. Third, the teacher assumed that Presha understood the terminology she used and the instructions she gave out. Fourth, asking to repeat the task (line 2) was unproductive as it was observed students reiterated the same mistake (line 17). It also points to the fact that syntactical mistake involving notations was not easy to ascertain. Fifth, the teacher ignored Presha's response. The teacher also ended the intervention prematurely.

However, Presha eventually succeeded in generating the graphs. The following extract of what she wrote in her work sheet shows the outcome of Presha's exploration. The examples that the teacher provided gave more direction with respect to the specificity of task expectation involving exploration of graphs of $y=mx+c$. It also indicates that the immediacy of the graphic display on the calculator confronted Presha with the connection between the numerical gradient in the equation and the slope of the line.

"These graphs are parallel because their gradients are the same."



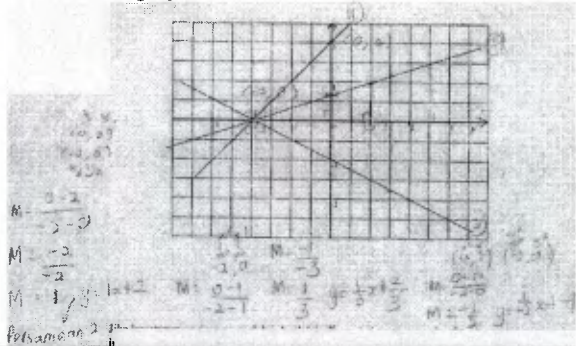
"These graphs are not parallel because their gradients are not the same."

However, Presha did not make the connection that the graphs of $y=mx+c$ as a two parameter family. From the written answer, Presha was fixed to only one aspect of the graph, that is the gradient, which was reflected in the teacher's emphasis on "m" in the (line 17). This also reflects the way Presha used the calculator, which could be described as a display tool. It was clear that teacher intervention was needed here to direct Presha to explore for graphs of different gradients with different intercepts. The teacher intervention only helped to the extent of getting Presha to see that equations of the same "m" were parallel to each other, and those of differing "m" were not parallel to each other. Presha did not appropriate the parameter "intercept".

It must be pointed out that Presha missed one session of the lessons involving tasks that investigated on how to determine the gradient of the lines (the earlier section of Lesson 11). This meant she had more to cope in the subsequent lesson that involved

determining the equations of straight-line graphs. Arguably this might have contributed to the fragmentation in her understanding, but Presha's confusion in answering Question 10 could be more of the consequence of the teacher intervention she experienced when solving similar problem as shown in the following excerpt.

Excerpt 7.10 Intervention that focuses on obtaining the correct answer

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1 5 10	T: this is line 1, this line 2, this line 3.	Teacher inspects Presha's work. Teacher labels each line on Pre's graph.	
13 15 20 25 30 35 40	<p>T: Where is the equation of your first line?</p> <p>Pre: Here..</p> <p>T: -2..why -2?</p> <p>Pre:....O ya, ya...</p> <p>T: That is your first mistake, okay name this, this is equation 1.</p> <p>T: Where is the second equation?</p> <p>How did you get 1.3?</p> <p>T: This line passes through Here...we use the point given...</p> <p>T: The clue is given at the x-axis.</p> <p>T: The equation of the line.. $y=mx+c$...passes through this point..what is the coordinate?</p> <p>Pre: (-2,0)</p> <p>T: The value of y is given..0. m you have found just now, m for this...what is the gradient you found just now?</p> <p>Pre: $\frac{1}{3}$</p> <p>T: Okay.., x is -2, now find c.</p> <p>T: y is equal to zero....negative 1 times negative 2....then ... find the equation.</p>	<p>Pre points to the equation</p> <p>Teacher hints that the gradient is incorrect.</p> <p>Pre tries to explain, points to the values in the figure, discovers her mistake and corrects it.</p> <p>Pre points to her working.</p> <p>Teacher refers to the intercept.</p> <p>Teacher did not wait for Pre's explanation and proceeds to explain.</p> <p>Teacher points to the point (-2,0) in the diagram.</p> <p>Teacher writes:</p> <p>$Y=mx+c$</p> <p>Teacher points to the coordinate.</p> <p>Teacher points to the graph.</p> <p>Teacher continues to write:</p> <p>$0=\frac{1}{3}(-2)+c$</p> <p>$0=-\frac{2}{3}+c$</p> <p>$c=\frac{2}{3}$</p> <p>Teacher leaves, Presha gives an exasperated look.</p>	

The manner in which the teacher intervened was clearly characterised by emphasis on the correct solution rather than why the task was solved in that particular way, for example the teacher was quick to highlight the number of mistakes Presha made (line 20). The teacher did not wait to hear how Presha obtained 1.3 for the intercept (line 23), (presumably determined by estimation from the diagram as the intercept is not an integer). The teacher continued with her solution plan and directly told Presha each step of the solution (line 27-43).

The teacher also made no comment on Presha's use of point-slope formula. Notably in the diagram, Presha wrongly chose the point (0, 1) instead of (0, -1) and the point (-2, 0) to determine the gradient for the third line resulting in $\frac{0-1}{-2-1}$ which would have given the answer as $\frac{1}{3}$ but Presha inserted a negative sign in front of the gradient as $-\frac{1}{3}$. While this indicates she was aware that the negative sign denotes a line in the negative direction, it also shows that she did not understand how the point-slope formula worked. She was merely applying it mechanically.

The above teacher intervention that emphasised on the correct solution without soliciting from Presha to reason the solution steps precluded Presha from making the connection that the x and y value in the equation $y=mx+c$ is any point (x,y) that lies on the line. As shown in the way Presha solved the above problem, Presha clearly memorised the point-slope formula and did not connect this as equivalent to the right-angle triangle method and how the "m" in the equation could be represented in the ratio form diagrammatically.

In conclusion, what seems to emerge from the video transcripts was Presha's continuing struggle with the overall functionality of the calculator including the functionality of the GRAPH Mode she used to execute the tasks. They competed with underlying mathematical ideas embedded in the tasks. Consequently, she had difficulty in making sense of the graphic display. The role of the graphing calculator in Presha's learning was restricted as a display tool. Nevertheless, with careful teacher intervention, the calculator facilitated her to view the global aspect of the graph. The visual display of graphing calculator facilitated Presha to see that equations of the same "m" were parallel to each other and those of differing "m" were not parallel to each other.

Presha's learning activities highlight the fact that correcting syntactic errors was not easy. They also highlight several aspects of inappropriate teacher intervention, which subsequently led to Presha being unable to bridge the gaps in her knowledge construction. One aspect was the teacher's lack of awareness of the errors that Presha made in operating her graphing calculator to carry out the tasks. This unawareness saw the teacher in giving "incorrect" feedback, which resulted in Presha not resolving her errors and difficulties with the related functionality of the calculator. The second aspect was the tendency for the teacher not to complete the intervention. The incomplete teacher intervention also resulted in Presha not resolving her difficulties. Another aspect was the teacher intervention that emphasised on correct solution without soliciting from Presha to reason the solution steps. This precluded Presha from making the connection that the x and y value in the equation $y=mx+c$ is any point (x,y) that lies on the line. It also resulted in Presha "memorising" (with no understanding) the point-slope formula and the steps to the solutions.

EMY (LOW ATTAINING GIRL)

The post-test shows that Emy's understanding of the straight-line concept was still transitory. In the post-test, Emy was able to solve Question 10, Question 15 and Question 17. She ignored Question 11 and Question 12 and attempted Question 14. Her answers show that she had difficulty with algebraic manipulation. Both Question 11 and Question 12 require algebraic manipulation to make y the subject of the equation.

It is very encouraging to note that Emy was successful in answering Question 17 indicating her understanding of the origin concept, and the concept of parallel lines. This indicates that Emy's understanding of the straight-line concept was still at the "fragile" state. Given the extent of Emy's progress, the analysis will focus on how Emy arrived at such level of understanding.

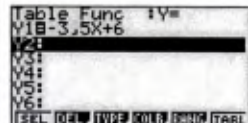

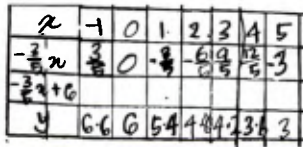
Role of the calculator

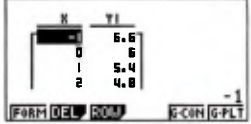


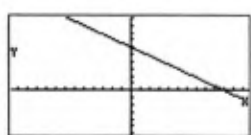
Emy's lack of computational skills was evident in her attempt to hand-graph

$y = -\frac{3}{5}x + 6$. The following excerpt shows how difficult it was for Emy to construct a

straight-line graph. It also depicts how the teacher helped Emy to use the graphing calculator as a resource tool to help with her hand graphing.

Excerpt 7.11 Emy's difficulty in hand graphing

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	Emy: What to do? T: Calculate first... here $-1 \dots -\frac{3}{5}$ times $-1 \dots$	Emy has finished graphing for $y=4x+9$ and is trying to hand graph the line $3x+5y=30$. Teacher has demonstrated how to rewrite it to $y=-\frac{3}{5}x+6$ and suggests the range for x from -1 to 5 . Teacher shows how to substitute the value of x .	
5	Emy: Then what? T: Do the table, get the value of x , the value of y , fill in your table.		
10	Emy: Ooh, like this?	Emy shows the table for the previous graph of $y=4x+9$	
11	T: The table..get the value of x , the value of x , draw it like this.	Teacher confirms.	
15	T: Did you get it? Emy: Why is it different?	Emy sees Nurul's table in the calculator is different.	
18	T: Nurul is doing in the dual screen. ...Look at your range...your x , what does it begin with?		
20	Emy: -1		
24	T: Ends at? Emy: 5		
25	T: The "interval" is one... Emy: the pitch? T: You put it at "intervals" of $1 \dots 1, 2, 3$ Emy: So it is 1 .		
29	T: Aaa..Execute, Table, get the values from the table...	When Teacher moves away, Emy was seen repeating the whole process of generating the table. She then fills the values of her table and begins to plot the graph on the graph paper.	Emy guessed the meaning of pitch.
30			
35			
40			Emy has difficulty in reading the scales of her graph when it involves

45	T: When $x=1$, $y=5.4$when $x=2$, $y=4.8$	decimals. Teacher marks the points on Emy's graph paper. Emy completes the plotting the points but finds they are not on a straight line! Teacher does not realise that Emy's scale is 2cm: 1 unit .	
50	Emy: Why is it not straight?		
	T: Selecting your scale...it must be the correct scale.		
55	Emy: After that?		
	T: Then, it must be correct.		
60	Emy: How?	Teacher ignores Emy's question. Emy is in a state of confusion. She joins the points anyway and produces a line graph but not a straight smooth one.	
65		Teacher sees Emy's incorrect graph, gives no corrective feedback but asks Emy to plot the graph with her calculator.	Emy's graph
70		Teacher points at the intercept at the graph.	
72	T: What is this coordinate?		
	Emy: 0,6		
75	T: Can it be in INIT..0,6?	Teacher probes whether the graph could be drawn on the INIT view-window.	
	Emy: Yes.	Teacher gives hint.	
80	T: Can it? (with surprise).		
81	Emy: Eh, no, cannot....		
82	T: Ha...STD, or do it in other view-window....VW1....VW2	Emy recalls the first and then uses the second view window.	

The excerpt above shows that Emy had difficulty in generating manually the table of values for the graph of $y=-\frac{3}{5}x+6$ because of her lack in computational skills. She successfully completed the table by obtaining the values from the calculator through teacher intervention (line 29) but despite this, examination of Emy's construction of the graph of $y=-\frac{3}{5}x+6$ produced a graph that was not straight. It is evident from the graph that Emy incorrectly read the scales on the y-axis as they involved decimals. This simply shows how difficult it was for Emy to construct a straight-line graph. It is here that the

calculator seemed to support Emy's learning of the straight-line concept. The graphic display of the graph of $y=-\frac{3}{5}x+6$ provided corrective feedback when Emy and the teacher could not find out why the graph Emy had hand graphed was not a straight line.

Role of the teacher

The above excerpt also indicates that Emy was gradually building up her knowledge of the functionality of the TABLE Mode. The teacher was also seen guiding Emy towards using the calculator meaningfully. In the intervention above, the teacher guided Emy to see that the table generated by the calculator was the same as the one constructed manually (line 11, 29) and the importance of choosing the view-window to enable to see the important features of the graph (line 75-82).

Notable in the excerpt was Emy's increasing understanding of the language of the calculator and how it relates to mathematical language, for example pitch and interval (line 24, 25). She also seems to understand the calculator and mathematical language that the teacher was using, for example range (line 18), execute (line 29), STD (line 81), and coordinate (line 72). She was seen to move between the screens indicating that she understood the operational syntax of the calculator.

The excerpt below shows another aspect of teacher intervention to guide Emy with exploration of the graphs of $y=mx+c$.

Excerpt 7.12 Teacher's scaffolding in Emy's exploration of graphs of $y=mx+c$

Line No.	Dialogue	Description/Action	Calculator Display/ Work sheet
1	T: Try $y=2x+2$	Teacher suggest to the girls and Emy of graphs that can be inputted eg. $y=2x+2$	
4 5	T: If $m=2$, for eg. $c=3$ what happens to the equation? Emy: $3x+\dots$		
10	T: What should come first? Emy: m T: So it is 2.. Emy: $2x$	Teacher gives corrective feedback.	
14	T: $2x$ plus		

15	Emy: $2x+3$		
20	T: This is the first equation ... try for the second equation, investigate the kind of graphs you get...	Emy listens to her friend and Teacher.	
25	Nurul: How is the second equation? T: Just put ... what do you want the m to be and what do you want the c to be. Nurul: Any value? T: Not the same values, change them...	Emy continues to listen.	
30	Zulfah: I got them...like this Nurul: I am going to do the graphs of these equations.	Zulfah shows her calculator display to Emy	
33	T: Let us make the value of m constant:	Teacher suggests to Emy.	
35	$Y1=2x+5$ $Y2=2x-3$ $Y3=2x-4$ $Y4=2x+4$		
40	Erase your equations just now and take these examples.... The value of m is always the same..	Emy is seen keying in the equation $Y2=2x+4$ in her calculator. Emy shows her calculator display to Nurul.	
41			

Emy was seen exploring for the properties of the graphs of $y=mx+c$ when the teacher suggested that she should try for the graph of $y=2x+2$ (line 1). The teacher was seen guiding Emy to the meaning of the parameters in the equation (line 4-14). The teacher continues to support Emy's exploration by suggesting to her to keep the "m" constant (line 33) and provided examples (line 35-41). This careful intervention might have enabled Emy to systematise her exploration of graphs of $y=mx+c$ as evidenced from her conclusion of the exploration written in her work sheet. It seemed Emy also extended her exploration to include other family of straight line graphs where the "c" was kept constant.

"parallel because the gradients are the same"

$y = 2x + 10$
 $y = 2x + 3$
 $y = 2x - 3$
 $y = 2x + 4$

$y = 3x + 2$
 $y = 4x + 2$
 $y = -4x + 2$

"not parallel because the gradients are not the same because it cuts at the point (0,2)"

However, her second answer that described the lines were not parallel because the “the gradients were not the same, because they intercept at (0,2)” indicates Emy’s fragility on the straight line concept as a two parameter family and further teacher intervention seems crucial to enable Emy to make the Cartesian Connection.

In conclusion, Emy’s activity indicates that her general understanding of the functionality of the calculator evolved as she progressed in her learning. Notably, Emy was able to relate that the table values generated in the calculator were the same table values she had to generate herself. Emy’s activities give a positive view of how the graphing calculator could support a low attaining student in the learning of the straight-line concept with careful teacher intervention. Her lack of algebraic, number, and computational skills did not preclude her from understanding the line as an object with the slope and intercept attribute. What seems to emerge from this case study was how careful teacher intervention was necessary to support a low attaining student like Emy to systematise her exploration with the calculator and to give specific guidance to ensure that the student sees the specific features of the displayed graph.

7.6 Conclusion on the concept of linear graphs in the Interpretative Learning Model

1. The calculator supports the learning of mathematical language for example the meaning of coordinates and range.
2. Detecting and correcting students’ syntactic errors is essential in enabling students to pursue their tasks as it is not easy for students to ascertain these type of errors.
3. An awareness of the commands in the calculator is essential in order to deal with semantic errors.
4. The teacher tended to use curriculum scripts in her interventions which seem to benefit high attaining students who understood the mathematical language used and the related mathematical knowledge.

5. Teacher intervention is necessary for low and medium attaining student with respect to the specificity of the task expectation involving exploration of graphs of $y=mx+c$. Low and medium attaining students had difficulty in structuring their investigation.
6. Clearly missing in students' activity was the use of the DYNA Mode to explore the properties of the linear graphs by varying the parameters in the equation.
7. Intervention with an emphasis on the correct solution could encourage memorization of procedure and preclude students from making the Cartesian Connection.
8. The display tool attribute of the calculator enabled students to see that graphs that have the same numerical gradients are parallel to each other, and that graphs that do not have the same numerical gradients intercept at a point.

CHAPTER 8

CASE STUDY 4: THE CONCEPT OF NON-LINEAR GRAPHS

The background to the non-linear graphs concept is described in section 8.1, including the performance of the three case study students in each learning model on the related test items in the post-test and Assessment 5. Section 8.2 describes the expected learning outcomes in relation to non-linear graph concepts using the graphing calculator. It begins with a brief outline of the mathematical tasks on the concept of non-linear graphs in each learning model, and follows with an explanation on how the graphing calculator is thought to assist in understanding the concept of non-linear graphs. Section 8.3 describes how the case study students in the StructLM appropriated the concept of non-linear graphs with the graphing calculator with a conclusion in section 8.4. Finally Section 8.5 discusses how the case study students in the InterLM appropriated the concept of non-linear graphs with the graphing calculator with a conclusion in section 8.6.

8.1 Background

The creation of a non-linear graph by hand is not easy because many points are needed for plotting. The computation process is tedious and time consuming if the equation is complex. The generation of table of values followed by the plotting of points on a suitably scaled graph may result in students losing sight of the meaning of the task itself (Swan, 1982). For a student who has poor basic operation skills, this can be a very daunting task. Avoidance of non-integers can result in creating incorrect graphs. Examples of this include a “flattened out” parabola at its vertex, or a reciprocal graph with a connecting line at the asymptote although the error in the latter graph may also originate from students’ conviction in continuity or connectedness without necessarily a correct understanding of this concept (Goldenberg, 1988). It appears that successful construction of a non-linear graph also involves an expectation of what the complete graph of the non-linear equation looks like. The complete graph of the non-linear equation is a graph that shows all its important features, for example its y-intercept, asymptote and zeros (Demana et al., 1993). This entails choosing the right scales for

both axes so that the whole graph can be constructed or viewed, and this means having to recognise which visual aspects of the graph will not change under the change of scales (e.g., the x-and y-intercepts).

The interpretation of non-linear graphs requires the understanding of the meaning of the variable and the parameters of the algebraic equation of the graph. For polynomials, it entails recognising that the highest power (the order) of the polynomial governs the general shape of the graph associated with it.

Graphing of non-linear equations necessarily involves attention to the changes of the shape of the graph and/ or position of the graph. There are inherent conflicts of absolute value and sign in graphs. The convention is that values get bigger as the focus moves up and to the right, and values get smaller to the left and down. However, competing with this vision is things increase in absolute size as they move symmetrically out from the origin. Therefore, students have to simultaneously track two features of magnitude, the size and the direction, which may pose problems for some students who have problems of fixation (Bell et al., 1987) and have a tendency to concentrate on one aspect and exclude the coordination with other aspects of the graph.

The following issues were examined in the tests: whether students could

- (i) identify which graphs are cubic and reciprocal graphs from the shape of the graph, and recognise the quadratic graph by looking at the form of the equation (Question 7),
- (ii) differentiate the form of equation between different straight lines and between the straight line and the parabola (Question 13), and
- (iii) recognise the different forms of equation to the general shape of several non-linear graphs and differentiate between the equations of two parabolas that had undergone different transformations (Question 2 in Assessment 5).

In general, all the case study students were able to name the graphs correctly except for the low attaining student, Emy. Similarly, all students were able to recognise to differentiate the form of the equation for the straight line and the parabola. What was remarkable was that all students were able to recognise the reciprocal graph and the

equation associated with it. Tables 8.01 and 8.02 below summarise how the case subjects performed in test items in each of the learning model.

Table 8.01 Students' performance on non-linear graphs in StructLM

Name of Student	Concept / Tasks		
	Q.7 Choose which graph fits the statement: (i) cubic graph (ii) graph of $y=1-x^2$ (iii) reciprocal graph	Q.13 Which graph most likely represent the graph for the equation $y=3/2x+1$? Why?	Q.2 (Assessment 5) Match the following sketch with the following equations: $Y=2x^2+3$ $Y=5/x$ $Y=x^3-x$ $Y=5-x^2$
Wan (High Attaining Boy)	All correct.	Correct. The graph moves up and the intercept is above the x-axis.	All correct.
Ng (Medium Attaining Boy)	Only the reciprocal graph matched. (seems Ng did not understand the question)	Correct but no reason given.	Only two are correct. The cubic graph and the upright parabola are interchanged.
Amin (Low Attaining Boy)	All correctly matched	Correct but no reason given.	The reciprocal and the cubic graphs are correct but the two parabolas are in reverse order.

Table 8.02 Students' performance on non-linear graphs in the InterLM

Name of Student	Concept / Tasks		
	Q.7 Name the graph that fits the statement: (i) cubic graph (ii) graph of $y=1-x^2$ (iii) reciprocal graph	Q.13 Which graph most likely represent the graph for the equation $y=3/2x+1$? Why?	Q.2 (Assessment 5) Match the following sketch with the following equations: $Y=2x^2+3$ $Y=5/x$ $Y=x^3-x$ $Y=5-x^2$
Grace (High Attaining Girl)	All correctly matched.	Correct but did not reason.	All correctly matched.
Presha (Medium Attaining Girl)	All correctly matched.	Correct but did not reason.	All correctly matched.
Emy (Low Attaining Girl)	Only reciprocal graph is correctly named.	Correct but did not reason.	All correctly matched

However, there were inherent weaknesses in these test items. They did not discriminate the varying degrees and diverse forms of understanding that students have about non-

linear graphs, specifically parabola transformations. Therefore, in the cases presented, I shall attempt to outline these differences by examining what students did in their learning activities including what they had recorded in their worksheets.

8.2 Expected learning outcomes with the graphing calculator on non-linear graphs

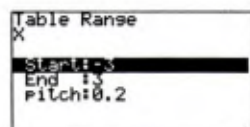
It was expected by the time students reached this stage of learning, they had the knowledge to work in at least the GRAPH Mode and TABLE Mode of the graphing calculator. Therefore the tasks were designed with an expectation that students could use these facilities in their graphing work. The tasks included the drawing of the same quadratic graph in different view-windows so that the students were confronted with the effect of “scales” and “pixels” on the shape of the graph. Several tasks were also designed so that the students were confronted with the fact that the highest power (the order) of the polynomial governed the general shape of the graph. These polynomials were restricted to “simple” polynomials in which they contained only one parameter in the leading term in order the students were not confounded with other parameters. Students explored several families of the quadratic graphs so that they were acquainted with how the other parameters influenced the shape and the position of the graphs. Hand graphing of a quadratic graph was included to confront the students with “roundness” at the vertex. Some tasks involved the sketching of the polynomials to confront the students the importance of “connectedness and smoothness” of the curves.

The mathematical tasks in both learning models involved examining the characteristics of the graph of $y=x^2$ and other quadratic graphs of different families. The tasks also involved generating “simple” graphs of other polynomials up to the degree 6. By “simple” was meant no other terms in the polynomial except the leading term. Exploration of some reciprocal graphs and conic graphs was included. The tasks in the InterLM were more open in that students had to assign their own parameter values in their investigation. In many cases, they had to decide themselves which facility of the calculator they were going to use in their investigation.

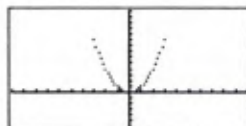
By entering the algebraic expression into the graphing calculator, its graph is immediately created. With the graphing calculator, it was expected that students would

be able to view many graphs and their corresponding equations, and thus recognise the general shape of the graphs associated with the equations that represents them.

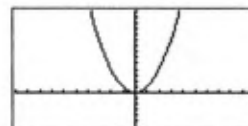
The TABLE Mode of the calculator displays values for x and y for more than one equation. This gives student a means to examine and compare the values between x and y of each equation and between the different equations. Students need to specify the range of values of x which determines the points that are plotted in the G-PLT command. It enables students to view values of the domain x and how it affects the overall shape of the graph. This was expected to confront students that the range of values of x is not bounded by values within the view-window – the graph that is generated in the connected form using the G-CON command “extends” towards the boundaries of the view-window. This is illustrated in figure 8.01



The “Table Range” values



G-PLT command



G-CON command

Figure 8.01 The parabola generated in the G-PLT and G-CON command with range from $x=-3$ to $x=3$

With the graphing calculator, students can also then begin to examine the relationship between the graphical characteristics and algebraic parameters. It was expected that the great speed of plotting would enable students to study families of graphs and the overall behaviour of each family of graphs. In the DYNA Mode, the graphs are dynamically generated allowing the user to see how the shape of a graph is affected as the value assigned to one of the coefficients of the equation changes.

A parabola graphed using different scales appears to change shape. The graphing calculator enables the quick generation of different view-windows thus allowing the user to immediately see the effect of scales on the shape of the graph. It was expected that students would be able to differentiate this from the transformation of the parabola arising from the change of the parameter in front of x^2 .

By pressing the cursor button on the graphing calculator, students could shift the x -axis further down or further up to view the same graph. This facility was expected to confront

students that two congruent parabolas positioned at different heights in the same viewing window also appear to have different shapes. Similarly, the relevant cursor button allows the y-axis to be shifted to the left or to the right allowing the student to see that the range of values of x is unlimited.

The graphing calculator gives students greater dexterity to investigate dynamically the effects of the changing parameters and variables and arguably should confront them with the fact that the variables in the equation gives the shape of the graph, and the parameter determines the position of the graph.

8.3 Analysis of case study students in the Structured Learning Model

WAN (HIGH ATTAINING BOY)

As Wan's tests results show, he not only recognised that the highest power of the polynomial governed the general shape of the graph, he was able to distinguish between the different families of parabolas and recognise the equations that represented them.

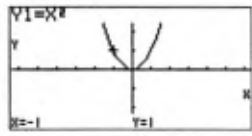
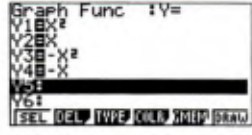
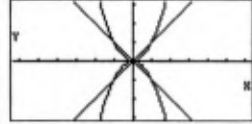
This case study will focus on the interplay between Wan's manipulation of the calculator facilities and his own mathematical knowledge in exploring the various non-linear graphs and their transformations.

Role of the calculator

Analysis of the videotape shows that Wan exploited the different facilities of the graphing calculator. One example was using the "cursor" facility to push the x-axis further down and further up to see how it affected the shape of the parabola. In another example, Wan used the "table" facility to obtain the values to construct the graph by hand. He was also seen exploring the family of graphs of $y=x^2+a$ and using the trace facility to see how the y-values in each graph differed.

The following excerpt exemplifies how Wan used the graphing calculator and how the graphing calculator itself shaped Wan's mathematical activity.

Excerpt 8.01 Transformation in Wan's activity

Line No.	Action	Calculator Display/ Worksheet
1	As Teacher is explaining on the board how to key in x^2 , Wan has already generated the graph and seen to be tracing it!	
5	A little while later, Wan is seen exploring for these different graphs.	
10		

Wan was making comparisons and trying to connect the existing concepts of graphs of $y=mx$ to the new concepts of graphs of $y=ax^2$. He systematically carried out his exploration by carefully choosing the values of the parameter and then extended it to include negative parameters. The calculator made it possible for Wan to test his conjectures. The excerpt shows how the graphing calculator not only enabled to integrate his previous knowledge with new knowledge. It also shows a transformation in Wan's mathematical activity in which he was going beyond the mathematical task given. This was also observed in Wan's exploration with cubic graphs. The calculator enabled him to recognise the sketch in his worksheet as $y=-x^3$. He was also able to give a correct hypothetical equation to the other sketch of the cubic graph as $y=x(x+4)(x+1)$.

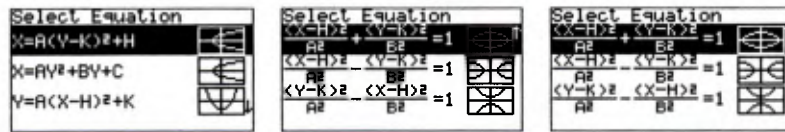
Wan's well developed conception of different aspects of the functionality of the calculator enabled him to discriminate which properties of the parabola were preserved in different situations. Wan wrote in relation to his exploration of graphs of $y=x^2$ in different view windows that

Bentuk masih sama tetapi kecuraman bentuk berubah

"the general shape remains the same but the steepness of the shape changes"

His understanding of the concept of the view-window of the calculator enabled him to discern the effect of the view-window on the shape of the parabola.

His worksheet indicates that Wan also explored graphs of $x=y^2$, $x=2y^2$, $x=3y^2+7$ in the CONIC Mode. The text screen of the CONIC Mode is as follows.



This was what Wan wrote in his worksheet on his exploration on conic graphs.

Bentuknya sama tetapi kecerunan dan pintasan y berlainan
 $y=x^2$ menggunakan kuadran 1 & 2 manakala $x=y^2$, $x=2y^2$
 dan $x=3y^2+7$ menggunakan kuadran 1 & 4.
 Mereka membentuk satu lengkok.

"The shapes are the same but the gradients and the intercepts at y are different. $y=x^2$ is in quadrant 1 and 2 whereas $x=y^2$, $x=2y^2$, and $x=3y^2+7$ are in quadrant 1 and 4. They are all parabolic curves".

This not only indicates that Wan was able to generate the conic graphs but he fully understood in general how the calculator operated.

Role of the teacher

Analysis of the videotape indicates that individual teacher intervention was absent in these activities. However, it is suspected that Wan benefited from the teacher's whole classroom teaching, for example induction to new terminology such as symmetry axis and quadratic graphs. As Wan had attempted most of the tasks himself with the graphing calculator, it is reasonable to infer that some of the teacher's classroom explanation might have provided him with some kind of confirmatory feedback and scaffold to help Wan with the co-construction of new knowledge. It would seem that the knowledge the teacher was trying to put across was not too far from Wan's existing concept. This was what Wan wrote on the difference between the graph of $y=x$ and $y=x^2$.

Bentuknya berbeza
 $y=x$ bergaris lurus dan tidak simetri manakala $y=x^2$ bergaris lengkok
 dan ada paksi simetri
 $x=0$

"Their shapes are different. $y=x$ is a straight line with no symmetry axis whereas $y=x^2$ is a curve with a symmetry axis $x=0$."

As Wan had a well-developed notion of vertical and horizontal lines, the concept of symmetry axis that are vertical lines seemed not too far from his existing knowledge. Wan was able to extrapolate his existing knowledge of the vertical line $x=0$ to describe the symmetry axis which the teacher had introduced. In his exploration of exponential graphs, his worksheet indicates that Wan was able to deduce that the graph of $y=1/(3+x)$ was different from the graph of $y=1/x$ in that its symmetry axis is at $x=-3$. This was indeed a very high level of comprehension. He also described that the x and y values at the ends of the exponential graph did not “stop”.

In conclusion, the emergence of such a high level of concept development could be attributed to Wan’s understanding of the overall functionality of the calculator, which by this time was extremely good from the way he used the graphing calculator. What was apparent in Wan’s activity was that individual teacher intervention had become almost non-existent. He seemed to be directing his own learning trajectory and the calculator provided him with a medium to explore or test his conjectures. By doing so, the calculator not only enabled to integrate his previous knowledge with the new knowledge but what seemed to emerge was Wan’s mathematical activity itself had been transformed. He was going beyond the mathematical task given into greater level of independent competence with many characteristics of higher order thinking (Resnick, 1987). It seems Wan’s knowledge of the functionality of the calculator enabled him to exploit the available features of the graphing calculator in his exploration and to modify or extend the task accordingly. It also enabled him to discriminate what graphical properties to observe on the screen. Wan was able to discern how the shape of the parabola was affected by the scale or the view-window values from that as a result of the transformation. The graphing calculator, for example, enabled him to immediately see that the same parabola appeared differently when the x or y -axis shifted up or down within the view-window. It also afforded him to see that graphs of the form $y=x^2+a$ are the same graphs positioned at different parts of the y -axis. His familiarity with the mathematical terms enabled him to understand the teacher’s whole class explanation and to extend it to connect with ideas he encountered in his constructional activity with the graphing calculator.

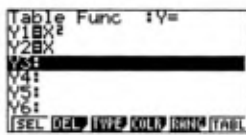
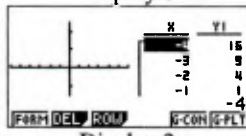


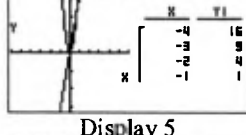


NG (MEDIUM ATTAINING BOY)

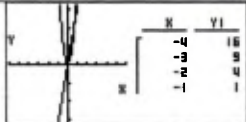
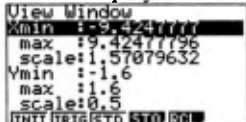
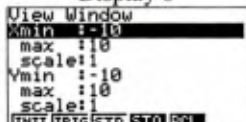
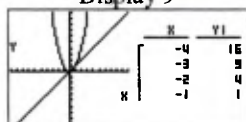
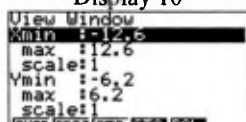
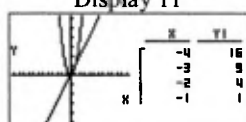
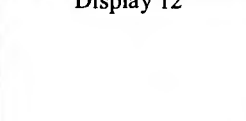
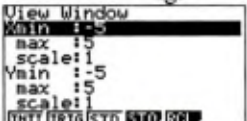
The tests results show that in general, Ng could distinguish the linear equations from the non-linear equations but had difficulty in discerning between equations of cubic and quadratic graphs. This case study will therefore explore how Ng made use of the calculator facilities in his activities and how they impinged on the extent of Ng's concept development.

Role of the calculator

The following excerpt documented how Ng used the calculator's TABLE Mode facility. It also shows his partial understanding of the view-window concept.

Excerpt 8.02 Inappropriate use of calculator facility

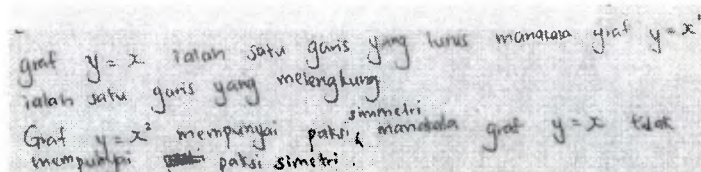
Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: Use the TABLE Mode to examine the values of y in both graphs	Teacher instructs to all students. Ng keys in the equation in the TABLE Mode.	
5		He is seen using the TABLE Mode and dual screen facility and playing with the cursor moving along the table up and down.	Display 1 
10		Teacher suggests to all students.	Display 2 
15		Teacher hints to all students but Ng is still busy exploring his tables.	Display 3 
20			Display 4 
22	T: Check what happens if you graph $y=x^2$ on different scales. T: It will still form a curve but there is a difference	He then generates the graphs (after a while).	Display 5 
25			Display 6 
28		Ng then checks his view window and looks surprise at this very "absurd" setting.	
30			

35		He graphs again.	
40		He rechecks the view-window.	Display 7 
45		He changes his view window to the STD window.	Display 8 
50		He graphs again using the STD window.	Display 9 
54			Display 10 
55		He selects another view window he has previously stored and generates to see the graph.	Display 11 
60			Display 12 
	T: Try to select the “view-window 5”	Teacher hints to all students to use this setting: 	

The excerpt shows that Ng experienced difficulties in using the facilities in relation to the task. In particular, the use of the dual screen did not provide additional information about the table, and using this facility in the first screen that appeared prevented him from seeing the whole table (Display 2 onwards). Ng was not aware of some of the limitations of the calculator as indicated by his failure to see that using the dual screen which depicts the graph and table side by side had reduced the space for drawing the graph considerably (Display 5). This also meant he did not appreciate that the calculator automatically alters the scale of the x-axis to fit the graph although examining the view-window setting shows no change in the scales (Display 6) resulting two graphs drawn in different view-windows using this facility looked seemingly the same in size (Display 7, 9, 11). However, the following actions by Ng indicate his disregard on the importance of view-window before the start of his task. Ng had started with a view-window acquired

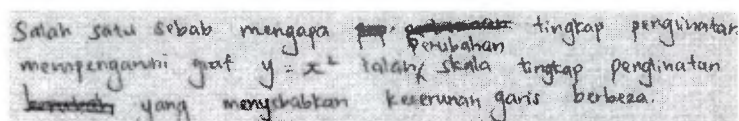
from his previous activity (line 28). His surprise indicates that he was not aware where the value setting came from. His choice of the view-window after generating the graph in the STD window (line 54) also indicates his partial understanding of the view-window concept.

What did Ng learn from this activity? This was what he wrote in his work sheet.



graf $y=x$ ialah satu garis yang lurus manakala graf $y=x^2$ ialah satu garis yang melengkung
 Graf $y=x^2$ mempunyai paksi simetri manakala graf $y=x$ tidak mempunyai paksi simetri.

"graph of $y=x$ is a straight line and graph of $y=x^2$ is a curved line, graph of $y=x^2$ has a symmetry axis and graph of $y=x$ does not have a symmetry axis"



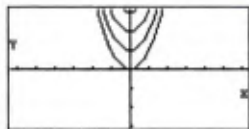
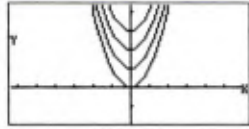
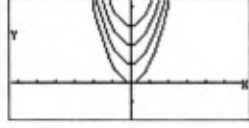
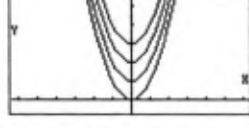
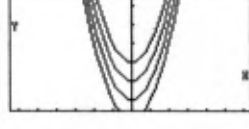
Salah satu sebab mengapa ~~perubahan~~ tingkat penglihatan mempengaruhi graf $y=x^2$ ialah skala tingkat penglihatan ~~berubah~~ yang menyebabkan keserunan garis berbeza.

"one of the reason why the view-window influence the graph of $y=x^2$ is that the scale of the view-window changes resulting in the gradient of the line to vary"

It seemed the visual display of the calculator did facilitate Ng in distinguishing between the two forms of graphs and the symmetry axis. Despite his partial understanding of the view-window, Ng did arrive at the understanding that the view-window settings influenced the "concavity" of the parabola. Noticeable was his generalisation of the terms or concept that he had encountered in his learning of linear graphs to that of the parabolas. He described the "curved line" to refer to the quadratic curve and "gradient of the line" to refer to the concavity of the parabola.

The excerpt also shows that Ng's partial grasp of the functionality influenced the way that Ng used the calculator. He was using the calculator according to the teacher's instruction and according to the instruction in the mathematical task. This was evident from the way he continued to generate the graph in the TABLE Mode using the dual screen because the teacher did not specify to change the mode or to remove the dual screen set up. The same situation was observed when Ng was using the DYNA Mode to investigate graphs of $y=ax^2$. His incomplete understanding of how to operate in the DYNA Mode resulted in confusion as to which button to press to execute certain commands in some steps. The use of the calculator as a display tool to generate the graphs, or as instructed by the teacher or in the mathematical task precluded him from deriving meaning from his action. The following excerpt illustrates this.

Excerpt 8.03 Using the graphing calculator as a display tool

Line No.	Action	Calculator Display/ Worksheet
1	Ng is observed graphing the graphs of $y=x^2$, $y=x^2+1$, $y=x^2+2$ and $y=x^2+3$ again in the INIT view window (after the STD view-window). He shifts the x-axis below a few times.	
5		
10		
		
		

In Ng's worksheet, he sketched these graphs describing them as graphs that “intercept at the y-axis”. The description fits what he literally viewed on the screen. He did not view the graphs as graphs of $y=x^2$ translated on the y-axis. His sketches of graphs of $y=x^3$, $y=x^4$, and $y=x^5$ were without any description and they were all sketched on the same graph making it difficult to discern between them. The graph of $y=x^4$ also looked seemingly similar to graph of $y=x^2$. It is not surprising that Ng was uncertain how the forms of non-linear equations relates to the general shape of the graph except for reciprocal graphs whose equations and graphs are very peculiar.

In conclusion, examination of the transcripts indicates that Ng's development of the non-linear graph concepts is closely linked to his knowledge of the functionality of the TABLE Mode and DYNA Mode which he used in his activities. Included was his uncertainty of how the view-window affected the display. It seems that his knowledge of the functionality of the calculator was still in the transitory stage. His lack of understanding of functionality of the DYNA Mode including forgetting some of the

operational commands and resulted in disruption of his activity flow. He sometimes used inappropriate facilities like the “dual screen” which effectively reduced the graphing space and prevented him from seeing the “actual shape” of the graph. The lack of understanding of the functionality of the calculator also affected the way he used the calculator. The result was that he tended to use it as a display tool, and he tended to remember some of the displays without making the connection to the mathematical ideas that produced them. However, the immediacy of the generation of the graphs enabled him to recognise some shapes and relate them to the forms of the equation.

Notably absent was individual teacher intervention during his activity. To a certain degree it could be inferred that Ng was directing his own learning trajectory by working independently. However, Ng was not able to progress much and make the conceptual connection on his own, in particular the idea of graph transformations.

AMIN (LOW ATTAINING BOY)

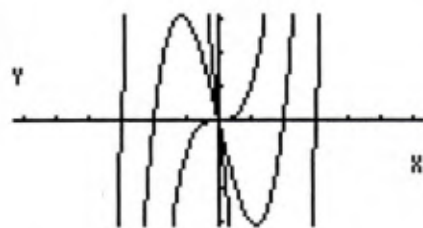
The results of the tests shows that Amin was able to recognise that the highest power to the x variable governs the general shape of the graph. He was also able to identify the shape of the cubic graph, the graph of $y=1-x^2$ and the reciprocal graph, but when two parabolas of different transformations were presented, he was unable to distinguish between the two equations that represent them.

It should be pointed out that Amin did not attend the first lesson when the activity on non-linear graphs began. Amin's encounter with non-linear graphs was restricted to just one lesson of about an hour. Amin was observed exploring the activities himself with the calculator. The analysis will focus on what Amin recorded in his worksheet of the graphs that he explored in his activity to gauge the extent to which the calculator assisted his concept development.

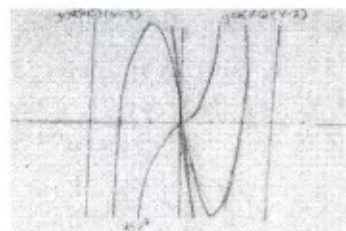
Role of the calculator

Amin's knowledge of the general functionality of the calculator was still in the transitory stage. This was very evident when he generated the graphs of $y=x^3$, $y=x(x+2)(x-2)$ and

$y=x(x+3)(x-3)$ simultaneously in the INIT window and repeated it a few times. He was seen drawing the following graphs as he looked on to the screen of the calculator.



Graphing Calculator

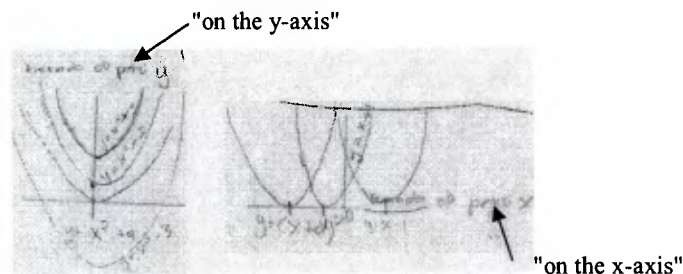


Amin's sketch

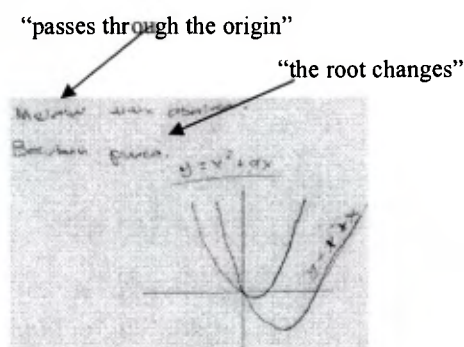
He did not attempt to change the view-window settings or use the zoom feature to capture the full graph. However, his sketch shows that Amin ignored the pixels that were evident on the graphs generated by his graphing calculator. Another evidence of his partial understanding of the functionality of the calculator was his inability to generate the given graphs in the CONIC Mode. Wan was seen explaining to Amin how to substitute the values in the equation $X=A(Y-K)^2+H$ to obtain the equation $x=y^2$, $x=2y^2$ and $x=3y^2+7$.

However, the graphing calculator could be a powerful tool to aid Amin's learning. Amin was observed generating the graphs of $y=x^2$, $y=x^3$ and $y=-x^3$ consecutively and was seen repeating them. He identified the latter graph of $y=-x^3$ as the sketch he saw in his worksheet. It was astonishing to see that Amin was also able to give a correct hypothetical equation to the other graph sketch as $y=x(x+1)(x+4)$. The graphing calculator in this instant was used to support his thinking by enabling him to test his conjecture. The whole graph could be viewed by drawing it on the STD window, a view-window that Amin was now familiar with but not necessarily a full understanding of it.

The level of understanding that was emerging in Amin's learning of non-linear graphs was closely linked to his ability to make sense of the graphic display. The following extract depicts a section of Amin's work with families of parabola. He described the parabolas of the form $y=x^2+a$ as being positioned along the y-axis and parabolas of the form $y=(x+a)^2$ as being positioned on the x-axis.



Amin's description shows a strong element of interpreting the graphs in terms of what he literally saw on the graphic display. Nevertheless, the graphic display enabled Amin to see that the graphs generated were parabolas. It also enabled him to discern between the two forms of the parabola. Absence of teacher intervention however, restricted Amin's discovery to pattern recognition rather than the graphs as translations along the y-axis and x-axis respectively. The following extract of his work aptly describes this notion of pattern recognition.



The above extract shows an erroneous graph of $y=x^2+x$. It is plausible that Amin generated a few graphs of this family and when reproducing them made the mistake. It is clear that he was memorising what he saw on the graphic screen. He described that all the graphs passed through the origin and "the root changes" but did not really understand how the coefficient "a" influenced the graph transformations.

Role of the teacher

Individual teacher intervention was evidently absent when Amin carried out his tasks. He randomly investigated a few parabolas and disregarded the family of graphs of $y=ax^2$. Inevitably, families of graphs with negative values of "a" were overlooked.

In conclusion, the graphing calculator clearly helped Amin to recognise that the highest power assigned to x in the equation affected the general shape of the graph with x^2 giving the shape of the parabola, x^3 the form of “cubic graphs” and $1/x$ the form of “reciprocal graphs”. Given his poor computational skills and difficulty in constructing graphs in general, it would have been impossible for Amin to construct all these non-linear graphs manually. It is suspected that the absence of teacher intervention precluded him from bridging the conceptual link between the graphic display and the equations that produced the different transformations.

8.4 Conclusion on the concept of non-linear graphs in the Structured Learning Model

1. There was a lack of teacher intervention in students’ activities, from which one could infer that at this stage, students had developed some confidence to carry out the tasks with the graphing calculator. However, it could also infer a lack of pedagogical content knowledge of the teacher to introduce the idea of transformation to students.
2. The immediacy with which the graphing calculator creates the graphs of particular forms of equations enabled even the medium and low attaining students to recognise some of the general shape of the graphs associated with the equations that represent them. In general they were able to recognise that the variables in the equation gives the shape of the graph. In particular, they were able to discern between equations that represent linear and non-linear graphs.
3. Medium and low attaining students had difficulty in articulating how the parameters affect the position of the graphs. In particular, understanding the transformation of parabolas was problematic and required teacher intervention to make the conceptual link between the various transformations.
4. Only the high attaining student was able to distinguish that the variables in the equation gives the shape of the graph and the parameter determine the position of the graph.

5. It is possible for students to recognise the shape of graphs of certain equations and develop the idea of graph transformation without an understanding of the scale concept.

8.5 Analysis of case study students in the Interpretative Learning Model

GRACE (HIGH ATTAINING GIRL)

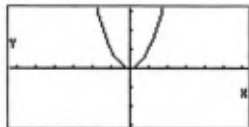
Grace successfully answered all the questions in the tests. She could distinguish between the cubic and reciprocal graphs and could recognise the quadratic graph by looking at the form of the equation. Grace could recognise the different forms of equation to the general shape of several non-linear graphs and differentiate between the equations of two parabolas that had undergone different transformations.

This case study will focus on aspects of her development in relation to her use of the calculator and the teacher intervention. In particular it will illustrate the way Grace attempted to interpret the graphic displays that were generated and the way teacher intervened which subsequently affected the kind of knowledge she attained.

Role of the calculator

The excerpt below shows Grace asking specific questions to the teacher in trying to make sense of the graphic display.

Excerpt 8.04 Grace's specific questions

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	Grace: How does the view window influence the shape of the graph? T: Do it also in the STD, compare with INIT. Grace: No difference. T: No difference? What view window is that? Grace: STD.	Grace had already drawn the graph of $y=x^2$.	 INIT view-window
2		Grace generates on both view-windows and could not see the difference.	
5			
10		Grace checks the view window again. Executes again.	


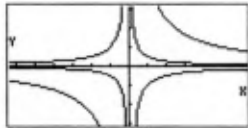

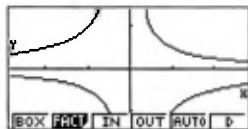
	Grace: Look at this.	Grace shows her display to the Teacher.	STD view window
15	T: There is difference...big...small.. Grace: This..is this bigger? T: aaa Grace: So how do you explain Teacher?		
20	T: No...how is the graph in STD, how is the graph in INIT? However the graph remains the same.	Grace writes in her work sheet: "The minimum and maximum values of x and y influence the size of the graph".	
25		She asks the Teacher again.	
26	Grace: But Teacher, the y is much bigger...	Grace is referring the y value compared to the x.	
30	T:Look..it depends on your scale...remember our choice of scale affect the shape...	Grace scratches her head and erases her answer.	

Grace specifically asked the teacher how the view-window influenced the shape of the graph (lines 1-2), as the graphs generated in the STD window and in the INIT window looked seemingly congruent to her. She went to the extent of showing the teacher her display (line 12). The teacher hinted at the size (line 14), and Grace began to focus on this aspect of the graph and subsequently sought confirmatory feedback from the teacher (line 15). Grace was persistent in continually questioning to extract explanation from the teacher that made sense to her. She was confused why the graph would appear different as the value of y-coordinate increased faster than x, and expected that the view-window would not affect the shape of the graph (line 26).

The above excerpt also shows the teacher was specific about what aspect of the graph to focus on (that is the shape of the graph). It is curious that the teacher did not ask Grace to try in another view-window besides the STD window to show a distinct contrast between the shape of the graphs. However, it seems probable that a high attaining student like Grace was able gradually to make sense herself of how the view-windows affected the shape of non-linear graphs as, as the next excerpt shows.

The following excerpt shows how Grace's knowledge of the zoom facility enabled her to successfully generate a family of reciprocal graphs. It also shows Grace relating what she keyed in to the output display.

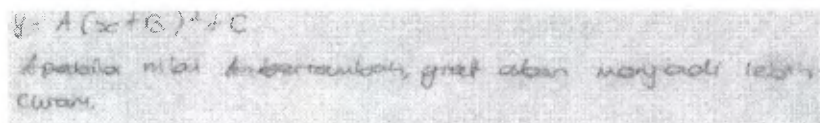
Excerpt 8.05 Interpretation of the graphic display

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1		Grace was seen graphing for reciprocal graphs.	
5		She used the zoom facility and was smiling at the displays. She looked engrossed for a while.	
10			
12		She zooms out.	
15			
18		She zooms in.	
20			
25		Teacher directed Grace to draw these graphs separately.	
26	Grace: What do you call this number in front?	Grace points to coefficient in front of x: $y=x^{-1}$, $y=10x^{-1}$ and $y=-2x^{-1}$.	
29	Grace: The value of the parameter?		
30	T: negative 2		

Grace was using the zoom facility of the calculator to generate a family of reciprocal graphs. She was seen zooming out (line 12) and zooming in (line 18) to look at the graphs further out and nearer respectively. By using this facility she knew that the fundamental shape of the graph was retained. This is also an indication that she was aware that the view-window affects the shape of the graph.

The above extract also shows Grace's attempt to clarify the meaning of the coefficient in front of x (line 26). She saw that this coefficient affected the graph, but in a different way to the one she encountered in linear graphs. She was very specific about what she wanted to know, and Grace ensured that the teacher understood her question by asking for the value for the parameter (line 29).

Grace's understanding of how the view-window affected the shape of the parabola helped her to distinguish it from the parabola derived from the coefficient of x^2 , as shown in the generalisation she made in the excerpt below. Grace recognised that as the coefficients increased the shape of the parabola also changed to become "steeper".



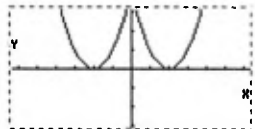
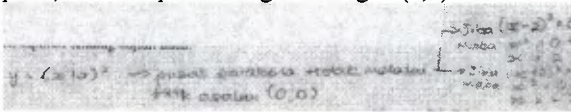
"When the value of A increases, the graph becomes steeper."

Noticeably, Grace generalised what she learnt about gradient in the straight-line to the parabola. She described the stretching in terms of the "steepness".

Role of the teacher

The excerpt below shows teacher's lack of knowledge on graph transformation and how it impacted on Grace's concept development.


Excerpt 8.06 The quality of teacher's knowledge


Line No.	Dialogue	Action	Calculator Display/ Worksheet
1 3 5 10 11 15	<p>Grace: Teacher, why is the intercept if it is positive, it goes to negative..if it is negative, goes at positive?</p> <p>T: This one And this one.. what is the difference...</p> <p>Grace: The intercept...this..x=2 and this....-2, it is the reverse</p> <p>T: aha, okay, it is the reverse.</p>	<p>Grace looks very hard at her display and seeks explanation from Teacher after exploring graphs of $y=(x+a)^2$. She pointed to the display on her calculator. $Y=(x+2)^2$ and $y=(x-2)^2$.</p> <p>Teacher picks up Grace's calculator and points at the minimum. Teacher pauses for a while. Grace explains again and points at the position. Teacher confirms what Grace is telling and left. Grace writes her conclusion for this family of graphs and giving her reason:</p>	 <p>From the interaction this must be display.</p>
20		<p>"The centre of the parabola (to refer to the minimum point) does not pass through the origin. (0,0)"</p> 	

Here, Grace sought explanation from the teacher about why the parabola intercepts at different places on the x-axis (line 5). Grace clearly was trying to impose meaning and structure to what she saw on the screen. This excerpt reveals that the teacher intervention did not add to Grace's existing knowledge. The teacher only gave confirmatory feedback of what Grace already knew (line 11). She did not answer Grace's question about the graph transformation (line 1-3). The teacher's reaction indicates that she was unprepared for Grace's request for more advance explanation beyond that required by the task. The teacher did not induct Grace into thinking of the graphs of $y=(x+a)^2$ as graphs of x^2 translated horizontally. This was reflected in Grace's answer (line 18) that shows that she did not think of the graph displayed as the graphs of x^2 translated along the x-axis. Grace ingeniously tried to relate it to her knowledge of $y=0$ and equate $(x-2)^2=0$ (and incorrectly solved it). The excerpt also reveals that, up to this stage of work, Grace continued to use the term "intercept" to refer to the point where the graph touched or cuts the x-axis. In earlier work with graphs of $y=x^2+a$, when Grace asked the teacher about the point where the graph cuts the y-axis, the teacher continued to use the term intercept and did not introduce the term "minimum point", which was a new term to Grace. This exemplifies that new mathematical conventions have to be acquired through mediation. More importantly the teacher's inadequate response could be interpreted as the teacher's lack of knowledge on non-linear graph transformations, which constrained Grace towards advancing to a higher level of knowledge.

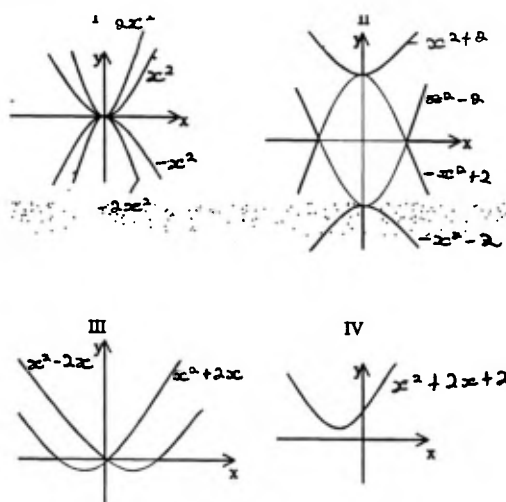
The following excerpt shows teacher's inductive reasoning in classroom teaching and Grace's active participation. It also shows how Grace's related mathematical knowledge enabled her to follow through the teacher's explanation.

Excerpt 8.07 Knowledge source from classroom teaching

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	T: What is you get a graph like this? What graph is this?Grace? Grace: negative	Grace is listening to explanation on black board. Teacher first drew the graph of $y=-x^2$. 	
5	T: What negative? In front of x is Grace: $y=-x^2$.	Teacher then sketches the graph that intercepts at the negative value for y.	

10	T: If this? Grace: -1,...-2..	 <p>Teacher draws another graph that intercepts at a positive value for y. Grace is then seen keying in the equations and generating the graphs in her calculator.</p>	
12	T: Maybe -1....so y=... Grace: $y=-x^2-1$		
15	T: If this? Grace: $y=-x^2+1$		

Here the teacher is structuring her explanation using a curriculum script, and Grace was actively taking part. Grace understood the information that the teacher requested (lines 5, 12). Her understanding of the negative number and where they were positioned along the y-axis enabled her to follow through the teacher's inductive reasoning. Notably the interaction involved a series of question-answer, in which it was observed that in each turn the level of knowledge was increasing in sophistication. The above excerpt shows the teacher was inducting Grace in two kinds of transformations, first the reflection of $y=x^2$ along the x-axis followed by the translation along the y-axis, but without introducing these terms to her. This followed with Grace using the graphing calculator to verify these ideas and expanding her activity to generate parabolas of the same family and other families. In the extract from her worksheet shown below, Grace could assign hypothetical equations for the different transformations of the parabola presented in the exercises.



In conclusion, Grace's high level of conception of transformation of parabolas and several other graphs of non-linear equations could be traced to her overall knowledge of the functionality of the calculator, her perception of the role of the graphing calculator in her learning and her related mathematical knowledge. For Grace, the graphing calculator provided her with opportunities to engage in dynamic mathematical activity to construct meaning and doing interpretative work rather than routine manipulations. This could be observed from the specific questions Grace asked the teacher and the confirmatory feedback she sought from the teacher when she was attempting to make sense of the graphic display. Grace always tried to relate what she keyed into the calculator to the output display.

Grace's mathematical knowledge of algebra and negative numbers enabled her to follow through the teacher's classroom instruction, and provided Grace with an extra source of knowledge. What seems to emerge from Grace's learning was that her potential for higher levels of concept development was restricted by the teacher's lack of knowledge on non-linear graph transformations. The graphing calculator enabled Grace to see the different types of "transformation", but without any teacher intervention, Grace's understanding of the parabolas that went through different transformations was still confined to the local feature of the graph (for example the y-intercept, the x-intercept). She was unable to see them in terms of the same graph being translated, reflected or stretched. Grace's understanding of the view-window enabled her to recognise different families of parabolas through pattern recognition of the graphs generated for each type of equation. Her understanding of how the view-window was related to the appearance of the graph enabled her to recognise how the parameters in the equations affected the position and the "steepness" of the graphs in the confined graphical space.

PRESHA (MEDIUM ATTAINING GIRL)

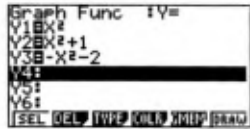
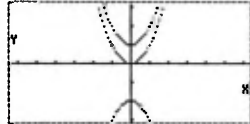
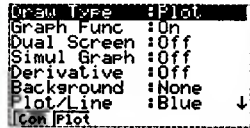
The results of the test shows that Presha was successful in all the test items, indicating that she was able to distinguish between equations that represent linear graph and non-linear graphs. Although Presha was also able to discern the equations of two parabolas that had undergone different transformations, Presha was not able to distinguish the different types of transformations of the parabola.

A key focus in Presha's concept development is the role of the teacher in her activities and the interplay of her grasp of the functionality of the calculator.

Role of the teacher

The following excerpt documents how the teacher intervened when Presha made a semantic error. It also shows Presha's difficulty in appropriating exploratory tasks.

Excerpt 8.08 Intervention on semantic error and Presha's difficulty with the task

Line No.	Dialogue	Action	Calculator Display/ Worksheet
1	Pre: What is there to write Teacher?	Students are to explore different families of graphs of parabolas. Presha is fiddling with her calculator. She is not sure what to do. Presha's text screen shows that she has generated the following graphs.	
2	T: You need to do it systematically	Teacher returns and saw Presha drawing graphs with plotted points.	
3	T: Pre, these are dotted, how do you graph it so that it is continuous?	Teacher points to the text screen that show that the "Graph Func:Y=" indicates it is in the GRAPH Mode.	This is probably the "dotted" graphs generated from the equations above.
5	Pre:Table	Presha points to the Type menu on the screen.	
6	T: Table?...Table is to obtain table. Look, this graph is now drawn in Graph (to mean Graph Mode).. If you want to draw it in Graph...	Pre then points to GMEM.	
10	Pre: Type		
13	T: Type means the kind of graph..		
14	Pre: Ooh.. Gmem what is it?		
15	T: That is for memory. Now if we want to determine the picture, use SET....		
18	Pre: Sketch?		
20	T: Not Sketch, use SET UP, look... Pre: Aah..SET UP T: Now ...Draw Type... your drawing is now ... Plot. Here there is connect and plot. If you want it to be continuous, what do you choose?	Presha suddenly remembers and activates the SET UP. Teacher points to the calculator display.	
23	Pre: Connect		This is probably the screen as from the conversation, Teacher is pointing to "Plot".
25	T: Eem.		
26	T: Do you see it connected...continuous?	Presha generates the graphs.	
28			
30	T: If you do so many graphs, what do you see? Pre: I cannot see.		
34	T: Then you must investigate for each family. This is one family, this is one family and this is one family.	Teacher points to the respective equations in the worksheet: $y=x^2+a$;	
35			

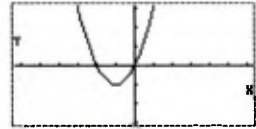
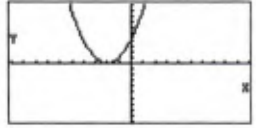
	Pre: This is one family, this is one family and this is one family.	$y=(x+a)^2$; $y=x^2+ax$. Presha repeats after Teacher and points to the respective equations.	
--	---	--	--

Presha was generating the parabolas as disconnected graphs (line 3). The teacher alerted Presha to generate them as connected graphs. Generating non-linear graphs in the continuous form is an important aspect of enabling students to view the global property of these graphs. It is clear that Presha was also confused about the difference between the disconnected graph plotted in the TABLE Mode using the G-PLT facility and the disconnected graph that could be generated in other modes (RUN Mode, DYNA Mode and GRAPH Mode) by changing the settings in the SET UP menu (line 6). The source of Presha's error was in the SET UP menu. Presha was unsure how to resolve the error (line 12). In her attempt to correct her semantic error, she encountered other calculator commands and menus. The teacher guided her to change the setting of her graphic display with explanations as to what each command or menu did (line 13-25) and also questioned Presha (line 23, line 28). In the process, Presha was confronted with other facilities of the calculator. Notably, Presha was questioning as to what each command did (line 14, 18), indicating her attempt to build an understanding of different aspects of the functionality of the calculator. The teacher's intervention ensured that Presha generated connected graphs throughout her exploration with non-linear graphs.

The text display (the first figure of the above excerpt) also shows that Presha lacked the capability to organise her exploration. She was exploring the parameters randomly. It seems the general instruction given by the teacher (line 2) was not sufficient in getting Presha to continue with her task. When the teacher elaborated to investigate for each family (line 34), Presha was still not able to continue her task. The following excerpt shows how the above general intervention that the teacher provided was not sufficient to guide Presha to organise her exploration. It also shows the quality of the teacher's knowledge on graph transformations and how it impacted Presha's understanding.

Excerpt 8.09 The impact of the quality of the teacher's knowledge

Line No.	Dialogue	Action	Calculator Display/ Work Sheet
1	Pre: How to do Teacher? T: Look at their	Presha is still not clear about what she has to do. Teacher is trying to locate	

5	differences....where are your graphs?	Presha's sketches.	
6	Pre: Do we have to draw one by one?	Presha points to the 2 (iii) $y=x^2+ax$	
	T: Yes....		
	T: Now draw...What you see?	Presha generates the graph and points at the display.	
10	Pre: It passes through the first, second and third quadrant.		
12	T: Not that.. it cuts at two points at the axis.	Teacher points at the graph.	
	Pre: the y-axis...eeh, the x-axis, what then....it cuts at...where does it cut Teacher?		This is probably the graph as this is the simplest one to start with and fits the ensuing interaction.
15	T: At the two points.... Which is this graph for....this is an example of graph (iii).	Teacher shows again. Teacher clarifies which family of graph Presha is drawing and points to the equation: $y=x^2+ax$	
20		Presha asks whether to draw four of these graphs. Presha immediately proceeds to draw other graphs but Teacher asks her to record what she has done before she moves on to the next graph.	
22	Pre: Do four?		
	T: Aaa...(yes)		
25			
26	T: Record what you have done , then you move on.		
30		Presha graphs the graph of $y=(x+2)^2$ and shows the graph display to the Teacher.	
34	T: Where does it cut?		
35			
36	Pre: It cuts at the y-axis.		

Here, Presha was able to continue only when she herself enquired of the teacher precisely what she had to do, in this case to generate a few graphs of the same type of equation (line 6 and line 22). The teacher cautioned Presha to record each observation (line 26). The teacher also focussed Presha on looking at where the graph intercepts the x-axis (line 12) and the y-axis (line 34). Notably the teacher did not give any corrective feedback on Presha's partial answer (line 36), which resulted in Presha missing the crucial feature that $(-2,0)$ is the minimum value for the graph of $y=(x+2)^2$. Indeed in her conclusion, the focus was only on the y-intercept. It was interesting, however, that Presha displayed her graph in the STD to include the y-intercept, indicating that she understood which aspect of the graph was preserved when the view-window changed. In her summary of her exploration with parabolas, as shown in the extract below, Presha emphasised the intercept feature of the graph to discern each family from the other.

$$\begin{array}{ll}
 y = x^2 + a & \text{Ia juga di paksi} \\
 \downarrow & y \\
 \text{"It is also at y"} \\
 (i) y = (x+a)^2 & \text{Ia memotong di paksi} \\
 \downarrow & y \\
 \text{"It cuts at the y-axis"} \\
 (ii) y = ax^2 & \text{Ia memotong di titik} \\
 \downarrow & \text{asalan} \\
 \text{"It cuts at the origin"} \\
 (iii) y = x^2 + ax & \text{Ia memotong} \\
 \downarrow & \text{2 titik di paksi} \\
 \downarrow & x \\
 \text{"It cuts at 2 points on the x-axis"}
 \end{array}$$

Presha's answer clearly was a reflection of what the teacher explained to her in the intervention. The emphasis was on the local aspect of the graph. The teacher did not induct Presha into thinking about the parameter in terms of transformations, indicating the teacher's lack of understanding of graphical transformations. The teacher's non-corrective feedback also led Presha to the misconception that "a" characterised the place where graph cut the axis. Presha's answer also indicates that she only tried specific values of "a" and generalised from these special cases. Presha's activity clearly shows that making sense of the graphic display was important to discriminate between the different transformations of the parabolas, which required teacher intervention.

In conclusion, Presha's activity was greatly influenced by her overall knowledge of the functionality of the calculator and her ability to make sense of the graphic display. Presha's overall knowledge of the functionality of the calculator continued to develop as evidenced from her choice of the STD window to include viewing the y-intercept of the graph. However, there were instances where she was unsure of some of the operations of the calculator and how they affected the graphic display. Teacher intervention was important in providing support towards developing her understanding of the functionality of the calculator. The way in which the teacher intervened to correct the semantic error she made, enabled Presha to use the appropriate features of the graphing calculator in her investigation of non-linear graphs, for example, drawing the graphs in connected form.

However, what emerges is that the quality of the teacher's knowledge of quadratic graph transformations affected the way she intervened, which subsequently affected Presha's understanding. In one example, it led Presha into developing the misconception of the y-

intercept of the parabola as the feature that changed when the “a” in the quadratic equation changed.

What also emerges is that presenting the tasks to investigate different families of parabolas by asking Presha to vary the values of “a” in the different forms of the quadratic equations was confusing. Her repeated request for specific task procedures indicates her lack of experience undertaking this kind of activity that required her to systematise her exploration.

EMY (LOW ATTAINING GIRL)

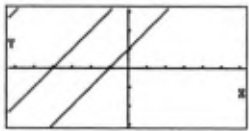
The results of the tests show that Emy was able to match all graphs of non-linear equations including the two quadratic equations with the respective graphs correctly. However, she was able to recognise only the reciprocal graph and could not distinguish between the cubic graph and the quadratic graph when given in a different situation, indicating her grasp of how the form equation relates to the shape of the graph is still at the fragile stage.

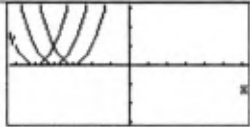

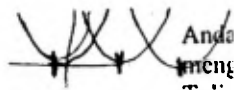
A key focus of this case study is Emy’s mathematical knowledge which underpins the way she internalised the teacher invention and the way she used the calculator.

Role of the teacher

The following excerpt shows Emy’s lack of mathematical knowledge and how the teacher intervention influenced her mathematical activity.

Excerpt 8.10 The influence of lack of mathematical knowledge and teacher intervention

Line No.	Dialogue	Action	Calculator Display/ Work Sheet
1	Emy: aaa... what is this?	Emy generates straight line graphs for $y=(x+a)^2$. (She had actually keyed in for $y=x+1^2$, $y=x+2^2$, and $y=x+3^2$.) Teacher observes Emy's equations.	
5	T: What did you use?		
7	T: Bracket all of it. T: Are the brackets not there?		
10	Emy: It is not there. T: Find the brackets,....brackets...		
		After successfully	

12	Emy (laughing): Is there such thing as negative axis?	generating a few graphs of $(x+a)^2$, Emy is laughing when she describes that the graphs are on the “negative axis”.	
15	Emy: If it is inside the boxes, what do you call that?		
18	T: Quadrant..		
20	Emy: Haa...quadrant.	Emy writes down that the graph is on the “negative quadrant”. Emy nods.	
25	T: did we learn about negative and positive quadrant? It did not ask you to look at quadrants...is there?		
26	T: What if “a” changes? Let me give you an example. If $a=0$., if $a=1$, if $a=-1$..., if $a=-2$,...	Teacher is pointing at the graphs at the same time	Teacher’s examples:
30	T: What is this? $y=x^2$ If here... $y=x^2-2$...		
32	T: If this? $y=(x+a)^2$?		$y = x^2 - 2$
33	Emy: It will move to the side. T: Aaa, it will move.... Look at these values.		$y = x^2 - 1$
35	T: It is the same for $y=ax^2$. What happens to your graph? Draw several graphs.	Teacher points to where the graph touches the x-axis.	
40		This is what Emy concluded of her “investigation” of graphs of $y=(x+a)^2$, $y=ax^2$ and $y=x^2+ax$:	
44		(i) It goes further to the side = $y=(x+5)^2$,	
45		(ii) It passes through the origin = $y=2x^2$	
50		(iii) It cuts at two points on the x-axis = $y=x^2+4x$	
51		(iv) It cuts at the axes x and y = $y=x^2+(-4)$	

Emy’s uncertainty with negative numbers predisposed her to limit her exploration to positive numbers only except for graphs of $y=x^2+ax$ where she also tried $a=-4$. It is highly probable that for the graphs of $y=(x+a)^2$, Emy only tried a few values from 1 to 5. Hence she saw the graph had “moved more and more to the side”(line 44), resulting only in partial understanding.

Although the teacher intervened (line 33) to focus Emy to observe the value where the graph touched the x-axis, it was evident that she ignored this directive. Her writing showed that she was focussing on the “movement” of the graph. The teacher did not give her a corrective feedback when Emy described the graphs “will move to the side” (line

32). This is partially correct but because she restricted her parameter to positive values only, she saw the graphs of $y=(x+a)^2$ as moving from right to left instead of each graph being a translation of the graph $y=x^2$ along the x-axis defined by the parameter a . However, the teacher did not induct Emy into the idea of a horizontal translation.

In Emy's summary of her investigation, she used a particular case to describe the characteristic of each family of graphs (line 40-51). In her exploration with graphs of $y=(x+a)^2$, she gave the example $y=(x+5)^2$. Noticeably Emy considered only positive integers except for one family in which Emy chose a negative parameter for the graphs of $y=x^2+a$: $y=x^2+(-4)$. It was interesting that the teacher had already given her some examples for these graphs and they involved only negative values for " a ": $y=x^2-1$ and $y=x^2-2$. The teacher's long didactical explanation (line 26-30) did not build Emy's understanding. Emy did not make the connection between the " a " that the teacher mentioned and the intercept on the y-axis. In her worksheet, she concluded that these graphs cut at the x and y-axes. She did not conceive that this graph is a translation of the graph $y=x^2$ along the y-axis. Her focus was where the graph cuts the x and y-axis.

The teacher told Emy to draw several graphs for the family $y=ax^2$ (line 35). Again here the teacher's focus was on the graph passing through the origin. It seemed the hint to draw several graphs and to look for "what happens to your graphs" was too general and not sufficient to confront Emy that each of these graphs is a stretch of the graph of $y=x^2$ defined by the parameter.

The excerpt clearly shows that Emy found it difficult to relate the quadratic equation and the graph when it involves transformation, more so if the transformation was more than once. Although some of these graphs necessarily involved only one button push, the distinction between the parameters in the equation was problematic for Emy. Understanding the influence of each parameter in the equation was confusing, for example, when did the graph go up, when did it go down, when did it not touch the x-axis and when did the graph cut the x-axis once or twice. The ineffective nature of the teacher intervention did not elevate her from the confusion. Instead the pinpointing as to where the graph touched or cut the x and y-axis respectively gave the mistaken notion that the graph was governed by this property (line 26-30). It is therefore not surprising that Emy's worksheet indicated that she omitted the tasks on parabolic graphs with

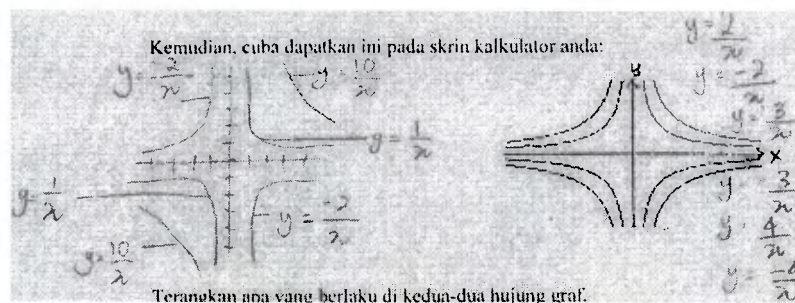
equations in the form $y=(x+a)^2 + b$. She also ignored tasks that demanded her to reproduce several graphs that involved parabolas.

However, for Emy, the calculator did assist in developing her understanding of algebraic equivalence. In the excerpt above, the calculator confronted Emy with the need to rethink the syntax of algebra. When the teacher mentioned to Emy to “bracket all of it” (line 7), it confronted Emy with the fact that the algebraic expression $(x+a)^2$ and $x+a^2$ do not carry the same meaning. Two things are happening here. First the display on the calculator alerted Emy that something was wrong. Second, the teacher intervention focussed Emy that her algebraic expression was wrong.

The calculator also seemed to support the learning of mathematical language, particularly the language of graphs (line 12 and line 18). However, it was the teacher who pointed out the meaning of quadrant (line 18).

Role of the calculator

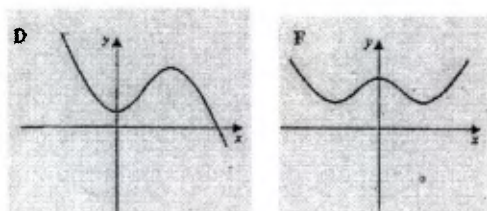
Given her difficulty with algebraic expressions, it is surprising that reciprocal graphs whose algebraic expressions looked very complicated, seemed to appeal to Emy. It was observed that she spent considerable time exploring graphs of this type and in her worksheet, she could even assign hypothetical reciprocal equations to the graphs given and reproduce them on her graphing calculator. Below is an extract of the work she did on reciprocal graphs:



A possible explanation as to why these graphs appealed to Emy was, because, although the algebraic expression looked more complicated, it involved only one key or one step calculator operation. The calculator immediately displays the graph for the equation that was keyed in, giving a direct connection between the shape and the equation. Although

the parameter changed, the general shape of the graph remained the same and the position of the graph changed only with respect to the distance from the origin following a "simple" pattern. It is interesting to note that in this activity, the calculator seems to be used as a mediational tool to support her thinking.

Emy also generated graphs of $y=x^3$, $y=x^4$, $y=x^5$, and $y=x^6$ with her graphing calculator. Graphs of $y=x^4$ and graphs of $y=x^6$ look seemingly similar to graphs of the parabola. It is not clear whether she could distinguish these graphs from graphs of the parabola. However, her limited exploration of cubic graphs might explain why she chose the graph (F) as the cubic graph rather than the correct graph (D) as shown in the excerpt below. It is highly likely that she guessed.



In conclusion, Emy's development of non-linear graphs was influenced by her lack of mathematical knowledge. Her lack of mathematical knowledge on negative numbers predisposed her to work almost exclusively on positive integers. Teacher intervention was an important aspect in Emy's learning as she also had difficulty in dealing with the exploratory nature of the task. However, the teacher intervention that failed to induct Emy into choosing the appropriate values for the coefficients resulted in only partial understanding of the parabola. The type of knowledge that the teacher imparted to Emy, which focused on the local feature of the parabola, reduced Emy's learning to memorisation of how the coefficients in the equations affected the appearance of the graphs.

What seems to emerge in Emy's learning was that the graphing calculator enabled Emy to explore non-linear graphs that were difficult to construct. For example, reciprocal graphs that arguably are difficult, proved not to be complicated to understand after all. Emy's activities also show that her algebraic difficulty was no barrier for her to advance her exploration with non-linear graphs. Indeed, the visual display of the graphing

calculator confronted Emy with the importance of following the mathematical conventions in algebraic manipulations.

8.6 Conclusion on the concept of non-linear graphs in the Interpretative Learning Model

1. In general, the calculator seemed to support the exploration of non-linear graphs in that it facilitated the quick and easy (no computation) generation of graphs, enabling students to make the connection between the general shapes of the graphs and the form of equations that represent them. The one button link between the equation and the graph facilitate students in seeing the relationship between the form of equations and their graphs.
2. Students needed to have confidence in exploration, which is lacking in low and medium attaining students. Students were not using the DYNA Mode to explore the parameters and variables. They also found difficulty in structuring their exploratory activities and randomly investigated the parameters. For the low attaining student, this is closely linked with her lack of the related mathematical knowledge, in particular in negative numbers, resulting in confining exploration to positive numbers.
3. The quality of teacher's content knowledge has an impact on the students' knowledge development. The non-introduction of the terminology of transformation to describe the movement of the parabolas limited students to description of the graph transformations in terms of pattern recognition.
4. The graphing calculator can support the understanding of algebraic equivalence. The graphic display immediately alerts the user whether two equations are equivalent.
5. Although students need to know how the scales affect the shape of the graph, lack of understanding of the scale concept does not preclude students from making the connection between the shape of the graph and the form of the equation.

6. The high attaining student asked specific questions in relation to the graphic display created by the varying parameters in the equation.
7. Classroom teaching provided the critical source of knowledge for the high attaining student to concretise her emergent knowledge, and to move her to higher levels of knowledge construction.

CHAPTER 9

FINDINGS AND DISCUSSION

The purpose of this exploratory study was to probe the potentiality for using graphing calculators in the learning of a few selected graphical concepts, namely continuity, gradient, linear graphs and non-linear graphs, among 14 year-old students. Two learning models based on two different interpretation of Vygotsky's zone of proximal development informed the study. In each learning model a high attaining, a medium attaining and a low attaining student were identified as case subjects. The presentation of the findings and discussion is made against the backdrop of the research questions outlined below within the theoretical considerations of the study.

1. To what extent do students of different mathematical attainment learn the graphical concepts with the graphing calculator according to each learning model?
2. What characterises the transitions in students' understanding of the graphical concepts with the graphing calculator?
3. What forms of teacher intervention emerge when graphing calculators are integrated in students' learning?

The findings and discussion are divided into four sections. The first section presents the extent to which the mathematical knowledge of students developed by using the graphing calculator, and how the graphing calculator impinged on the development of their understanding. The second section is based on two major issues that emerge from the analysis, namely the functionality of the calculator and the nature of teacher intervention that influences students' learning with the graphing calculator. The third section considers the issues surrounding the implementation of the two learning models used in the study. The last section encapsulates the findings and discussion in the form of two models of student learning. It concludes with a view on the complexity of the teaching and learning situation using the graphing calculator.

9.1 Mathematical knowledge

It is not surprising that only high attaining students understood the concept of continuity in both learning models, confirming the difficulty of the concept even in a technology-supported environment (Kerslake, 1981; Goldenberg, 1988; Leinhardt et al., 1990). Learning the concept of continuity using the graphing calculator calls for understanding the view-window concept, and most students only developed a partial understanding of this. The view-window's relation to the movement of the pointer on the graphic screen was problematic for most students. They were aware that the distance the pointer moved from one point to the next point varied according to the view-window values. However, it was difficult for them to decipher how the invariant number of presses on the left/right or up/down buttons for any view-window setting was related to the distance between consecutive marks on the axes. The difficulty arose from the non-obvious value, in the case of the graphing model used in this study, which involved 63 presses and 31 presses to cover the length of the x and y-axis respectively from the origin in either direction. Although students in the StructLM were specifically asked to explore how much the pointer moved for each press on the cursor in different view-windows, relating this piece of information to the distance moved in the view-window defined proved to be very difficult.

An absolute understanding of scale rather than relative understanding of scale dominated among students in this study as evident from observation of students' avoidance to change the scale value of the view-window throughout their tasks. The syntax of the calculator for "scale" determines what the tick marks on each axis represent (i.e. the spacing of the x-axis increments and the spacing of the y-axis increments), not the absolute meaning of scale which most students understood. Therefore, the distance between two consecutive markings on each axis may not be the same although the scale value shown is the same. However, the limited understanding of the scale concept did not preclude students from understanding other graphical concepts, such as gradient. The built-in view-windows and the various view-window settings suggested in the tasks gave the students several choices of different view-windows which could be suitable to draw the complete graph of the equations that were given in the tasks. The ease in which the graphing calculator allowed students to choose different view-windows concealed the need for a full understanding of the scale concept, a concept suggested by many to be

crucial in graphing (Goldenberg, 1988; Leinhardt et al., 1990; Dion; 1990; Dick, 1992; Cavanagh and Mitchelmore, 2000).

The study highlights the great difficulty that low attaining students had even in constructing linear graphs. The graphing calculator enabled them to construct the correct graphs and supported the global view of the linear graph as an “object” with the attributes of slope and direction. The easy generation of linear graphs enabled students to see that lines that were parallel to each other had the same gradient, and lines that were not parallel to each other did not have the same gradient. The form in which the expression had to be entered into the graphing calculator raised the awareness and reinforced the conception that the general equation of the straight line is in the form $y=mx+c$. However, low and medium attaining students found difficulty in understanding the straight-line as a two-parameter family. This evolved from uncertainty about which features of the graph to notice on the graphic display when each of the parameters was investigated. In the InterLM, this was also closely related to the difficulty students had in systematising their investigation. What was prevalent for these students was looking randomly at the features of the graph that appealed or appeared distinct to them, with little consciousness of the overall goal of the activity. The failure to think of gradient in the reverse also contributed to their partial understanding of the straight-line concept. Medium and low attaining students had difficulty in thinking in the reverse (Dubinsky, 1991) about the gradient. This necessarily involves thinking of the numerical gradient in the equation in both the ratio form and in the geometric form, indicating that the global understanding of the graph as an object with the “slope” and “direction” attributes is not sufficient for students to gain a full understanding of the gradient concept.

The analysis supports the view that the quick and easy generation of graphs by technology enabled students to have a global conceptualisation of non-linear graphs, (Heid, 1988; Demana et al., 1993; Smart, 1995). Students in this study not only were able to make the connection between the graphs and the forms of equations that represented them, but in general were also able to recognise that the highest power of the polynomial governed the general shape of its graph. Notably, they were all able to distinguish between the equations that represented linear graphs and non-linear graphs. A lack of algebraic skills did not preclude students from having a global view of the complete graph (Demana et al, 1993) of non-linear equations that are known to be

difficult to construct. It seems the graphing calculator made the global conceptualisation of graphs accessible even to low attaining students to the extent they were also able to recognise and explore families of reciprocal graphs.

However, only high attaining students were able to discern the different transformations of the parabola. The tendency for students to generalise what they encountered during their learning of linear graphs to non-linear graphs (e.g., Tall, 1987, Orton, 1983) persisted in the graphing calculator environment. For example, in describing the stretch of the parabolas, the students described the “gradient” of the parabola changing as the magnitude of the coefficient changed. Systematising the exploration of different families of parabolas was problematic for low and medium attaining students. In the InterLM, a lack of knowledge about negative numbers for the low attaining student exacerbated the situation, resulting in her working only with parameters that had positive values. Low attaining students in both learning models tended to focus on the local features of graphs in discerning and describing the different transformations.

Clearly, the object conceptualisation (thinking of the graph as an entity that can be manipulated as a whole translated and rotated) (Sfard, 1991), remained difficult for medium and low attaining students despite the presence of technology. This challenges the claim that the object conceptualisation can be made accessible through technology-supported environments (Kieran, 1993; Mosckovich et al., 1993; Yerushalmy and Schwartz; 1993). While students in this study did conceive straight-line graphs as objects with certain attributes, it was confined to the “static” form of object conceptualisation rather than the dynamic form as described by Sfard. For example relating the “m” in the equation $y=mx+c$ to the “slope-related” graphical property of the line. This indicates that the accessibility of such concepts in a technology-supported environment is not straightforward.

The analysis also reveals that the graphing calculator has the potential to support students’ learning of some aspects of algebra, mathematical terms, conventions and notations related to graphical concepts. The graphing calculator confronts the user with the necessity to use the syntax correctly. The non-equivalence of algebraic expression, for example, the difference between the $x+a^2$ and $(x+a)^2$ is immediately pointed out from the calculator’s graphic representation. It simultaneously confronts the meaning and use

of the brackets. The induction of new mathematical terms, conventions and notations can be strengthened if they correspond with the calculator syntax and notations. This is very significant for low attaining students who have great difficulty with algebra and in remembering some mathematical terms. Noticeably in the study, low attaining students developed a firm understanding of the concept of origin known to be problematic for even high school seniors (Moschkovich et al., 1993). The invariant nature of the graphic screen where the x and y-axis always intercept at zero might have reinforced this fact.

The pattern of students' learning of graphical concepts that emerged seems to indicate that Vygotsky's notion of the zone of proximal development is useful in describing development of their understanding. Students could develop a mathematical concept presented to them if it was not too advanced from their actual (existing) mathematical concepts. High attaining students tended to have a rich conceptual knowledge base, which enabled them to appropriate more of the mathematical ideas that were presented to them. In other words, the mathematical ideas presented to them, and their own understanding of the related mathematical ideas were not very distant in terms of the students' zone of proximal development. The graphing calculator afforded them a support structure to connect what they already knew and what the learning tasks expected them to discover or establish. The calculator therefore functioned as a *mediational tool* that supported their mathematical thinking. That is, using the graphing calculator as a tool to direct one's own mental processes characterised by activities such as testing out conjectures, making guesses and extending as well as articulating what one is thinking. The graphing calculator, in other words, is used as means of focussing one's attention and directing one's mental operations toward the solution of a problem. For low attaining students, the distance between their actual knowledge and the knowledge they were expected to come to know was much greater. Therefore the potentiality of going wrong or not discovering the mathematical ideas embedded in the learning tasks was greater. Indeed, in some instances, they had difficulty in making sense of the learning activity. Knowledge of the functionality of the calculator plays an important role here. If they understood the functionality of the calculator that was related to the task, the calculator could mediate their understanding of the related concept. Otherwise, the calculator was limited as a static display tool, or to carry out procedures of the task. It seems the transition from using the calculator as a display device to using the calculator as a mediational tool marked the beginning of students developing their own

understanding of the mathematical concepts embedded in the tasks and self-regulating their learning. This was the point where the mathematical thinking played a central role in their activity, with the graphing calculator acting as a tool to support thinking.

9.2 Functionality of the calculator

Knowledge of the functionality of the calculator seems to be an important aspect of students' learning as it determines how the students use the calculator, and in turn how they structure their learning activity and thinking. This includes knowing which calculator button to press and what sequences of button presses are needed to retrieve the required menu or commands to perform the desired operations. It also involves an expectation of what is to be displayed on the screen as well as developing a reasonable scepticism about the graphic display or calculator result when certain commands are executed or operated in certain modes. In short, having an awareness of some aspects of the potentiality and limitations of the graphing calculator. This kind of knowledge has many similarities with the instrumentation process described by Guin and Trouche (1999, 2000).

The view that the constraint-support structure embodied in the tool affects learning (Kaput, 1992; Goldenberg, 1988) was found in this study. In the case of the graphing calculator, one example is the default behaviour of the tracer that automatically begins to trace from the minimum value of x on the line (in a specific view-window) which does not necessarily appear on the screen. The expectation is to see the tracer on the screen immediately it is activated. Another example is that the calculator automatically plots a point within the limits of its "readability" resulting in integers being plotted as decimals. The built-in characteristic of the calculator that automatically resets the y -axis when the value being traced is out of the value of the view-window is also a source of difficulty and confusion for medium and low attaining students.

High attaining students were able to recognise the potentiality and the limitations of the graphing calculator. Medium attaining and low attaining students had variable understanding of its potentiality and limitations, and a limited knowledge of the functionality. This was apparent when tasks involved many manipulations or action sequences, for example in operating the TABLE Mode and the DYNA Mode. A source

of confusion here was the calculator syntax. For example, the term “range” used in the TABLE and the DYNA Modes refers to different meanings in each mode, likewise the term “pitch”. In the former mode, pitch refers to the variable x value increment, whereas in the latter, it refers to the dynamic coefficient value increment.

Medium and low attaining students had different degrees of understanding about the functionality of the calculator related to the task, and this had implications for what they learnt. There was a limited, procedural understanding of the functionality of the calculator leading to a fragmented understanding of concepts. In some instances, this caused misconceptions through visual illusions. One particular example was the equidistant jumping of the pointer from one point to the next point, which gave the illusion that points lying on the line are discrete. Another problem was seeing the graph generated in the G-PLT facility as short and dotted whereas the graph generated in the G-CON facility was long and continuous. The dynamic generation of the graphs by the DYNA Mode gave the visual illusion that the graph was turning, or rising and falling instead of having different gradients. Graphs below the line $y=x$ were seen as getting shorter when the values of “ m ” started to decrease below 1. Partial understanding of the functionality of the calculator also resulted in medium and low attaining students not taking full advantage of the technology. Noticeable was their not using the parametric facility of the DYNA Mode to generate graphs with more than one parameter.

How students developed a knowledge of the functionality of the calculator was greatly influenced by the way teacher responded to student’s difficulties or errors when operating with the calculator. There was a lack of consistency in the way teachers in this study dealt with students’ errors. Two types of errors could be identified using Merrill et al.’s (1995) classification of errors when using technology: syntactic and semantic. With respect to the graphing calculator environment, syntactic errors arise from not following the calculator’s syntax, or its rules for entering expressions. One example is keying in $3 \div 8x$ instead of $(3 \div 8)x$ for $\frac{3}{8}x$. Semantic errors consist of errors made on the calculator’s commands that mostly are contained in menus, for example, the low attaining student who confuses between the commands “range” and “pitch”. Embedded in these menus are the limitations and the potentiality of the calculator. Both types of errors were prevalent in low attaining students who have significant difficulty with their

algebra, and coordinating between graphical language and the calculator language. Consistent with Merrill et al.'s (1995), syntactic errors were not easy for students to correct themselves but in fact they had little significance in students' learning. Semantic errors could be easily overlooked, and demanded a great deal of teacher foresight about the possibilities of student errors and student thinking. Telling students directly how to correct semantic errors predisposed students towards a procedural understanding of the functionality of the calculator. Students needed to understand the reason behind the operation of the graphing calculator, otherwise they merely viewed the graphing calculator as a machine for doing mathematics instead of a tool for learning (Adams, 1997).

9.3 Teacher intervention

Consistent with some other studies on graphing calculators, (Hennessy et al., 2001; Lawler, 2000; Adams, 1997), this study revealed that intervention is a vitally important component in learning. Intervention was important in introducing students to the main ideas of the domain in two ways. Firstly, it inducted them into new mathematical terms and conventions. Secondly, teacher intervention helped in making the link between the different concepts that they were developing from the different tasks. One example was making the connection between the different ways of thinking about gradient. Connecting the idea of gradient as the ratio of increase in y to increase in x to the "m" in the equation, and to the "global" gradient associated with the slope-related graphical property was problematic for many students and required intervention of the teacher.

Typically, the teachers were always operating within the high attaining students' zones of proximal development in terms of the language and the leading questions or mathematical knowledge presented. Therefore, for high attaining students, the continual teacher interventions are always at the "cutting edge" of their competencies in their continually changing zones of proximal development (Clay and Cazden, 1990). This enabled the high attaining student to repeatedly bridge his/her zone (Jones and Thornton, 1993) and to successively assume "advance" tasks and reach a heightened state of understanding. Low attaining students on the other hand, benefited very minimally from classroom instruction as the knowledge that the teacher presented was usually not within their zones of proximal development. In other words, the teacher was not functioning

within their “region of sensitivity to instruction” (Rogoff, 1991). Typically too, the teachers were always using curriculum scripts (Graesser, 1995, Chi, 1996) in their interventions. The strict adherence to curriculum scripts sometimes resulted in rote and compartmentalised understanding.

The study reveals that medium and low attaining students tended to need more specific kinds of intervention. For example, it was insufficient to tell them to look at what happened to the line when “ m ” changed. A more productive approach was to direct attention to the steepness when the magnitude of “ m ” changed or to the direction when the sign of “ m ” changed as this kind of intervention would be targeted closer to their actual developmental level. If the teacher intervention was not effective, usually in terms of incomplete intervention, the gap between the student’s actual developmental level and the level that teacher was inducting was too wide for the student to make the connection or to use the calculator to make the connection.

The study reveals that the quality of the teachers’ content knowledge influenced the quality of teacher’s explanations, therefore the type of knowledge that teachers imparted and how they intervened. Teachers’ impoverished conceptualisation of some graphical concepts was evident from their avoidance of discussions with students of some of these concepts, for example, the properties of graphs in terms of transformations. The teachers also did not advance the idea of turning points and did not assist students towards formalising some students’ naïve description of transformations, suggesting that the teachers were operating at a level of instrumental rather than relational understanding (Norman, 1993). The way the teachers in the study intervened when students determined the straight-line equation also explained why some students developed partial understanding of the straight-line concept. The teachers’ focus was on procedures and obtaining the right answer and did not help students to overcome their fixation tendency (Bell et al., 1987). The teacher’s limited understanding of the graphical concepts may also have affected the ways they used the graphing calculator. For example, in the learning of graph transformations in the case study involving non-linear graphs, the idea of the “movement of objects” in the graphical space was not explained by the teacher.

9.4 The learning models

Although theoretically the two learning models could be constructed and were distinguishable, in practice many interventions characteristic of each learning model, were found in both case study groups, StructLM and InterLM. However, whether the interventions were process or goal type, they did not impact on the overall theoretical construct of the learning model. The learning tasks and the classroom instructional setting, one being teacher-led and the other being activity-led, distinguished and maintained the differentiation of the two models. The nature of intervention that the students demanded further showed that the goal and process type interventions did not influence their development of graphical concepts. Students in this study tended to request a more directional type of intervention regardless of the learning environment they were working in. This confirms Hoyles and Sutherland's (1989) finding that students preferred directional type rather than the reflectional (process or goal) type of interventions. What emerges from this study is that it was not useful to specify or prescribe the types of intervention allowed. This evolves from issues involving students' views of how they themselves should be learning, and teachers' understandings of how teachers should teach. Students in this study were more attuned to prescriptive instruction methods with expository type and directive intervention, and highly structured tasks. The excerpt in Appendix 1 neatly describes how a student in the InterLM expressed his exasperation and dislike of the uncertainty and lack of structure for this type of learning approach. The student considered that the teacher was not teaching "properly" and expressed that he wanted the instruction to be overt and directive!

Due to unfamiliarity with an open style of task, medium and low attaining students in the InterLM found difficulty in systematising their exploration. This was clearly observed in their exploration of the concept of continuity. Changing different values of the "pitch" until the dots appeared very close to become a seemingly continuous line similar to the one generated by the G-CON command, was clearly missing from students' activities. Students tended to restrict the pitch value to 1 or 0.5 according to the teacher's suggestion. In investigating linear graphs as a two-parameter family, students randomly assigned the values of the parameter. The same behaviour was observed when they investigated quadratic graphs with varying values of the parameters. This is consistent

with Adams (1997), whose study found that students who used the graphing calculator as an add-on feature to their existing learning performed better than those who used the graphing calculator together with specifically designed instructional materials which students were not familiar with.

As expected, students always pressured teachers to provide explicit procedures for continuing or completing tasks. This made it difficult for teachers to follow closely the suggested interventions in each learning model although they fully understood the theoretical underpinnings of each learning approach. This seems to concur with the literature on how difficult it is for teachers to change the way they teach (e.g., Lubinski and Jaberg, 1997, Simmt, 1997). The teachers' lack of using the graphing calculator to support their explanations during intervention also suggests that the teachers' own mental model of how knowledge should be presented to students is significant. The teachers in this study were used to the expository and didactic style of instruction. It is therefore not surprising that curriculum scripts were prevalent in many interventions which seemed to profit high attaining students who understood the mathematical language, and the mathematical concepts that the teachers presented to them. For low attaining students, it could go in either direction. If their actual level of knowledge was not too far from the knowledge that the teacher was trying to induct, they also benefited from the instruction. More often than not, this was not the case. Their lack of understanding of the mathematical language that the teacher used, and the lack of knowledge of the functionality of the calculator, made it impossible to maintain the flow of the curriculum script. The intervention either was left uncompleted or reduced to the teacher "transmitting" knowledge, which was evident from the many long didactical explanations.

The analysis shows that in general whether the learning tasks were open or structured, development of students' graphical concepts was contingent on the efficacy of teachers' interventions. Low and medium attaining students required specific intervention in relation with the difficulty they have encountered, whether it is the task itself or the functionality of the calculator. Regardless of the learning model, the level of mathematical knowledge that teachers had, greatly influenced the way they intervened, and the extent to which students advanced their mathematical concepts. For example, the way that students in this study distinguished the different families of graphs in terms

of their local properties rather than viewing or describing them as transformations suggests that teachers had restricted content knowledge about graph transformations. More importantly it highlights that teacher intervention is necessary towards students' formalisation and induction of new knowledge.

9.5 Models of students' learning

The study reveals two distinct profiles of high attaining students, versus medium and low attaining students. High attaining students seemed to be able to build their own understanding of the functionality of the calculator. This understanding enabled them to exploit its features to use it as a mediational tool to support their mathematical thinking. It enabled them to transform their learning to include conjecturing and generalising, and in the process, to make the connection between the mathematical ideas that they had, and what they were supposed to encounter themselves if the new mathematical ideas were slightly beyond their actual level of development. Teacher's classroom instruction and individual intervention provided them with a critical source of opportunities to learn and bridge their continually changing zones of proximal development. In the former, their understanding of the mathematical language that the teacher used and the related mathematical knowledge the teacher presented, enabled them to participate actively, an important factor in knowledge construction (Chi, 1996; Demana and Waits, cited in Farrell, 1996).

Through active participation, the high attaining students were supported by the task structure and teachers' questions, compiling and organising the pieces of information they needed to construct meanings. They also became enculturated into the accepted conventions for communicating and verifying knowledge claims. Their understanding of the mathematical language that the teacher used and mathematical knowledge the teacher presented enabled them to follow through teachers' curriculum scripts. Characteristic of the interactions was their interactive nature, usually with additional information in each conversational turn. This tutorial-like interaction structure enabled the high attaining students "to take stock of processes (of development) that are now in the state of coming into being, that are only ripening or developing" (Vygotsky, cited in Wertsch and Tulviste, 1996, p. 57). Questions being asked by high attaining students were usually of specific knowledge pieces they did not know. Although in some cases when the teacher

intervention was not effective, the students nonetheless managed to continue the tasks. The interaction process assisted them to articulate their ideas as it requires the creation of a common framework for the coordination of action and the exchange of information (Rogoff et al., 1991).

The zones of proximal development of low attaining and medium attaining students are small compared to high attaining students, that is what they can do without assistance is less. The differential readiness to perform competently in the task domain is a function of such factors such as prior mathematical knowledge and the understanding of the functionality of the calculator. They had variable understanding of the functionality at different stages of their learning with marked difficulty in integrating all the different functions of the calculator. This precluded them from using it as a mediational tool and predisposed them to different kinds of visual illusions from the graphic display. They depended on the teachers to tell them what to do, resulting sometimes in procedural understanding of the functionality of the calculator, which resulted in using the calculator as a static display tool, or to carry out task procedures. Using the graphing calculator as a display tool restricted how they could structure their learning activity. The teacher's classroom instruction did not have much impact on these students' learning as very often they could not understand the teacher's mathematical language, which is crucial in concept formation from the Vygotskian perspective. The teacher did not play a mediating role. In other words, the means of interpersonal communication required of the transformation for the intrapersonal processes of the student was severed. Therefore the "potential concepts", which are the precursors of true concepts (Vygotsky, 1986) of the students, did not advance to the level of genuine concepts.

Low and medium attaining students had difficulty in following through teachers' curriculum scripts if the mathematical ideas that the teachers tried to initiate were too far from their current mathematical knowledge. According to Vygotsky, a certain minimal "ripeness" of higher mental functions (perception, attention and memory) is required, that is, it is equally important to determine the lowest threshold at which instruction may begin. The missing pieces of relevant mathematical knowledge sometimes diverted students from the main idea of the interaction. The lack of understanding of the functionality of the calculator frequently posed another layer of complexity. Pervasive in the interaction was the teacher directly telling the students what to do, resulting in rote

learning and fixation towards certain aspects of information, in particular when there was high concept density. This is consistent with the Vygotskian approach where the structure and content of a particular interaction constitute the form and content of conceptual development (Waschescio, 1998). Students' questions were directed towards executing or completing the task, either in terms of the calculator functionality or inadequate prerequisite mathematical knowledge that were related to the task. Ineffective teacher intervention, usually in terms of incomplete explanations gave rise to problematic gaps (Mayer, 1989) that result in fragmented understanding where only parts of the explanation may be understood. These problematic gaps exemplify what Sweller (1994) calls "imposed instructional" load, that is the load created by the teacher in addition to that supplied by the topic. They led students to enduring misconceptions and unproductive floundering.

Overall, what emerges from the study is the complexity of the teaching and learning situation when technology is integrated. A student's learning of graphing concepts with the graphing calculator is a function of his/her zone of proximal development, existing mathematical knowledge, knowledge of the functionality of the calculator and the nature of teacher intervention. Intervention targeted within the zone of proximal development appears to help students to develop their understanding but what constitutes this zone differs for each student and each graphical concept.

Implementing a more open approach is constrained by the teaching and learning culture that students and teachers experience at school. The difficulties observed in full understanding of the straight line concept were related to the subject of mathematics, as taught in school, which favours the algebraic (including the use of standard formulae and computation) rather than the graphic representation of problems. It seems that the success of implementation of a more open approach also depends on how well the student can systematise their exploration, and this is closely linked to the prerequisite knowledge needed to carry out the task. However, using the structured learning method did not warrant a greater facility towards developing the graphical concepts. As shown in case study of the concept of continuity, it also depended on the concept density itself and the student's related understanding of the functionality of the calculator in use. Developing an understanding of the calculator takes time and how

students develop an understanding is greatly influenced by the way the teacher responds to students' difficulties and errors when using the calculator.

CHAPTER 10

CONCLUSION

This chapter presents the conclusion of the thesis that centres on Vygotsky's notion of the zone of proximal development to investigate the potentiality of using the graphing calculator in 14 year-old Malaysian students' learning of graphical concepts. The first section of this chapter highlights the central findings of the study, focussing on the impact of technology on students' learning. The second section considers the value of positioning students' learning with technology within the Vygotskian notion of the zone of proximal development. In the third section, implications for the use of graphing calculators in student' learning of graphical concepts are presented. Included in these implications is the use within the context of Malaysian schools. The last section presents the contribution and the limitations of the present study. Further research in graphing calculator use is suggested.

10.1 The impact of technology on students' learning

This study serves to further confirm the view that learning in a technology supported environment is not simply a matter of an individual working with the tool. The mediation of technology requires other mediations by the teacher (Hoyles and Sutherland, 1989; Noss and Hoyles, 1996; Hudson, 1995; Somekh, 1997, Guin and Trouche, 1999; 2000). Teachers are pivotal mediators of the technicalities of the language, the mathematics embedded in the learning activity, and the formalisation of novel mathematical knowledge that students have not encountered before. However, using technology in learning changes the role of the teacher into a more challenging one. The managerial, task setter and explainer roles of the teacher in classrooms without technology (Farrell, 1996) are still important, but this study shows that other roles (e.g., guide, facilitator) occur concurrently when technology is used.

Graphing without technology needs to be examined carefully as evidently students in this study advanced much more than what was thought possible by the curriculum. The "affordances" of the graphing calculator as described by Hennessy et al. (2001) would

not have emerged in paper-and-pencil graphing activities. In particular, the capability of the graphing calculator to manipulate and generate multiple graphs easily provided opportunities for exploring properties of, and relationships between, graphs. The immediate visual representations of graphs facilitated students to develop their own intuitions about what a graph looks like. The use of a graphing calculator impacts on the progression of learning of students of different mathematical attainments according to the mathematical knowledge that they had. The lack of algebraic and computation skills in low attaining students was *not* an inhibiting factor in their learning of graphical concepts that were related to the form of the equation and the shape of the graph, and graph transformations. This seems consistent with the view on using technology in learning that it will change the manner in which mathematical concepts are usually learnt (e.g., Palmiter, 1991; Goldstein, 1994; Dunham, 2000). The learning of graphical concepts which follows a hierarchy, usually beginning with point plotting to graphing of linear equations, and advancing to graphing of polynomials of higher degree, seems no longer warranted in a technology supported learning environment. Students can learn some graphical concepts before any formal knowledge of graphing. The learning of graphical concepts could also be integrated with the learning of algebra, for example the concept of equivalence, transposition of equations, and algebraic manipulations. However, hand graphing is still an important aspect of students' learning, as this study clearly shows that the concept of gradient related to the geometric representation of the numerical value of "m" in the equation was not facilitated by the graphing calculator. Manually constructing graphical representations themselves helps students to make explicit links between different representations in that graphs supply the variation that is hidden in an equation and make the structure visible (Cox and Brna, cited in Hennessy et al., 2001).

This study highlights technology's integral role in influencing mathematical thinking and shaping activity. Incorporating technology into learning changes the way students learn in that it facilitates an active approach to learning, where their role can be one of questioning rather than merely accepting. However, students have to make sense of the calculator's graphic display in order to use it as a mediational tool.

This study points to the fact that modelling the learning environment which uses technology is complex. There were three major factors that influenced students' learning

in the graphing calculator environment: knowledge of the functionality of the calculator, the existing mathematical knowledge and the nature of teacher intervention. Arguably there may be other factors that influence students' learning depending on the technology deployed and the knowledge domain chosen. However, what is clear from this study is that we need to examine the interplay between these factors in modelling the learning environment.

10.2 The value of using Vygotsky's idea of the zone of proximal development

This study reveals that describing students' learning with Vygotsky's theoretical construct of the zone of proximal development makes it possible to frame students' learning with the graphing calculator in various settings, and simultaneously to explore how they impinge on learning. The construct of the zone also enables us to discern how each of the three factors identified (the knowledge of the functionality of the calculator, mathematical knowledge and the nature of teacher intervention) operates to influence what students learn or do not learn. It offers a coherent explanation of how each of this factor interacts to model students' learning with technology, as the zone informs us of the importance of the relative "intellectual abilities" of the students. Students of different mathematical attainment appropriate the graphing calculator and the structures of teacher intervention differently in their learning.

Vygotsky's idea of the zone of proximal development illuminates not only the importance of the teacher's role but also informs us how the role of the teacher impacts on students. In particular, integrating a different instructional approach from that which students and teachers have been enculturated is not easy, and it depends on teachers' efficacy to deal with the situation. It illuminates how the structure and content of teacher intervention through mathematical language and content knowledge influences the kinds of understanding that students develop. The idea of the zone of proximal development helps to explore why only some students could use the graphing calculator as a mediational tool and how this use impinged on whether they developed a partial, or compartmentalised understanding of the graphical concepts.

Vygotsky's idea of internalisation, and his insistence on the need to concentrate on the very process by which higher mental functions are established (that is, by psychological

tools and means of interpersonal communication) is important in this study. It enables us to discern that it is the role of the technology in students' learning that is crucial rather than how it is deployed, either as an add-on or with a specifically designed instructional material. It also helps to understand how the role of technology impacts on students' learning. A shift in the role of the graphing calculator from a static, display tool to a mediational tool is necessary. This sees students' learning shifting from being a passive learner dependent on the teacher or instructional tasks to someone having a "mathematical disposition" (Henningsen and Stein, 1997). The teacher and the students in their interaction define the role of the graphing calculator. The teacher is crucial in guiding and shaping the structure and the content of the interaction, and thereby the kind of understanding which students develop about the functionality of the calculator and the graphical concepts. The students, in their interaction with the graphing calculator also define its role in their learning, and this largely depends on whether the tasks, mathematical knowledge and knowledge of the functionality of the calculator are within their zones of proximal development.

10.3 Implications for the integration of graphing calculators in students' learning

The study clearly reveals that using the graphing calculator as a mediational tool supports and transforms students' learning, whereas using it as a static display tool or to perform task procedures predisposes towards a fragmented and compartmentalised kind of understanding of graphical concepts. This suggests a need for directing students towards a mediational use of technology, which means students need to understand the reasoning behind the functioning of the calculator related to the mathematical activity. In turn, this points towards a need for teachers to develop a framework within which to think about which functions of the calculator are necessary in executing the task, and ensuring that they do not obscure students from executing the task. Other considerations include deciding how the task is related to the prerequisite mathematical knowledge, so that the task is set not too far beyond students' existing knowledge, and ensuring that teachers are aware of the possibility of some low attaining students might not have this knowledge. For example, the exploring activities as depicted in the study that involve parameterisation are clearly too difficult for low attaining students and, to a certain extent medium attaining students, particularly when it includes more than one parameter.

The study shows that there is no preferred or linear manner in which students should learn graphical concepts. Graphs of polynomials that are difficult to hand graph are accessible even to low attaining students with the graphing calculator. This has direct implications on which kinds of graphs students should learn and what graphical concepts are assessed. It raises the question of whether hand graphing of certain families of graphs is still desirable with the use of technology in the learning process. With reference to the Malaysian education setting, the results of the study also suggests that the order in which the graphing concepts are presented in the current curriculum design and implemented “hierarchically” year by year throughout the four years of the students’ secondary schooling needs to be evaluated. Clearly, it could be a restricting structure for students, in particular low attaining students. The “non-linear” manner in which the students develop graphical concepts also suggests the potentiality of the graphing calculator to advance a learner-centred approach as recommended by the curriculum.

Teachers’ mathematical knowledge is clearly challenged when technology is used suggesting the need not only for teachers to be aware of this fact but also the relevant authorities to provide support for teachers in terms of training and courses for teachers to advance their mathematical knowledge in the domains where technology is involved in the teaching and learning process.

Students’ understanding of the functionality of the calculator is built over time suggesting the need for teachers to be aware of the evolution of the instrumentation process (Guin and Trouche, 1999; 2000). The way that teachers deal with students’ errors and difficulties influences the nature of students’ understanding of the functionality of the calculator and the graphical concepts. The inconsistencies that teachers displayed when assisting students suggest a lack of knowledge in how to effectively deal with this situation. This suggests that teachers need to be encouraged to reflect on their role when technology is used, and develop forms of reflective classroom practice such as making observation to link what he/she attempts during the intervention and the subsequent learning. Therefore, it is important for educational authorities to realise that preparing teachers to teach with technology involves more than superimposing some knowledge of technology on teachers’ current teaching knowledge and practices. Using technology not only places new demands on their mathematical knowledge and on their facility with technology, but also on their understanding of what

it means to learn and understand mathematics, and on their ability to use these understandings to structure meaningful instructions with technology. This suggests a need to support teachers in developing their pedagogical knowledge of the technology in use.

The study clearly identifies two types of errors that students made when using the graphing calculator: semantic and syntactical errors. How teachers deal with these errors has implications on whether students develop a mediational use of the graphing calculator, or otherwise. This suggests there is a need for teachers to recognise the types of errors that students make when using the graphing calculator in their mathematical activities, and consequently provide the appropriate intervention.

10.4 Limitations of the study and suggestions for further research

As with all case studies, the results of this study are limited in terms of their generality. It would be interesting to include more students in the analysis to see whether the results of similar study could be made more general. However, case studies do offer deep insights in students' learning using the graphing calculator. The visual illusions would be difficult to realise otherwise. In particular, the revelation that the nature of the graphing calculator, in not working with continuous quantities and thereby plotting some points within the limits of its readability, is confusing for many students, needs further investigation. It seems that understanding how much the pointer moves exactly on the graphic screen for each push on the cursor is important in students' understanding of the behaviour of the pointer in different view-window settings. It would be interesting if future studies that highlight this issue could make a difference to students' learning; in particular, to the suitability of the graphing calculator in learning the concept of continuity.

The analysis reveals the importance of student-student interaction on learning. This supports other graphing calculator studies (Mok, 1999; Hennessy et al., 2001) which found evidence that the interaction between students and their peers affects the effective use of the technology in the mathematics classroom. It would be interesting to identify which graphical concepts students could learn collaboratively with the graphing calculator. This would be informative for teachers to manage their time efficiently to

devote more time to students who need more careful intervention, and to focus on providing interventions for graphical concepts that are difficult for students to conceptualise without their intervention.

Although some studies in other technology supported environments have warned to be mindful of the constraint-support structure of the technology, this study shows that the constraint-support structure is related to the graphical concepts being investigated. For example, the “discreteness” of the graphing calculator appears to have a strong effect on students’ conception of the continuity concept but not on the idea of transformation. Students in this study were oblivious to the fact that the graphing calculator did not work with continuous quantity. The problem of discreteness²¹ suggests that an awareness of the constraint-support structure associated with the graphical concept is warranted if the graphing calculator is to be realised as a mediational tool. Therefore, it would be informative if further research is undertaken to identify which graphical concepts are more “susceptible” to the specific constraint-support structure of the graphing calculator.

The quality of teachers’ content knowledge clearly influences the way teachers intervene. Teachers’ content knowledge on transformation in this study was clearly lacking, thus it would be interesting to look at the forms of understanding that students could have developed had the teacher been more competent. This would give further insight on the potentiality of the graphing calculator in learning.

Teachers’ “pedagogical technology knowledge”, that is, pedagogical knowledge on how to use technology, clearly influenced the manner in which they intervened. The training of the teachers in this study was conducted over a very short period of time. It would be interesting to look at the extent to which the training of teachers conditions the ways teachers intervene, and whether they could provide interventions that were productive in students’ learning with the graphing calculator as found in the teacher of Doerr and Zangor’s (2000) study.

²¹ The graph display screen of the model that was used in this study is made up of 127 (width)×63 (height) dots. Therefore when a point is plotted (or traced) on a specific view-window, the calculator automatically plots (or traces) the point within the limits of its “readability”. It is confusing when plotting (or tracing) a point, say (3,2) on the STD window, the value read on the screen is (3.0158730158, 1.9354838709) when visually the maximum and minimum x and y-values are 10 and -10 with 10 division markings along each side of the axes. The expectation is for the point to be on the grid.

Further research would be helpful to outline specifically the kinds of students' difficulties and errors when using the graphing calculator in relation to the graphical concepts. This would be useful to inform teachers on the kinds of errors that students made in relation to the graphical concepts and subsequently how to deal with such situations.

10.5 Concluding remarks

This study highlights how technology can play an integral role in influencing mathematical thinking and in shaping learning activity. It also shows that mathematical learning involving technology cannot be analysed only at the cognitive level. The mediation of technology in students' learning requires other mediations by the teacher. Students' mathematical backgrounds need to be considered in appropriating the task and the interventions in their learning. There is no definitive role for the teacher when technology is used; the role as a guide or as a facilitator could be productive depending on the task at hand, implying that the teacher's role cannot be rigidly pre-specified and then applied in every situation. At certain times, a more specific intervention is required. However, the key principle is that teachers provide assistance within the students' zone of proximal development. It is also important to get the student to play an active role in the student-teacher interaction so that the students too contribute in the concept formation process. There is also a need to develop students' instrumentation processes in order to shift the role of the graphing calculator from a static, display tool to a mediational tool.

BIBLIOGRAPHY

- Abuloum, K.M. (1996). *Graphing calculators: teacher perceptions, training, and attitude*. Doctoral dissertation, Dissertation Abstracts International, 57/03, p. 1063, The University of Nebraska.
- Adams, T.L. (1997). Addressing students' difficulties with the concept of function: applying graphing calculators and a model of conceptual change. *Focus on Learning Problems in Mathematics*, 19(2), 43-57.
- Ainley, J., & Pratt, D. (1993). Portable mathematics and integrated technology. L. Burton & B. Jaworski (Eds.), *Proceedings of the Technology and Teaching Conference* (pp. 101-108), University of Birmingham.
- Amarel, M. (1984). Classroom and computers as instructional settings. *Theory into Practice*(22), 260-266.
- Anderson, M., Bloom, L., Mueller, U., & Pedler, P. (1999). The impact of the graphics calculator on the assessment of calculus and modelling. *International Journal of Mathematical Education in Science and Technology*, 30(4), 489-498.
- Ary, D., Jacobs, L.C., & Razvich, A. (1990). *Introduction to research in education*. London: Holt, Rinehart and Winston.
- Asp, G., Dowsey, J., & Stacey, K. (1993). Linear and quadratic graphs with the aid of technology. B. Atweh, C. Kanes, M. Carss, & G. Booker (Eds.), *Proceedings of the 16th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 51-56), Brisbane: Queensland University of Technology.
- Averbeck, P.J. (2001). *Student understanding of functions and the use of the graphing calculator in a college algebra course*. Doctoral dissertation, Dissertation Abstracts International, 61/11, p. 4315, Oregon State University.
- Balacheff, N., & Kaput, J.K. (1996). Computer-based learning environments in mathematics. In A.J. Bishop, K. Clements, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education, Part One*. (pp. 469-501). Dordrecht: Kluwer Academic Publishers.
- Barclay, T. (1987). A graph is worth how many words? *Classroom Computer Learning*, 7(5), 46-50.
- Barling, C. (1997). How technology impacts on curriculum, *Paper presented at the Asian Technology Conference in Mathematics*, Malaysia.
- Barr, G. (1980). Graphs, gradients and intercepts. *Mathematics in School*, 9(1), 5-6.

- Barr, G. (1981). Some student ideas on the concept of gradient. *Mathematics in School*, 10(1), 14-17.
- Becker, B.A. (1992). *The concept of function: misconceptions and remediation at the collegiate level*. Doctoral dissertation, Dissertation Abstracts International, 52/08, p. 2850, Illinois State University.
- Bell, A., Brekke, G., & Swan, M. (1987). Misconceptions, conflicts and discussion in the teaching of graphical interpretation, *Proceedings on the 2nd International Seminar on Misconceptions and Educational Strategies in Science and Mathematics*, Vol. 1 (pp. 46-58): Cornell University.
- Berger, M. (1998). Graphic calculators: an interpretative framework. *For the Learning of Mathematics*, 18(2), 13-20.
- Blease, D., & Cohen, L. (1990). *Coping with computers. An ethnographic study in primary classroom*. London: Paul Chapman Publishing Ltd.
- Bliss, J. (1994). From mental models to modelling. In H. Mellar, J. Bliss., R. Boohan, J. Ogborn, & C. Tompsett (Eds.), *Learning with artificial worlds: computer based modelling in the curriculum* (pp. 27-32). London: The Falmer Press.
- Bloom, B.S. (1956). Taxonomy of educational objectives: The classification of educational goals. Handbook 1: Cognitive domain. New York: McKay Co.
- Borba, M., & Villarreal, M.E. (1998). Graphing calculators and reorganization of thinking: the transition from functions to derivative, *Proceedings of the 18th Conference of the International Group for the Psychology of Mathematics Education*, 2, 136-144.
- Broadfoot, P. (1996). Assessment and learning: power or partnership? In H. Goldstein & T. Lewis (Eds.), *Assessment: problems, developments and statistical issues* (pp. 21-40). Chichester ; New York: John Wiley & Sons Ltd.
- Brown, A.L., & Ferrara, R.A. (1985). Diagnosing zones of proximal development. In J.V. Wertsch (Ed.), *Culture, communication and cognition: Vygotskian perspectives*. Cambridge: Cambridge University Press.
- Brown, A.L., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, 18(1), 32-42.
- Brown, C.A., Stein, M.K., & Forman, E. (1996). Assisting teachers and students to reform the mathematics classroom. *Educational Studies in Mathematics*, 31, 63-93.

- Burton, L., & Jaworski, B. (1995). Introduction. In L. Burton & B. Jaworski (Eds.), *Technology in mathematics teaching - a bridge between teaching and learning*. Bromley: Chartwell-Bratt.
- Carey, D., Fennema, E., Carpenter, T., & Franke, M. (1995). Equity and mathematics education. In W.G. Secada, E. Fennema, & L.B. Adajian (Eds.), *New Directions for equity in mathematics education* (pp. 93-125). New York: Cambridge University Press.
- Carpenter, P.C., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T.A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19-32). Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Cavanagh, M., & Mitchelmore, M. (2000). Student misconceptions in interpreting basic calculator displays, *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, 161-168.
- Cazden, C. (1981). Performance before competence: assistance to child discourse in the zone of proximal development. *Quarterly Newsletter of the Laboratory of Comparative Human Cognition*, 3(1), 5-8.
- Chi, M.T.H. (1996). Constructing self-explanations and scaffolded explanations in tutoring. *Applied Cognitive Psychology*, 10, S33-S49.
- Clay, M.M., & Cazden, C.B. (1990). A Vygotskian interpretation of reading recovery. In L.C. Moll (Ed.), *Vygotsky and education. Instructional implications and applications of sociohistorical psychology* (pp. 206-222). Cambridge: Cambridge University Press.
- Cobb, P., Wood, T., & Yackel, E. (1989). Young children's emotional acts while engaged in mathematical problem solving. In D.B. McLeod & V.A. Adams (Eds.), *Affect and mathematical problem solving: a new perspective* (pp. 117-148). New York: Springer-Verlag.
- Cobb, P., Wood, T., & Yackel, E. (1991). Learning through problem solving: A constructivist approach to second grade mathematics. In E. vonGlaserfeld (Ed.), *Constructivism in mathematics education* (pp. 157-156). Dordrecht: Kluwer Academic Publishers.
- Cobb, P. (1995). Cultural tools and mathematical learning: a case study. *Journal for Research in Mathematics Education*, 26(4), 362-385.
- Collins Dictionary (1995). *Collins Concise Dictionary and Thesaurus* (2nd ed.). Glasgow: Harper Collins Publishers.

- Confrey, J. (1995). How compatible are radical constructivism, sociocultural approaches and social constructivism. In L.P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 185-226). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Cooney, T.J. (1999). Conceptualizing teachers' ways of knowing. In D. Tirosh (Ed.), *Forms of mathematical knowledge* (pp. 163-187). Dordrecht: Kluwer Academic Publishers.
- Crawford, K.P. (1986). Cognitive and social factors in problem solving behaviour, *Proceedings of the 10th Conference of the International Group for the Psychology of Mathematics Education*, 415-421.
- Crawford, K. (1996). Vygotskian approaches in human development in the information era. *Educational Studies in Mathematics*, 31, 43-62.
- Dalton, D.W., & Hannafin, M.J. (1987). The effects of word processing on written composition. *Journal of Educational Research*, 80(6), 338-342.
- Daniels, H. (1996). Introduction: psychology in a social world. In H. Daniels (Ed.), *An introduction to Vygotsky* (pp. 1-27). London and New York: Routledge.
- Davydov, V.V., & Zinchenko, V.P. (1993). Vygotsky's contribution to the development of psychology. In H. Daniels (Ed.), *Charting the agenda. Educational activity after Vygotsky* (pp. 93-106). London and New York: Routledge.
- Dawes, L. (2001). What stops teachers using new technology? In M. Leask (Ed.), *Issues in teaching using ICT* (pp. 61-79). London and New York: Routledge Falmer.
- Dean, P.G. (1982). *Teaching and learning mathematics*. London, England: The Woburn Press.
- Demana, F., & Waits, B.K. (1988). The Ohio State University calculator and computer precalculus project: the mathematics of tomorrow today! *AMATYC Review*, 10(1), 46-55.
- Demana, F., Schoen, H.L., & Waits, B. (1993). Graphing in the K-12 curriculum: the impact of the graphing calculator. In T.A. Romberg, E. Fennema, & T.P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 11-39). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Dick, T.P., & Shaughnessy, J.M. (1988). The influence of symbolic/graphic calculators on the perceptions of students and teachers towards mathematics, *Proceedings of the 12th Conference of the International Group for the Psychology of Mathematics Education*, 327-333.

- Dick, T.P. (1992). Symbolic-graphical calculator. *Teaching tools for mathematics*, 92(1), 1-5.
- Dion, G. (1990). The graphics calculator: a tool for critical thinking. *Mathematics Teacher*, 83(7), 564-567.
- Doerr, H.M., & Zangor, R. (1999). Creating a tool: an analysis of the role of the graphing calculator in a pre-calculus classroom. *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*, 2, 265-273.
- Doerr, H.M., & Zangor, R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics*, 41(2), 143-163.
- Dorfler, W. (1993). Computer views and views of the mind. In C. Keital & K. Ruthven (Eds.), *Learning from computers: mathematics education and technology* (pp. 159-186). Berlin, Germany: Springer.
- Dreyfus, T., & Eisenberg, T. (1983). The function concept in college students: linearity, smoothness and periodicity. *Focus on Learning Problems in Mathematics*, 5, 119-132.
- Drijvers, P., & Doorman, M. (1996). The graphics calculator in mathematics education. *Journal of Mathematical Behaviour*, 15(4), 425-440.
- Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 95-126). Boston: Kluwer Academic Publishers.
- Dugdale, S. (1993). Functions and graphs - perspectives on student thinking. In T.A. Romberg, E. Fennema, & T. P.Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 101-130). Hillsdale, New Jersey: Lawrence Erlbaum Associates Publishers.
- Dunham, P.H., & Dick, T.P. (1994). Research on graphing calculators. *The Mathematics Teacher*, 87(6), 440-445.
- Dunham, P.H. (2000). Graphing calculators: issues affecting secondary school teachers and professors: Texas Instruments Incorporated (On-line: <http://www.t3www.org/t3/>).
- Ehrmann, S.C. (1995). Asking the right questions: what does research tell us about technology and higher learning? *Change*, 27, 20-27.
- Emese, G.L. (1993). *The effects of guided discovery style teaching and graphing calculator use in differential calculus*. Doctoral dissertation, Dissertation Abstracts International, 54/02, p. 450, The Ohio State University.

- Entwistle, N. (1987). *Understanding classroom learning*. London: Hodder and Stoughton.
- Ernest, P. (1999). Forms of knowledge in mathematics and mathematics education: philosophical and rhetorical perspectives. In D. Tirosh (Ed.), *Forms of mathematical knowledge. Learning and teaching with understanding* (pp. 67-83). Dordrecht: Kluwer Academic Publishers.
- Even, R., & Lappan, G. (1994). Constructing meaningful understanding of mathematics content. In D.B. Aichele & A.F. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 128-143). Reston, Virginia: The National Council of Teachers of Mathematics.
- Farrell, A.M. (1996). Roles and behaviours in technology integrated pre-calculus classrooms. *Journal of Mathematical Behaviour*, 15, 35-53.
- Fennema, E., & Franke, M.L. (1992). Teachers' knowledge and its impact. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan Publishing Company.
- Fleener, M.J. (1995). The relationship between experience and philosophical orientation: a comparison of preservice and practicing teachers' beliefs about calculators. *Journal of Computers in Mathematics and Science Teaching*, 14(3), 359-376.
- Flyfield, J.A., & Blane, D. (1992). *The school mathematics dictionary*. London: Collins Educational.
- Forman, E.A. (1996). Learning mathematics as participation in classroom practice: implications of sociocultural theory for educational reform. In L.P. Steffe, P. Nesher, P. Cobb, G.A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 115-130). Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Fox, L.F. (1998). *The effect of a graphing calculator used in an active learning environment on intermediate algebra students' achievement and attitude*. Doctoral dissertation, Dissertation Abstracts International, 59/03, p. 761, University of South Florida.
- Friedler, Y., & McFarlane, A.E. (1997). Data logging with portable computers: a study of the impact on graphing skills in secondary pupils. *Journal of Computers in Mathematics and Science Teaching*, 16(4), 527-550.
- Fry, E. (1984). *A theory of graphs for reading comprehension and writing communication*. New Brunswick, New Jersey: Rutgers University.
- Gagne, R.M. (1967). *The conditions of learning*. New York: Holt, Rinehart and Winston.

- Gallimore, R., & Tharp, R. (1990). Teaching mind in society: teaching, schooling and literate. In L.C. Moll (Ed.), *Vygotsky and education. Instructional implications and applications of sociohistorical psychology* (pp. 175-205). Cambridge, United Kingdom: Cambridge University Press.
- Galloway, D., Leo, E., Rogers, C., & Armstrong, D. (1996). Maladaptive motivational style: the role of domain specific task demand in English and mathematics. *British Journal of Educational Psychology*, 66, 197-207.
- Giamati, C.M. (1991). *The effect of graphing calculator use on student's understanding of variations on a family of equations and the transformations of their graphs*. Doctoral dissertation, Dissertation Abstracts International, 52/01, p.103, University of Michigan.
- Goldenberg, E.P. (1988). Mathematics, metaphors, and human factors: mathematical, technical and pedagogical challenges in the educational use of graphical representation of functions. *Journal of Mathematical Behaviour*, 7(2), 135-173.
- Goldstein, R. (1994). Mathematics education, computers and calculators. The next ten years. In M. Selinger (Ed.), *Teaching Mathematics* (pp. 169-181). London: Routledge.
- Gomez, P., & Fernandez, F. (1997). Graphics calculators use in precalculus and achievement in calculus, *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education*, 3, 3-1- 3-7.
- Good, T.L., & Bophy, J.E. (1997). *Looking in classrooms*. New York: Longman.
- Goos, M., Galbraith, P., & Renshaw, P. (1999). Establishing a community of practice in a secondary mathematics classroom. In L. Burton (Ed.), *Learning mathematics: from hierarchies to networks* (pp. 36-61). London: Falmer Press.
- Goulding, M. (1997). Pupils learning mathematics. In S. Johnston-Wilder, P. Johnston-Wilder, D. Pimm, & J. Westwell (Eds.), *Learning to teach mathematics in the secondary school*. London: Routledge.
- Graesser, A.C., Person, N.K., & Magliano, J.P. (1995). Collaborative dialogue patterns in naturalistic one-to-one tutoring. *Applied Cognitive Psychology*, 9, 495-522.
- Graham, A. (1992). Too many buttons. *Micromath*, 8(1), 25-26.
- Graham, A., & Thomas, M. (1997). Tapping into algebraic variables through the graphic calculator, *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education*, 3, 9-16.

- Greenfield, P.M. (1984). A theory of the teacher in the learning activities of everyday life. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: its development in social context*. Cambridge: Harvard University Press.
- Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: the case of calculators. *International Journal of Computers for Mathematical Learning*, 3, 195-227.
- Guin, D., & Trouche, L. (2000). Thinking of new devices to make viable symbolic calculators in the classroom, *Proceedings of the 24th International Group for the Psychology of Mathematics Education*, 3, 9-16.
- Hatfield, M.M., & Bitter, G.G. (1994). A multimedia approach to the professional development of teachers: a virtual classroom. In D.B. Aichele & A.F. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 102-115). Reston, Virginia: National Council of Teachers of Mathematics.
- Hativa, N., & Lesgold, A. (1996). Situational effects in classroom technology implementations: unfulfilled expectations and unexpected outcomes. In K.J. Rehage (Ed.), *Technology and the future of schooling* (pp. 131-171). Chicago, Illinois: The University of Chicago Press.
- Hector, J.H. (1992). Graphical insight into elementary functions. In J.T. Fey & C.R. Hirsch (Eds.), *Calculators in mathematics education* (pp. 131-138). Reston, Virginia: The National Council of Teachers of Mathematics.
- Hedegaard, M. (1996). The zone of proximal development as basis for instruction. In H. Daniels (Ed.), *An introduction to Vygotsky* (pp. 171-195). London and New York: Routledge.
- Heid, M.K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education*, 19, 3-25.
- Hennessy, S., Fung, P., & Scanlon, E. (2001). The role of the graphic calculator in mediating graphing activity. *International Journal of Mathematical Education in Science and Technology*, 32(2), 267-290.
- Henningsen, M., & Stein, M.K. (1997). Mathematical tasks and student cognition: classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28(5), 525-549.
- Herring, P.M. (2000). *Teacher beliefs concerning the use of graphing calculators in the teaching of logarithmic functions in a community college algebra course: two case studies*. Doctoral dissertation, Dissertation Abstracts International, 61/05, p. 1773, The Florida State University.

- Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 60-86). Reston, VA/ Hillsdale, NJ: National Council of Teachers of Mathematics/ Lawrence Erlbaum Associates, Publishers.
- Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: an introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: the case of mathematics* (pp. 1-27). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Hiebert, J., & Carpenter, T.P. (1992). Learning and teaching with understanding. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-87). New York: National Council of Teachers of Mathematics.
- Hollar, J.C., & Norwood, K. (2000). The effects of a graphing-approach intermediate algebra curriculum on student' understanding of function. *Journal for Research in Mathematics Education*, 31(4), 221-226.
- Hoyles, C., & Sutherland, R. (1989). *Logo mathematics in the classroom*. London: Routledge.
- Hubbard, D. (1998). Improving student knowledge on the graphing calculator's capabilities: ERIC Document Reproduction Service No. ED422175.
- Hudson, B. (1995). Groupwork with multimedia in mathematics: the role of the technology and of the teacher?, *British Congress of Mathematics Education 3* (pp. 231-238), The Manchester Business School, UK.
- Hudson, B., & Borba, M. (1999). The role of technology in the mathematics classroom. *Micromath*, 15(1), 19-23.
- Ismail, Z.H., & Saleh, F. (1997). Graphing abilities of Malaysian secondary school students. *Journal of Science and Mathematics Education in Southeast Asia*, 20(2), 1-7.
- Janvier, C. (1983). Teaching the concept of function. *Mathematical Education for Teaching*, 4(2), 48-60.
- Janvier, C. (1987). Translation processes in mathematics education. In C. Janvier (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 27-32). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers
- Jaworski (1994). *Investigating mathematics teacher. A constructivist enquiry* (Vol. 5). London: The Falmer Press.

- Jones, G.A., & Thornton, C.A. (1993). Vygotsky revisited: nurturing young children's understanding of number. *Focus on Learning Problems in Mathematics*, 2 and 3, 18-28.
- Jones, P.L. (1993). Realising the educational potential of the graphics calculator. L. Lum (Ed.), *Proceedings of the 6th Annual International Conference on Technology* (pp. 173-177), Reading, MA: Addison-Wesley Publishing Company.
- Kaput, J.J. (1992). Technology and mathematics education. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). New York: Macmillan Publishing Company.
- Kerslake, D. (1981). Graphs. In K.M. Hart (Ed.), *Children's understanding of mathematics: 11-16* (pp. 120-136). London: John Murray.
- Kieran, C. (1993). Functions, graphing, and technology: intergrating research on learning and instruction. In T.A. Romberg, E. Fennema, & T.P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 189-237). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Kissane, B. (1997). *More mathematics with a graphics calculator*. Claremont: The Mathematical Association of Western Australia.
- Knuth, E.J. (2000). Student understanding of the cartesian connection: an exploratory study. *Journal for Research in Mathematics Education*, 31(4), 507-597.
- Kozulin, A. (1986). Vygotsky in context. In A. Kozulin (Ed.), *Thought and language* (pp. xi-lvi). Cambridge, Massachusetts and London, England: The MIT Press.
- Kuchemann, D. (1981). Algebra. In K.M. Hart (Ed.), *Children's understanding of mathematics concepts: 11-16* (pp. 103-119). Oxford, England: Alden Press.
- Lagrange, J. (1999). Learning pre-calculus with complex calculators: mediation and instrumental genesis, *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*, 3, 193-200.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.
- Lave, J., & Wenger, E. (1996). Practice, person, social world. In H. Daniels (Ed.), *An Introduction to Vygotsky* (pp. 143-150). London and New York: Routledge.
- Lawler, G.M. (2000). *The use of written scaffolds to instruct linear functions in a reformed algebra course using graphing calculators*. Doctoral dissertation, Dissertation Abstracts International, 61/06, p. 2168, State University of New York at Albany.

- Leinhardt, G., & Greeno, G., J. G. (1986). The cognitive skill of teaching. *Journal of Educational Psychology*, 78, 75-95.
- Leinhardt, G., Zaslavsky, O., & Stein, M.K. (1990). Function, graphs, and graphing: tasks, learning, and teaching. *Review of Educational Research*, 60(1), 1-64.
- Leontev, A.N. (1981). The problem of activity in psychology. In J.V. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp. 33-71). New York: M. E. Sharpe.
- Lepper, M.R., Aspinewall, L., Mumme, D., & Chabay, R.W. (1990). Self-perception and social perception processes in tutoring: subtle social control strategies of expert tutors. J.M. Olson & M.P. Zanna (Eds.), *Self-inference processes: The Sixth Ontario Symposium in Social Psychology* (pp. 217-237): Lawrence Erlbaum Associates, Publishers.
- Lerman, S. (1998a). Cultural perspectives on mathematics and mathematics teaching and learning. In F. Seeger, J. Voight, & U. Waschescio (Eds.), *The culture of mathematics classroom*. Cambridge: Cambridge University Press.
- Lerman, S. (1998b). A moment in the zoom of a lens: towards a discursive psychology of mathematics teaching and learning, *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education*, 1, 66-81.
- Lubinski, C.A., & Jaberg, P.A. (1997). Teacher change and mathematics K-4: developing a theoretical perspective. In E. Fennema & B.S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 223-254). Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Magidson, S. (1992). What's in a problem? Exploring slope using computer graphing software, *Proceedings of the 16th Conference of the International Group for the Psychology of Mathematics Education*, 2, 64-71.
- Mansfield, H. (1985). Points, lines, and their representations. *For the Learning of Mathematics*, 5(3), 2-6.
- Markovits, Z., Eylon, B., & Bruckheimer, M. (1983). Functions: linearity unconstrained, *Proceedings of the 7th Conference of the International Group for the Psychology of Mathematics Education*, 271-277.
- Martin, C.D. (1991). Stakeholder perspectives on the implementation of micros. In R.L.B. Jr. & C.D. Martin (Eds.), *Case studies in computer aided learning* (pp. 169-221). London: The Falmer Press.
- Mayer, R.E. (1989). Systematic thinking fostered by illustrations in scientific text. *Journal of Educational Psychology*, 81, 240-246.

- McKendree, J. (1990). Effective feedback content for tutoring complex skills. *Human-Computer Interaction*, 5, 381-413.
- Merrill, D.C., Reiser, B.J., Merrill, S.K., & Landes, S. (1995). Tutoring: guided learning by doing. *Cognition and Instruction*, 13(3), 315-372.
- Mesa, V.M. (1997). The use of graphing calculators in solving problems on functions, *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education*, 3, 240-247.
- Milou, E. (1999). Graphing calculator: classroom survey. *School Science and Mathematics*, 99(3), 133-139.
- Ministry of Education Malaysia (1997). *Smart School Flagship Application: The Malaysian Smart School*. Malaysia: Government of Malaysia.
- Mohamad, D.S.D.M. (1991). Malaysia: the way forward (Vision 2020), *Paper presented at The Inaugural Meeting of the Malaysian Business Council, Malaysia*, Malaysia: National Printing Department, Kuala Lumpur.
- Mok, I.A.H. (1999). Learning opportunities with graphing calculators: the case of asymptotes: ERIC Document Reproduction Service No. ED443730.
- Mokros, J.R., & Tinker, R.F. (1987). The impact of microcomputer based labs on children's ability to interpret graphs. *Journal of Research in Science Teaching*, 24(4), 369-383.
- Moll, L.C. (1990). Introduction. In L.C. Moll (Ed.), *Vygotsky and education. Instructional implications and applications of sociohistorical psychology* (pp. 1-27). Cambridge: Cambridge University Press.
- Moschkovich, J., Schoenfeld, A.H., & Arcavi, A. (1993). Aspects of understanding: on multiple perspectives and representation of linear relations and connections among them. In T.A. Romberg, E. Fennema, & T.P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 69-100). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Nelson, D. (1998). *Dictionary of mathematics*. London: Penguin.
- Newman, D. (1987). Functional environments for microcomputers in education. In R.D. Pea & K. Sheingold (Eds.), *Mirrors of minds: patterns of experience in education computing: papers from the center for children and technology, Bank Street College* (pp. 57-). Norwood, New Jersey: Ablex.
- Newman, D., Griffin, P., & Cole, M. (1989). *The construction zone: working for cognitive change in school*. Cambridge: Cambridge University Press.

- Nickson, M. (1992). The culture of the mathematics classroom: an unknown quantity. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 101-114). New York: Macmillan Publishing Company.
- Nohda, N. (2000). Teaching by open-approach method in Japanese mathematics classroom. *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, 1, 39-53.
- Norman, F.A. (1993). Integrating research on teachers' knowledge of functions and their graphs. In T.A. Romberg, E. Fennema, & T.P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 159-187). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Norton, S., McRobbie, C.J., & Cooper, T.J. (2001). Exploring secondary mathematics teacher's reasons for not using computers in their teaching: five case studies. *Journal of Research on Computing in Education*, 33(1), 87-109.
- Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: learning cultures and computers*. Dordrecht: Kluwer Academic Publishers.
- Oldknow, A.J. (1995). Personal technology and new horizons. In L. Burton & B. Jaworski (Eds.), *Technology in mathematics teaching - a bridge between teaching and learning* (pp. 97-108). Studentlitteratur, Lund: Chartwell-Bratt Ltd.
- Oldknow, A., & Taylor, R. (2000). *Teaching mathematics with ICT*. London: Continuum.
- Olsen, J.R. (2000). Two dynamic function representations to help student get to the big three. *Focus on Learning Problems in Mathematics*, 22(3 and 4), 55-66.
- Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics*, 14(3).
- Palmiter, J. (1991). Effects of computer algebra systems on concept and skill acquisition in calculus. *Journal for Research in Mathematics Education*, 22(2), 151-156.
- Paton, H.L., & Christofides, C. (1995). It's more ziggy. *Micromath*, 11(2), 30-33.
- Pea, R.D. (1987). Cognitive technologies for mathematics education. In A.H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89-122). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Penglase, M., & Arnold, S. (1996). The graphics calculator in mathematics education: a critical review of recent research. *Mathematics Education Research Journal*, 8(1), 58-90.

- Philipp, R.A., Martin, W.O., & Richgels, G.W. (1993). Curricular implications of graphical interpretations of graphical representations of function. In T.A. Romberg, E. Fennema, & T.P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 239-278). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Polya, G. (1981). Thinking about problems. In A. Floyd (Ed.), *Developing mathematical thinking* (pp. 117-119). Wokingham, England: Addison-Wesley Publishers Limited.
- Ponte, J. (1987). Geometrical and numerical strategies in students' functional reasoning, *Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education*, 423-418.
- Porzio, D. (1999). Effects of differing emphases in the use of multiple representations and technology on student's understanding of calculus concept. *Focus on Learning Problems in Mathematics*, 21(3), 1-29.
- Pusat Perkembangan Kurikulum (1989a). *Huraian Sukatan Pelajaran Matematik Tingkatan I*. Selangor Darul Ehsan: Dewan Bahasa dan Pustaka. (in Malaysian national language).
- Pusat Perkembangan Kurikulum (1989b). *Huraian Sukatan Pelajaran Matematik Tingkatan II*. Selangor Darul Ehsan: Dewan Bahasa dan Pustaka. (in Malaysian national language).
- Pusat Perkembangan Kurikulum (1989c). *Huraian Sukatan Pelajaran Matematik Tingkatan III*. Selangor Darul Ehsan: Dewan Bahasa dan Pustaka. (in Malaysian national language).
- Pusat Perkembangan Kurikulum (1990a). *Huraian Sukatan Pelajaran Matematik Tingkatan IV*. Selangor Darul Ehsan: Dewan Bahasa dan Pustaka. (in Malaysian national language).
- Pusat Perkembangan Kurikulum (1990b). *Huraian Sukatan Pelajaran Matematik Tingkatan V*. Selangor Darul Ehsan: Dewan Bahasa dan Pustaka. (in Malaysian national language).
- Pusat Perkembangan Kurikulum (1995). Laporan Keberkesanan Kurikulum Bersepadu Matematik Sekolah Menengah. Kuala Lumpur: Pusat Perkembangan Kurikulum. (in Malaysian national language).
- Resnick, L.B. (1987). *Education and learning to think*. Washington, D.C.: National Academy Press.

- Reusser, K. (1993). Tutoring systems and pedagogical theory: representational tools for understanding, planning, and reflection in problem solving. In S.P. Lajoie & S.J. Derry (Eds.), *Computers as cognitive tools* (pp. 143-177). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Reys, B.J., & Smith, N.L. (1994). Integrating calculators: how far have we come. *Teaching Children Mathematics*, 1(4), 206-207.
- Rich, B.S. (1991). *The effect of the use of graphing calculators on the learning of function concepts in precalculus mathematics*. Doctoral dissertation, Dissertation Abstracts International, 52/03, p. 835, University of Iowa.
- Rist, R.C. (1991). Introduction: computers, clay pots, and case studies. In J.R.L. Blomeyer & C.D. Martin (Eds.), *Case studies in computer aided learning* (pp. 1-7). London: The Falmer Press.
- Robinson, N., Even, R., & Tirosh, D. (1992). Connectedness in teaching algebra: a novice-expert contrast, *Proceedings of the 16th Conference of the International Group for the Psychology of Mathematics Education*, 2, 258-266.
- Rogoff, B. (1991). The joint socialization of development by young children and adults. In P. Light, S. Sheldon, & M. Woodhead (Eds.), *Learning to think* (pp. 67-96). London and New York: Routledge.
- Romberg, T.A., Carpenter, T.P., & Fennema, E. (1993). Toward a common research perspective. In T.A. Romberg, E. Fennema, & T.P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 1-10). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Ruthven, K. (1990). The influence of graphic calculator use on translation from graphic to symbolic forms. *Educational Studies in Mathematics*, 21, 431-450.
- Ruthven, K. (1992). Personal technology and classroom change: a British perspective. In T. Fey & C.R. Hirsch (Eds.), *Calculators in mathematics education*. Reston, Virginia: National Council of Teachers of Mathematics.
- Ruthven, K. (1994a). Supercalculators and the Secondary Mathematics Curriculum. In M. Selinger (Ed.), *Teaching Mathematics*. London: Routledge.
- Ruthven, K. (1994b). Computational Tools and School Mathematics. *Micromath*, 10(3), 28-32.
- Ruthven, K. (1995). Pressing on. In L. Burton & B. Jaworski (Eds.), *Technology in mathematics teaching - a bridge between teaching and learning* (pp. 231-259). Bromley: Chartwell-Bratt.

- Ruthven, K. (1996). Calculators in the mathematics curriculum: the scope of personal computational technology. In A.J. Bishop, K. Clements, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education, Part One*. (pp. 435-468). Dordrecht: Kluwer Academic Publishers.
- Salomon, G. (1993). On the nature of pedagogic computer tools: the case of the Writing Partner. In S.P. Lajoie & S.J. Derry (Eds.), *Computers as cognitive tools* (pp. 179-196): Lawrence Erlbaum Associates, Publishers.
- Sandholtz, J.H., Ringstaff, C., & Dwyer, D.C. (1997). *Teaching with technology: creating student centered classrooms*. New York: Teachers College Press.
- Schmidt, M.E., & Callahan, L.G. (1992). Teachers' and principals' beliefs regarding calculators in elementary mathematics. *Focus on Learning Problems in Mathematics*, 14, 17-29.
- Schmidt, M.E. (1999). Middle grade teachers' beliefs about calculator use: pre-project and two years later. *Focus on Learning Problems in Mathematics*, 21(1), 18-34.
- Schmittau, J. (1993). Vygotskian scientific concepts: implications for mathematics education. *Focus on Learning Problems in Mathematics*, 15(2 and 3), 29-39.
- Schoenfeld, A.H. (1989). Explorations of students' mathematical beliefs and behaviour. *Journal for Research in Mathematics Education*, 20, 338-355.
- Schofield, J.W. (1995). *Computers and classroom culture*. New York: Cambridge University Press.
- Selinger, M. (2001). The role of the teacher: teacherless classrooms? In M. Leask (Ed.), *Issues in teaching using ICT* (pp. 83-95). London: Routledge Falmer.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22(1), 1-36.
- Shoaf-Grubbs, M.M. (1995). Research results on the effect of the graphic calculator on female students' cognitive levels and visual thinking. In L. Burton & B. Jaworski (Eds.), *Technology in mathematics teaching* (pp. 213-230). Sweden: Chartwell-Bratt.
- Shulman, L. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, 15, 4-14.
- Shumway, R. (1990). Supercalculators and the curriculum. *For the Learning of Mathematics*, 10(2), 2-9.

- Simmt, E. (1997). Graphing calculators in high school mathematics. *Journal of Computers in Mathematics and Science Teaching*, 16(2/3), 269-289.
- Simonsen, L.M., & Dick, T.P. (1997). Teachers' perceptions of the impact of graphing calculators in the mathematics classroom. *Journal of Computers in Mathematics and Science Teaching*, 16(2/3), 239-268.
- Skemp, R.R. (1978). Relational understanding and instrumental understanding. *Mathematics Teacher*, 26(3), 9-15.
- Slavit, D. (1996). Graphing calculators in a "hybrid" algebra II classroom. *For the Learning of Mathematics*, 16(1), 9-14.
- Smart, T. (1995). Visualisation, confidence and magic. In L. Burton & B. Jaworski (Eds.), *Technology in mathematics teaching - a bridge between teaching and learning* (pp. 191-212). Bromley: Chartwell-Bratt.
- Smith, E., & Confrey, J. (1992). Using a dynamic software tool to teach transformations of functions. L. Lum (Ed.), *Proceedings of the 5th Annual International Conference on Technology in Collegiate Mathematics* (pp. 225-242), Reading, MA: Addison-Wesley Publishing Company.
- Smith, K.B., & Shotsberger, P.G. (1997). Assessing the use of graphing calculators in college algebra: reflecting on dimensions of teaching and learning. *School Science and Mathematics*, 97(7), 368-375.
- Somekh, B. (1997). Classroom investigations. Exploring and evaluating how IT can support learning. In B. Somekh & N. Davis (Eds.), *Using information technology effectively in teaching and learning* (pp. 115-260). London and New York: Routledge.
- Sowder, J. (1992). Estimation and number sense. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 371-389). New York: Macmillan Publishing Company.
- Steele, D. (1993). The graphics calculator: Friend or foe? In J. Mousley & M. Rice (Eds.), *Mathematics: Of primary importance* (pp. 50-56). Melbourne: Mathematical Association of Australia.
- Stevenson, I. (1990). *Expertise observed. A study of sixth formers sketching graphs*. Unpublished Masters Dissertation, Institute of Education, London.
- Strait, G.J. (1993). *A comparison of inductive and deductive teaching strategies utilizing graphing calculator capabilities*. Doctoral dissertation, Dissertation Abstracts International, 54/04, p. 1261, Texas Tech University.

- Swan, M. (1982). The teaching of functions and graphs. In G.v. Berneveld & H. Krabbendam (Eds.), *Conference on Functions* (pp. 151-165). Enschede, The Netherlands: National Institute for Curriculum Development.
- Swatton, P., & Taylor, R.M. (1994). Pupil performance in graphical tasks and its relationship to the ability to handle variables. *British Educational Research Journal*, 20(2), 227-243.
- Sweller, J. (1994). Cognitive load theory, learning difficulty and instructional design. *Learning and Instruction*, 4, 295-312.
- Tall, D. (1987). Algebra in a computer environment. *Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education*, 1, 262-271.
- Tan, R., Delgado, J., & Whelan, S. (1997). Graphing calculators: what research says, what we are doing, and what needs to be done to integrate its' use in the mathematics curriculum, *Proceedings of the 8th Annual International Conference on Technology in Collegiate Mathematics* (pp. 399-403).
- Tapson, F. (1996). *The Oxford mathematics study dictionary*. Oxford: Oxford University Press.
- Taylor, L. (1993). Vygotskian influences in mathematics education, with particular reference to attitude development. *Focus on Learning Problems in Mathematics*, 15(2 and 3), 3-17.
- Tharp, R., & Gallimore, R. (1991). A theory of teaching as assisted performance. In P. Light, S. Sheldon, & M. Woodhead (Eds.), *Learning to think* (pp. 42-61). London and New York: Routledge.
- Tharp, M.L., Fitzsimmons, J.A., & Ayers, R.L.B. (1997). Negotiating a technological shift: teacher perception of the implementation of graphing calculators. *Journal of Computers in Mathematics and Science Teaching*, 16(4), 551-575.
- Thomas, M.O.J., Tyrell, J.A., & Bullock, J.H. (1995). Using a computer in the mathematics classroom: the effect of teacher attitude, *British Congress of Mathematics Education* 3, 3, 303-314.
- Thompson, A.G., Philipp, R.A., Thompson, P.W., & Boyd, B.A. (1994). Computational and conceptual orientations in teaching mathematics. In D.B. Aichele & A.F. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 79-92). Reston, VA: National Council of Teachers in Mathematics.
- Trickett, L., & Sulke, F. (1988). Low attainers can do mathematics. In D. Pimm (Ed.), *Mathematics teachers and children*. Great Britain: The Open University Press.

- Tudge, J. (1990). Vygotsky, the zone of proximal development, and peer collaboration: implications for classroom practice. In L.C. Moll (Ed.), *Vygotsky and education. Instructional implications and applications of sociohistorical psychology* (pp. 155-172). Cambridge: Cambridge University Press.
- Tuska, A. (1993). *Student's errors in graphing calculator-based precalculus classes*. Doctoral dissertation, Dissertation Abstracts International, 53/08, p. 2725, The Ohio State University.
- Underwood, J.D.M., & Underwood, G. (1990). *Computers and learning*. Oxford: Blackwell.
- Usiskin, Z. (1993). Lessons from the Chicago Mathematics Project. *Educational Leadership*, 50(8), 14-18.
- van Oers, B. (1996). Learning mathematics as a meaningful activity. In L.P. Steffe, P. Nesher, P. Cobb, G.A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 91-113). Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.
- van Streun, A., Harskamp, E., & Suhre, C. (2000). The effect of the graphic calculator on student' solution approaches: a secondary analysis. *Hiroshima Journal of Mathematics Education*, 8, 27-39.
- Vazquez, J.L. (1991). *The effect of the calculator-based, graph-exploration method of instruction on advanced placement calculus achievement*. Doctoral dissertation, Dissertation Abstracts International, 54/11, p. 3660, The University of South Carolina.
- Vonder Embse, C.B. (1992). Concept development and problem solving using graphing calculators in the middle school. In T. Fey & C.R. Hirsch (Eds.), *Calculators in mathematics education* (pp. 65-78). Reston, Virginia: National Council of Teachers of Mathematics.
- Vonder Embse, C.V., & Engebresten, A. (1996). Friendly windows for graphing calculators. *The Mathematics Teacher*, 89(6), 508-511.
- Vygotsky, L.S. (1978). *Mind in society. The development of higher psychological processes*. Cambridge and London: Harvard University Press.
- Vygotsky, L.S. (1981). The instrumental method in psychology. In J.V. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp. 134-143). Armonk, New York: Sharpe.
- Vygotsky (1986). *Thought and language*. Cambridge and London: The MIT Press.

- Waits, B.K., & Demana, F. (1996). A computer for all students - revisited. *Mathematics Teacher*, 89(9), 712-714.
- Waits, B.K. (2000). Viewpoint: A vision of the future: the role of technology in the teaching and learning of mathematics. *Focus on Learning Problems in Mathematics*, 22(3 and 4), 185-187.
- Ward, R.A. (2000). Observing high school students' strategies and misconceptions as they use graphing calculators. *Focus on Learning Problems in Mathematics*, 22(3 and 4), 28-40.
- Waschescio, U. (1998). The missing link: social and cultural aspects in social constructivists theories. In F. Seegar, J. Voigt, & U. Waschescio (Eds.), *The culture of the mathematics classroom* (pp. 221-241). Cambridge, United Kingdom: Cambridge University Press.
- Wertsch, J.W., & Tulviste, P. (1996). L. S. Vygotsky and contemporary developmental psychology. In H. Daniels (Ed.), *An introduction to Vygotsky*. London and New York: Routledge.
- Wilson, B.G., Teslow, J.L., & Taylor, L. (1993). Instructional design perspectives om mathematics education with reference to Vygotsky's theory of social cognition. *Focus on Learning Problems in Mathematics*, 15(2 & 3), 65-86.
- Wilson, M.R., & Krapfl, C.M. (1994). The impact of graphics calculators on students' understanding of function. *Journal of Computers in Mathematics and Science Teaching*, 13(3), 252-264.
- Winograd, T., & Flores, F. (1986). *Understanding computers and cognition: A new foundation for design*. Norwood, NJ: Ablex.
- Wood, D.J., Bruner, J.S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, 17(2), 89-100.
- Wood, D. (1988). *How children think and learn*. Oxford: Basil Blackwell.
- Yerushalmy, M., & Schwartz, J.L. (1993). Seizing the opportunity to make algebra mathematically and pedagogically interesting. In T.A. Romberg, E. Fennema, & T.P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 41-68). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Zaslavsky, O. (1987). *Conceptual obstacles in the learning of quadratic functions*. Unpublished doctoral dissertation, Technion, Haifa, Israel.

Appendix 1 Excerpt of a student's disapproval of the open learning approach

Line No.	Dialogue
1	Teacher: You still do not understand?
2	Hazim: Aah... Understand...I understandlah but I do not like this, it is not clear.
3	Teacher: What can we say, you have to learn it whether you like it or not.
4	Hazim: ...huh, then teach properly...
5	Teacher: 1...so $2=x+c$...how much...this is the answer.
6	Hazim: How did you get it?
7	Teacher: Substitute the value of x, here the intercept at the x-axis. We want to find the value of c.
8	Hazim: Show it clearly, I do not understandlah....

Evidently, the student (Hazim) expressed his displeasure at the open learning approach (line 2) and considered that the teacher was not teaching “properly” (line 4). In the last line (line 8), Hazim expressed that he wanted the instruction to be overt and directive.

TEACHING MODULE A

- 1 -

ABOUT THE MODULE

1. This teaching module covers the topic on Equations and their Graphs.
2. The focus is to make use of graphing calculators to assist students to discover or develop the graphing concepts.
3. There are 15 lessons in this module.
4. Each lesson in the module is accompanied by a corresponding Student's Worksheet.
5. Each lesson in the module is divided into 4 parts which defines the aim of the lesson, the activities of the lesson, the interventions that can be used in the lesson and the teaching points or instructions that have to be carried out by the teacher.
6. The activities in each lesson are designed to help students discover or develop the graphical concepts that are outlined in the aim of the lesson.

HOW TO USE THE MODULE

1. Your role when using the module is a **guide** who engages student in learning tasks including when to use the graphing calculator.
2. The module provides the curriculum that should be covered in the topic on Equations and their Graphs.
3. Student's current performance can be assessed by the Assessment Tasks supplied with this module.
4. Students are supplied with the corresponding Student's Worksheet for each lesson in the Module and a Student's Manual on Graphing Calculator use.
5. The interventions suggested in this module are in the form of process-type questions. You may add your own interventions that you think are appropriate. As a guide, questions posed to students enables students to progress in their tasks or directs them towards tasks that enable them to confront with the mathematical concepts they are suppose to discover or develop such as:
 - Have you thought about this?
 - What happens when the value changes?
 - How about rearranging the points?
 - Would it help if you change the value of c ?
 - What other values have you tried?
 - Have you looked for a pattern?
6. The "Teaching Points" marked * should be introduced to the students at the start of the lesson. These are new information that is necessary to enable student to begin with the activities in their worksheets. Those unmarked are discussed at the end of the lesson so that students are brought to focus on the mathematical concepts they are expected to discover or develop in their learning activity.

- 2 -

LESSON 1

AIM

- (i) To show the different ways information can be represented.
- (ii) To introduce the idea of mapping, ordered pairs/Cartesian coordinates, hence Cartesian graph.

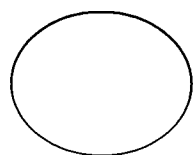
ACTIVITY

The table shows the rate of growth of a broad bean shoot.

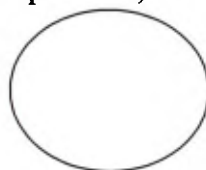
Age in weeks	1	2	3	4	5	6
Height in cm	2	4	6	8	10	12

1. Can you think of other ways to represent this information?
Discuss with a friend.
**Explain to students that this information can be represented in many forms, e.g., arrow diagram.

On your first BLANK PAPER, draw two sets. Fill in the data in each set.
Draw the arrows that show that “5 maps to 10”, etc.



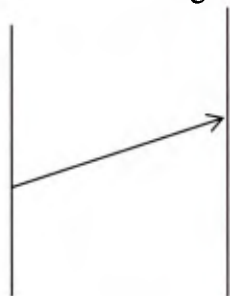
age in weeks



height in cm

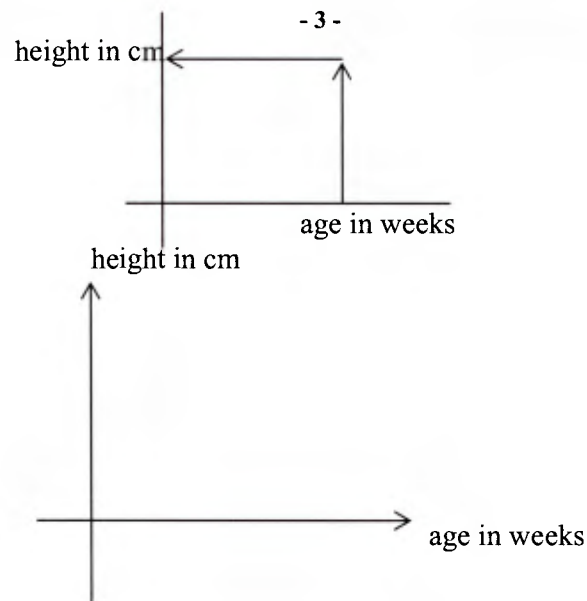
On your second BLANK PAPER, draw two parallel axes and mark them as shown:

age in weeks height in cm



Draw the arrow that show “5 maps to 10”, etc.

On your first GRAPH PAPER, draw two axes that are perpendicular to each other. Draw the mapping arrow with a right angle bend in it:



On your second GRAPH PAPER, draw two axes that are perpendicular to each other.

Mark the points where the mapping arrow “turns the corner”.

Emphasise that this point still means “5 maps to 10” or simply written as an ordered pair (5,10).

Complete the marking the points.

2. What can you say about the growth of the broad bean shoot?
3. Can the height of the broad beans be the same at a particular age?
4. Can the height of the broad beans be different at one particular age?
5. How can you predict the height of the broad bean shoot when it is 10 weeks old?
6. What can you say about the relationship between the age and the height of the broad bean?
7. If x is the age in weeks of the broad bean and y is the height in cm of the broad bean, can you write the relationship in terms of x and y ?

SUGGESTED INTERVENTIONS

1. How high does the broad bean grow each week?
2. What assumptions about the growth of the broad bean do you make when you join the points?
3. Do you get a straight line when you join the points?

- 4 -

4. Can you read from your graph the height of the broad bean when it is $2\frac{1}{2}$ weeks old?
5. Look at your table and compare the x (age in weeks) and y (height in cm) values.

TEACHING POINTS

1. The straight line represents the graph of the relationship between x and y. This type of relationship is called a linear relationship and the graph is called a straight line graph or linear graph.
2. If x is the age of the broad bean, then the height of the broad bean is $2x$.
3. Mathematically, if x is the age of the broad bean and y is the height of the broad bean, then the relationship between the age of the broad bean and the height of the broad bean can be written as the equation/formula $y=2x$.
4. The graph drawn is the graph of the equation $y=2x$.

****Presented in the form of discussion.**

LESSON 2

AIM

- (i) Familiarise with the conventions of the graphing calculator.
- (ii) Able to use the graphing calculator to plot points.
- (iii) Able to plot points and name / write them correctly as ordered pairs / use the convention that x-coordinate always comes first.
- (iv) Know that the graphing calculator has a finite viewing window, thus if you try to plot a point that does not fit on the viewing window, the calculator will not plot anything.
- (v) Able to differentiate (in position) between positive and negative values for x and y.
- (vi) Knowledge of origin.

ACTIVITY

1. In the RUN Mode what do you observe on the screen of your calculator?
2. Plot the following points on your graph paper as well as on your calculator in the RUN Mode.
 - (i) Plot the following point: (1, 2). What do you observe?
 - (ii) Use the SET UP command to insert grids on your screen. What do you observe about these grids? Explain.
 - (iii) Now plot the points (4, 1) and (1, 6). What do you observe? Explain what you observe.
 - (iv) Use the *pointer* (a flashing plus sign, +) by pressing the cursor keys [▲], [▼], [◀], [▶] to move around the screen.
What do you notice about the x and y value when the *pointer* is in the
 - (a) first quadrant?
 - (b) second quadrant?
 - (c) third quadrant?
 - (d) fourth quadrant?Plot some points and ask your friends to read these points. Make sure you deactivate (switch off) the coordinate display in the SET UP menu.
 - (v) What do you observe about the movement distance of the *pointer* when you press the cursor key once? Twice?
 - (vi) Plot the following points: (1, 2), (-3, 2), (-1, 2). What do you observe?
 - (vii) Plot the following points: (-1, -1), (-1, 2), (-1, 3). What do you observe?
 - (viii) What do observe when you move the *pointer* horizontally? Vertically?
3. What is the value of x and y where the axes intersect?

- 6 -

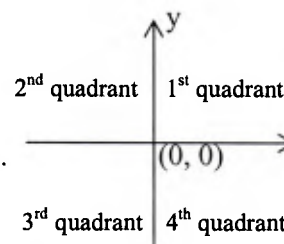
SUGGESTED INTERVENTIONS

1. Try to note down the coordinates in the first quadrant, the second, the third and the fourth. Compare them.
2. Where would a positive x-coordinate be?
Where would a negative x-coordinate be?
Where would a positive y-coordinate be?
Where would a negative y-coordinate be?
3. Use the cursor to find how many markings are there on the x-axis, is it possible to plot the point, say (7,2)? Why?
4. Use the cursor to find how many markings are there on the y-axis, is it possible to plot the point, say (4,5)? Why?
5. How do you tell from the coordinates if a point lies on the left-hand line/ right-hand line?

TEACHING POINTS

1. *Calculator's syntax on how
 - (i) to get started with the graphic calculator:
Press [AC/ON]
Select RUN Mode from the MAIN MENU and press [EXE]
 - (ii) to plot points:
Press [SHIFT]/Sketch/►/PLOT/Plot 3,2 (eg. for the point (3,2))
Press [EXE]
 - (iii) to insert grid:
Press [SHIFT]/ SET UP.
Press cursor key down [▼], select Grid On and press [EXE]
 - (iv) to clear screen:
Press Cls / [EXE]
Remind students to CLEAR their graphic calculator screen each time before starting a new task
2. *The value of the coordinate of the *pointer* / *tracer* are displayed at the bottom of the calculator screen.

3. The name of the four quadrants.
4. The x and y axes.
5. The origin (0, 0) is where the x and y intersect.



- 7 -

LESSON 3

AIM

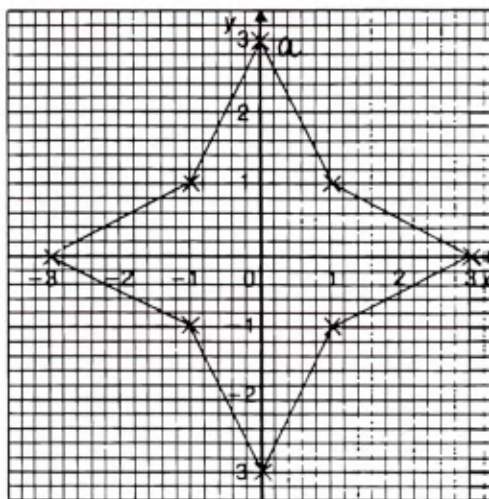
The main aim of this activity is to further induct students to using the graphing calculators and become well-verse with the conventions/ commands of the graphics calculator to plot points. Students are encouraged to explore the functions of [AC/ON], <OFF>, [EXIT], [SHIFT], [EXE] and <QUIT> commands. This activity can also be used to find out whether students are

- able to name the coordinates, and
- fluent with the conventions/ commands of the graphics calculator to plot points

so that appropriate interventions can be given.

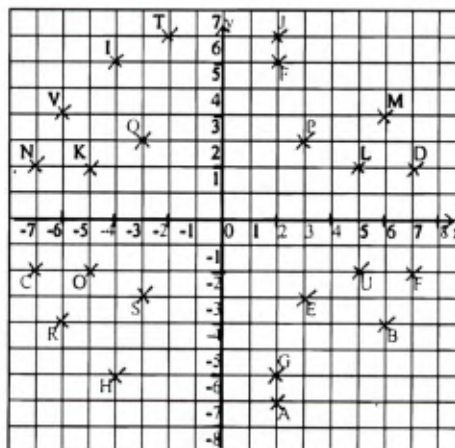
ACTIVITY

- On your calculator, plot these points which are corners of a square. Find the coordinates of the missing corner. Use the coordinates (3, 3), (1, 1), (3, -1).
- List the coordinates of every corner of the figure, starting at "a" and going clockwise.



- Which letter in the diagram opposite is at:

- (-5, 2)?
- (-2, 7)?
- (-6, -4)?
- (3, -3)?



- 8 -

Choose the points that can be plotted on your graphic calculator and plot them.

Think of how you can plot the other points.

SUGGESTED INTERVENTIONS

1. How does a square look like? Can you sketch a square on the paper?
2. Move your *pointer*, how does the value of x (or y) varies as it moves from left to right (or from top to bottom)?

TEACHING POINTS

1. *Calculator's syntax on how to cancel a point that is wrongly plotted:
Select Plt-Off / Press [EXE]
2. Position of positive and negative values for x and y-coordinates

- 10 -

Write down these values.

x =

y =

Explain your answer.

Store this setting in view-window V-W1 by pressing:
STO/ V-W1 / [EXIT] STD.

3. Press [SHIFT] ⟨V-Window⟩ and choose the STD view-window. What do you observe about values set in the STD view-window?

Write down the following values: $x_{\min} =$ $y_{\min} =$

$x_{\max} =$ $y_{\max} =$

Can the point (3, 2) be plotted exactly? Why? Try to store this view-window in V-W1 and continue to explore with other view-windows.

4. Change the view- window to $-31.5 \leq x \leq 31.5$ and $-15.5 \leq y \leq 15.5$. Press the cursor, what do you notice? (The tick marks are at every 5 units on each axis). Store this view-window in V-W2. Plot a point. Press the cursor keys, what happens?
5. Can you plot a point where both numbers are integers that is exact on the grid/ markings of the view-window? Explain how you do this.

SUGGESTED INTERVENTIONS

- Find how far the *pointer* moves after each press on the cursor.
- Press the cursor until the *pointer* is out of the screen. What is the maximum/ minimum value of x and what is the maximum/ minimum value of y that you can read?
- Try to insert the grids on your screen to help you see the position of the point.
- Is the distance/ length between two consecutive markings on the x-axis and the y-axis equal in the INIT view-window?
- In which view-window is the distance/ length between two consecutive markings on the x-axis and the y-axis equal?

TEACHING POINTS

- *Calculator's syntax on how to get into, to store and to recall the View-window.
 - ☐ to get into view- window: press [SHIFT]/ V-Window.
 - ☐ to store: select STO, then select the view-window you want to store in from V-W1 to V-W20.

- 11 -

□ to recall: select RCL, then select the view-window you want to recall.

2. *The INIT view-window gives the default window setting of the calculator.
3. The INIT view-window gives the actual size of the view-window that is 6.3cm \times 2 on the y-axis and 3.1cm \times 2 on the x-axis.

Each press on the cursor key in the INIT view-window moves the *pointer* 0.1 unit on the screen.

4. The STD view-window is the standardised view-window of the calculator with the following setting: $x_{\min}=-10$, $x_{\max}=10$ on the x-axis; and $y_{\min}=-10$, $y_{\max}=10$ on the y-axis.
5. The scale values for x and y tell what the tick marks on each axis represent. The scale values for x and y in the INIT view-window are the same: a unit on the x-axis is the same length as a unit on the y-axis. Thus the grids are squares.
6. The view-window setting may cause irregular scale spacing. The scale values for x and y may be the same but a unit on the x-axis is not the same length as a unit on the y-axis. In the STD view-window, a unit on the x-axis is not the same length as a unit on the y-axis. Thus the grids are rectangular, not squares.
7. The view-window area of the calculator is fixed. The view-window area of the graph paper can be changed according to suitability.
8. The grid on the INIT view-window is a square but the grid on the STD view-window is NOT a square.

LESSON 5**AIM**

- (i) Able to plot lines on the graph paper and on the calculator.
- (ii) Recognise infinite number of points passing through the line.
- (iii) Knowledge of pixels
 - (a) recognise the difference in “smoothness” between two different line segments drawn on the calculator.
 - (b) recognise the difference in “smoothness” between the line segment and the horizontal and vertical lines drawn on the calculator.
 - (c) recognise the difference in “smoothness” between the line segment drawn on the graph paper and those drawn on the calculator.
- (iv) Able to write the equations of the vertical and horizontal line.

ACTIVITY

Unless specified, use the INIT view-window.

1. Draw the line segment joining (3, 2) and (-5, 1) both on the calculator (in the RUN Mode) and on the graph paper. Compare them. What do you find?
2. Draw the line segment joining (-3, -2) and (4, 3) on the calculator. Compare this with the line segment you have previously drawn on the calculator. What do you find?
3. Draw the line segment joining (2, -2) and (5, -2) on the calculator. What can you say about this line and the points that lie on this line?
How is this line different from the lines you have drawn above?
What do you think the equation of this line would be?
4. Draw the line segment joining (3, -2) and (3, 5). What can you say about this line and the points that lie on this line?
How is this line different from the lines in Q.1, Q.2, and Q.3?
What do you think the equation of this line would be?
5. On your graph paper, draw and label the following lines:
 $x=-4$, $x=0$, $x=2$, $y=-1$, $y=0$, $y=2$
What are do you observe about the similarities and the differences between these lines?
6. You can draw the horizontal line and a vertical line on your calculator without plotting the points by selecting either the Vert or HztI command or by typing in the equation of the line in the GRPH command. Try this. Show to a friend how you draw the lines in Q.3 and Q. 4. Then draw the lines in Q.5.

Use the Trace command to move your *tracer/pointer* along the line. Notice the x and y values as the *tracer* trace the line. What does the equation of the horizontal or vertical line inform you about points on the graph?

- 13 -

SUGGESTED INTERVENTIONS

1. Is there any difference between the line segment and that you draw on the calculator and on the graph paper?
2. Why is the line drawn on the graph paper is smooth but the line drawn on the calculator is jagged?
3. Sketch a horizontal line on the graph paper. Write down a few of the coordinates of the points on the line. What do they have in common/ what do notice about the value of the x and y coordinates? Which value remains constant?
4. Sketch a vertical line on the graph paper. Write down a few of the coordinates of the points on the line. What do they have in common/ what do notice about the value of the x and y coordinates? Which value remains constant?
5. What can you say about the value of x and y coordinates when you trace the graph of $y=c$ and $x=c$? (c is any real number)
6. Does the horizontal line pass through the origin?
Does the vertical line pass through the origin?

TEACHING POINTS

1. *Calculator's syntax on how
 - (i) to draw a line segment:
Press [SHIFT]/ Sketch / ▸/ LINE/ F-LINE [3]/ [,]/ [2]/ [,]/ [-5]/ [,]/ [1]
(for the line segment joining the points (3,2) and (-5,1))/ [EXE]
 - (ii) to draw a vertical and horizontal line
 - ☐ Press [SHIFT]/ Sketch / ▸/ select either Vert or Hztl; OR
 - ☐ Press [SHIFT]/ Sketch / GRPH/ select either Y= or X=c
 - (iv) to use Trace
Press [SHIFT]/ Trace/ and one or more of these cursor keys:
[▲] , [▼] , [►] , [◄] until the *tracer* appears on the screen.
2. In the Trace command, the *pointer* is also referred as the *tracer*.
3. Recap that in the first activity, the straight line graph can be “represented” by the equation of the straight line. The equation of the horizontal line is $y =$ number and the equation of the vertical line is $x =$ number.
4. There is infinite number of points on the line.

- 14 -

5. The point plotted occupies one pixel. The pixel is rectangular in shape having horizontal and vertical sides.
6. The equation of the horizontal (vertical) line informs you of all the possible pairs of values on the line, however in this case the value of y (x) is unique.

- 15 -

LESSON 6**AIM**

- (i) Able to see the effect of view-window / scales on the pixels.
- (ii) Able to make connection between the points (ordered pairs), table of values, the graph and the equation of the line.
- (iii) Knowledge of straight line graph as a continuous graph.
- (iv) Knowledge of the concept variable.
- (v) Knowledge of the graph $y=x$ passes through the origin.
- (v) Recognise that any pair of values that fits the relationship/ the equation of the line will plot a point somewhere along the same line.

ACTIVITY

1. Draw a line segment joining the points $(-5, -2)$ and $(6, 3)$ on the INIT view-window. Repeat drawing the same line segment on different viewing windows, starting with the STD view-window, and V-W1 and V-W2 that you had earlier stored. How has your line changed when you draw them on different viewing windows? What difference(s) did you notice about the pixels and the length of the line segment in each of the viewing window?
2. Draw a horizontal and a vertical line on the INIT window. Then change the view-window to the STD and then the V-W1 window. What do you notice about the position of the line?
3. Here are some points: $(-2, 2)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$
What do you notice about the value of x and the value of y ?

Make a table of values for the above information.

x						
y						

4. Plot the above points on your on your graph paper.
What do you notice about these points?
Do these points appear to form a pattern?
Find a mathematical relationship between x and y (or the equation of the line).
Write down as many points as you can that passes through/ lie on the line.
Is it possible to list down all the values of x and y for this relationship?
How is this graph different from the graph of $y=3$?
How is this graph different from the graph of $x=4$?
5. With your calculator (in the RUN Mode), draw the graph whose equation is $y=x$ using the GRPH command. Using the Trace command, write down as many points that pass through the line. Can the *tracer* of the calculator trace every point on the line? Why?
Compare the points that you trace with those you obtain by drawing on the graph paper. What do you find?
Does the point $(2.5, 2.5)$ lie on the line?

- 16 -

Does the point (8, 8) lie on the line?

Does the point (20, 20) lie on the line?

Does the point (2, 1) lie on the line?

How can you determine whether a point lies on the line from the equation of the line? Give an example.

SUGGESTED INTERVENTIONS

1. How does the size of the pixel and the length of the line vary when the view-window changes? Would it help if you record and compare each of the graph?
2. Does drawing in different viewing window gives you a different line?
Do the lengths of the line change?
Does the size of the pixels change?
3. Make a table and compare those you obtain from the graph paper and the calculator:

x								
y (graph paper)								
y (calculator)								

Comment.

TEACHING POINTS

1. *Calculator's syntax on how to graph a line:
Press [SHIFT]/ Sketch/ GRPH/ Y=
Press the button [x,θ,T] to obtain the variable x.
2. The view-window setting affects the length of the line segment and the size of the pixel of the line. It also affects the "position" of the line.
3. x and y are called variables.
4. The graph of the equation $y=x$ passes through the origin.
5. The graph of a straight line is a continuous graph.
6. There are infinite points that lie on the line. Any points that fits (satisfies) the equation of the line will plot a point somewhere along the same straight line. Therefore to determine whether a point lies on the line, the value of x and y must satisfy the equation of the line.

LESSON 7**AIM**

- (i) Recognise the “number” in the equation of the straight line graph $y=x+\text{number}$ is the y intercept of the graph.
- (ii) Able to distinguish the difference between graphs drawn using the G-CON (continuous graph) and the G-PLT (graph consisting of discrete plot of table value) feature.
- (iii) Knowledge of range by operating in the GRAPH Mode and TABLE Mode.
- (iv) Able to operate using the Dual Screen in the GRAPH Mode and the TABLE Mode.

ACTIVITY

1. Plot these points on your graph paper: (-1.5, -2.5), (-1, -2), (-0.5, -1.5), (0, -1), (0.5, -0.5), (1, 0), (1.5, 0.5), (2, 1), (2.5, 1.5), (3, 2), etc.
Do these points appear to form a pattern?
Join these points.
Find a relationship or the equation of the line of the graph.
2. Draw the graph by putting in the equation of the line you obtain above in the RUN Mode.
What do you observe?
What is the value where the line cuts (intercepts) the y-axis?
Name the coordinate where the line cuts the y-axis?
Use the *tracer* to trace the points that lie on the line.
How is this graph different from the one you drew previously that is the graph of $y=x$? In what way/ ways are they the same?
3. Plot the graph for the above relationship in the GRAPH Mode. You can also draw the graph from in the TABLE Mode. Notice that in the TABLE Mode, the screen shows the table of values. Note them down. Observe too that you can draw the graph and the table side by side using the Dual Screen in the TABLE Mode.

There are two ways to obtain graphs in the TABLE Mode, either using the G-PLT command or the G-CON command.

Observe what kind of graphs you obtain when you use each of the command.

What difference(s) do you find between the graph plotted on the G-CON command and the graph plotted on the G-PLT command?

From these observations what can you deduce about the points on a straight line?

4. Determine whether these points lie on the line:
 - (i) (-2, -3)

- 18 -

- (ii) (4, 2)
- (iii) (0, 0)

Show how you do this.

5. Draw the graph for $y=x+1$ from the TABLE Mode. Select TABL command to view the table of values. Press [EXIT] to select RANG to view the range values. Try to “guess” what this range means. Draw the graph using the G-CON command. Observe carefully.
Now go back to change your range from -5 to 5 . Note down what happens to your table of values and your graph plotted using the G-CON command.
6. Change to the STD view-window. What happens to the graph?
To compare the effect of different view-windows, you can use the Dual Screen in the GRAPH Mode. This enables you to draw two similar graphs side by side with different view-window settings.
7. How is the graph of $y=x+1$ different from the previous two graphs of $y=x$ and $y=x-1$? How are they the same?
Play with the buttons to enable you to get the table of values simultaneously for all three graphs.
8. Predict how the graph of $y=x+3$ would look like. How would $y=x-3$ be? Test your prediction by graphing them on your calculator.

SUGGESTED INTERVENTIONS

1. Move the *pointer* to the point you want to plot (or press the cursor 5 times to the right and five times to the top). What happens when you press the [EXE] button?
2. Draw a table of values. As x increases, does the value of y increase or decrease?
What is the value of this difference?
3. Compare the table of values you obtain from the calculator (in the TABLE Mode/ Table command) and the table of values you obtain yourself.
4. Look at where the line intercepts the y -axis. What is the value of x coordinate?
Look at where the line intercepts the x -axis. What is the value of the y coordinate?
5. Which graph passes through the origin?
What is the equation of the graph that passes through the origin?
6. Is the equation of the graph for the graph plotted on the G-Con command and the graph plotted on the G-Plt command the same?

- 19 -

7. Do the value of x and y satisfy the equation of the line?

TEACHING POINTS

1. *Calculator's syntax on how to
 - (i) to operate in the GRAPH Mode
Select GRAPH from the MAIN MENU, press [EXE].
Type in the equation of the line at $Y1=$
Press DRAW
 - (ii) to operate in the TABLE Mode
Select TABLE from the MAIN MENU, press [EXE].
Type in the equation of the line at $Y1=$
Press TABL
Press [▼] to go down the table to look at other values of x and y
Press the G-CON or G-PLT
Press RANG to look at range

To get back to the first/default screen, press the button [EXIT]
2. *The term intercept: cross/ intersect/ cut.
3. The meaning of intercept: y intercept
 - ☐ from the graph, the y intercept is when $x=0$
 - ☐ from the equation $y=x+\text{number}$, the number is the y interceptRelate that when the y intercept is 0, the graph passes through the origin.
The coordinate of the y -intercept can be written as $(0, y)$
4. The meaning of range. The points plotted on the G-PLT include only those within the specified range and the points can be viewed only if it is within the range of the view-window.

LESSON 8**AIM**

- (i) Recognize the general equation of the graph of straight line as $y=mx+c$.
- (ii) Able to recognize some characteristics straight lines.
- (iii) Able to differentiate between variable, parameter and constant.
- (iv) Recognize the effect of varying the values of m in $y=mx$.
- (v) Able to use the Dual Screen and the Zoom feature of the calculator.
- (vi) Able to operate in the DYNA Mode.

ACTIVITY

1. Graphs of $y=x+1$; $y=x+2$; $y=x-1$ belong to the same “family”/ collection. Can you guess why?
Write down the equations of 3 more lines which belong to the same family/ “collection”.
2. Draw graphs of relationships with $y=\text{number} \times x$. Some examples to start with are $y=2 \times x$, $y=\frac{1}{2} \times x$ and $y=(-1) \times x$. Then predict what the graph of $y=4 \times x$ will look like; graph it to check your prediction.
Begin your investigation with the GRAPH Mode.
Repeat your investigation in the DYNA Mode. In the DYNA mode, specify the parameter (or in the graphic calculator language, the dynamic variable) range from -3 to 4 .
What can you deduce about the value of the number/ parameter in the equation?

What is the common characteristic of this family of graphs?
3. Graphs of $y=2x+2$, $y=3x+2$, $y=8x+2$, $y=-x+2$, $y=-2x+2$, $y=-\frac{1}{2}x+2$ also belong to another type of family. Draw them in the GRAPH Mode. Write down what you notice about this family of graphs.
Press Zoom and use IN to get a closer look where the lines intersect the y -axis. Note the value of the y -intercept. Identify this value with the value in the equation of the line.
Write down the coordinate of this point of intersection.
Press OUT to see the graph at a greater distance.

Write down the equations of 5 more lines which belong to the same family.

How is this family of graphs different from the family of graphs in Q.1 and Q.2?

You can change your SET UP to activate the Dual Screen feature of your calculator to get the table of values side by side with your graph. Use the Dual Screen together with the Trace command to help you distinguish between the graphs.

- 21 -

4. What can you say about the gradient of the family of graphs that are parallel to each other?

SUGGESTED INTERVENTIONS

1. Look at the direction of the line when m is positive.
2. Look at the direction of the line when m is negative.
3. What happens to the “steepness” of the line as the m increase (or decrease)?
4. Are graphs having different m parallel to each other?
5. What do the equations of lines that cuts/crosses/intercepts the y -axis at the same point have in common?
6. What do the equations of parallel lines have in common?

TEACHING POINTS

1. *Calculator’s syntax on how to
 - (i) operate the Dual Screen
 - ☐ Press [SHIFT]/ <SET UP>
 - ☐ Press [▼] to go down to Dual Screen
 - ☐ Select G to T to get table side by side with graph or
 - ☐ select GRPH to get two graphs simultaneously side by side
 - (ii) use the ZOOM
 - ☐ Press [SHIFT]/ Zoom
 - ☐ Select IN to get a closer look or
 - select OUT to get a distant view
 - (iii) operate in the DYNA Mode.
 - ☐ Select DYNA in the MAIN MENU, press [EXE]
 - ☐ Press the button <ALPHA> to select the letter “A”
 - ☐ Press the button [x,θ,T] to type in the variable x
 - ☐ Press VAR and the calculator screen shows Dynamic Var :
 - ☐ Press DYNA

Point out: in the calculator “A” is named as the “dynamic variable”

2. *Point out the difference between the meaning of the term “dynamic variable” used in the syntax of the calculator and the variables x and y .
3. m is a “parameter” that determines the “steepness” of the line.

- 22 -

4. The general equation of the straight line is given by $y=mx +c$ where m is the gradient (“steepness”) of the line and c is the y intercept.
When the intercept $c=0$, the general equation of the line is given by $y =mx$ and the line passes through the origin.
5. The direction of the line determines whether the gradient of the line is positive or negative.
6. The m and c values determine the characteristic of the graph, for example, graphs that have the same y intercept belong to the “family” having the same y intercept; graphs that have the same gradient belong to the “family” having the same gradient.
7. Straight lines having the same gradient are parallel to each other. Parallel lines do not intersect each other.
8. The equation of the line can be determined if you know the gradient of the line and where the line cuts/intercepts the y -axis.

LESSON 9**AIM**

- (i) Able to draw graphs using the paper and pencil method.
- (ii) Able to choose appropriate scales for the graph.
- (iii) Able to determine appropriate range for the graph.
- (iv) Able to determine, write and differentiate between the coordinates of the x and y intercept.
- (v) Able to determine whether a point lies on the line or not.

ACTIVITY

1. The Mathematics tuition charges, y , by Mrs. Zaiton is related to the teaching time x by the equation $y=4x + 9$. Copy and complete the table below for the equation $y=4x + 9$

x	0	1	2	3	4	5	6	7
$4x$	0	4						
$4x+9$	9	13						
y	9	13						

Subsequently, draw the graph of this equation on your graph paper. Note where the graph intercepts at the y-axis. Write down the coordinate of the y- intercept. Identify this value from the equation of the line.

Note where the graph intercepts the x-axis. Write down the coordinate of the x-intercept. Identify this value from the equation of the line.

Suggest what the intercept represents, that is, why is there a charge when Mrs. Zaiton is not teaching?

Check the graph you have drawn using your calculator. You can draw your graph from the RUN Mode, GRAPH Mode or TABLE Mode. Use the Trace command in your checking and to verify where the graph intercepts y-axis.

2. Draw the graph for the equation of $3x+5y=30$ on your graph paper. Determine your own range. You can use the calculator to help you obtain your table of values.

How many points do you need to plot a straight line graph? Why?

What points would you choose to plot a straight line graph? Why?

What is the difference between the intercepts at the x-axis and at the y-axis?

There is infinite number of points passing through the line. Without using your *tracer*, how would you determine whether a point lies on the line or not?

- 24 -

SUGGESTED INTERVENTIONS

1. Look at your axes. What values are you plotting for? What should the axes be? Is it $2x$? $3y$? What do you need to know?
2. Look at the values of x (and y) in your table and your x -axis (and y -axis) of your graph. Do they include all the values of x (and y) in your table?
3. What is the general equation of the straight line?
Is the equation $3x+5y=30$ in the form of $y=mx+c$?
Can you determine the values of y directly from the equation $3x+5y=30$?
4. How would you rewrite the equation so that y is in terms of x ?
5. Are two points sufficient to draw a straight line?
6. Can you show me where the line crosses the y -axis?
What is the value of y at the y intercept?
What is the value of x at the y intercept?
7. Can you show me where the line crosses the x -axis?
What is the value of x at the x intercept?
What is the value of y at the x intercept?

TEACHING POINTS

1. Any equation that can be expressed in the form $y=mx+c$ where m and c are any selected numbers will produce a straight line graph.
2. The minimum number of points necessary to fix a straight line is two, but usually at least three are plotted with the third serving as a check.
3. It is a good idea to use two extreme values of x and one other value so that we know what values need to be covered on y -axis.
4. There is no hard and fast rule for deciding the scales to be used on the axes. The aim is to choose a scale so that the graph fits the graph paper and have axes that are easy to read.
5. The coordinate on the y intercept is $(0, y)$ and the coordinate on the x intercept is $(x, 0)$.

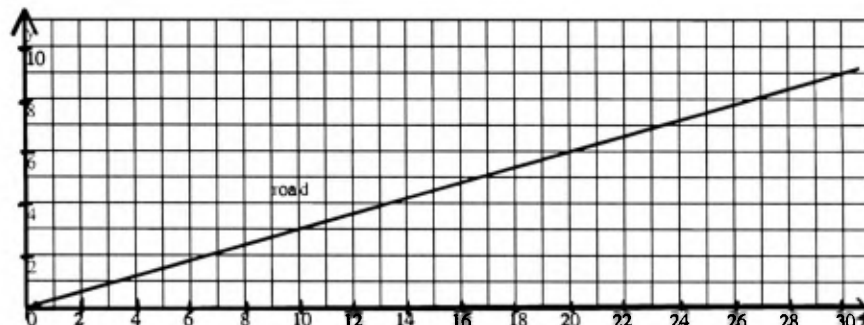
LESSON 10**AIM**

- (i) Able to determine the gradient of a straight line.
- (ii) Able to determine the equation of a straight line passing through any two points.
- (iii) Able to determine the equation of a straight line with given gradient and passing through a point.
- (iv) Able to recognise some effects of visual illusions (on gradients).

ACTIVITY

1. The equations of several lines are given below. For each one, write down the gradient of the line and give at least two points that pass through each line:
 - (i) $y=2x$
 - (ii) $2y=6x + 5$
 - (iii) $y=-4x + 3$
 - (iv) $x+y=4$
2. Choose any one equation above and investigate what happens to the related graph when you look in different viewing windows. How does the view-window affect the gradient of your straight line graph? Use the Dual Screen facility in the GRAPH Mode to help you. Write a short report on what you observe. Use the Trace to examine the value of the y intercept in each of the view-window.
3. The gradient of a line is a measure of how steep it is. One way of determining the gradient of the line is from the equation of the line. The next activity is to explore how to determine the gradient of a line.

A civil engineer makes a survey of a new road to be built from Kelana Road to Subang Road up a hill. She then makes a scale drawing on squared paper to show the road builders how steep to build the road:



The road is represented by a graph of straight line.
 What is the equation of the road, that is, what is the equation of the line y in terms of x ?
 Show how you determine this equation.
 From the equation, what is the gradient of the line?

- 26 -

Choose any two points on the line.

Calculate the increase in y (vertical distance) and calculate the increase in x (horizontal distance)

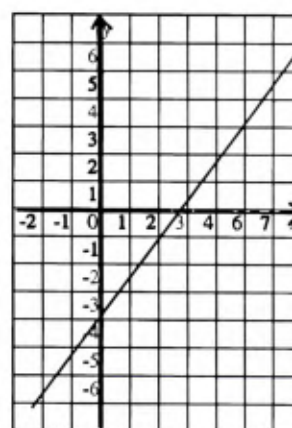
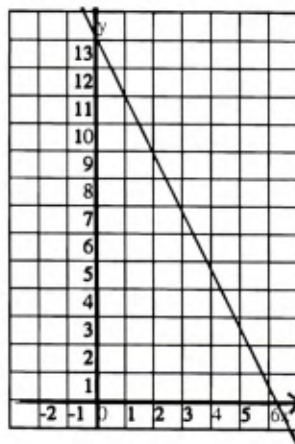
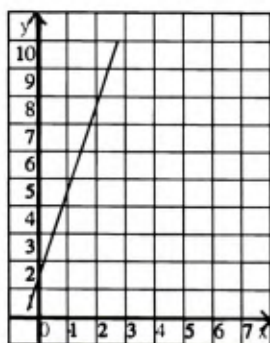
Then find the value of $\frac{\text{increase in } y}{\text{increase in } x}$.

Compare this value to the gradient of the line from the equation of the line you obtain previously. What can you say about this value?

Repeat by choosing any two other points on the line. What do you discover?

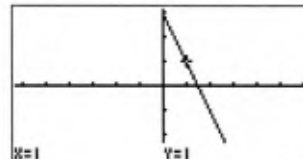
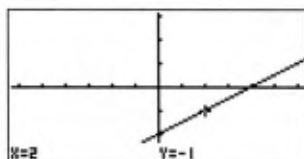
How do you determine the gradient of a line that passes through any two points that lie on the line? Explain.

4. Determine the gradient of each of the line given below:



What is the equation of each of the line?

5. Write down the equations of the graphs that have been graphed here on the INIT screen?



6. Find the gradient of the straight line segments joining each pair of given points:

(i) (5, 9) and (7, 17)

(ii) (-1, 5) and (-3, 19)

(iii) (-3, 7) and (2, 2)

(iv) (-2, -2) and (2, 0)

Subsequently, determine the equation of the straight line in each case.

7. A straight line of gradient -5 passes through the point (1, 2). Can you determine the equation of this straight line?

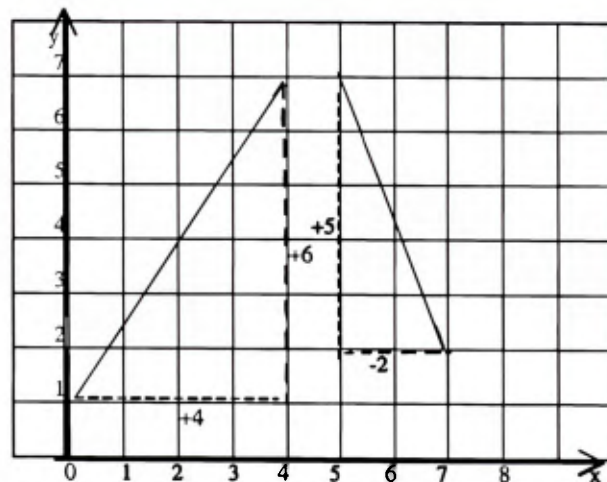
- 27 -

SUGGESTED INTERVENTIONS

1. What is the value of m in the equation?
2. Choose a point. If you substitute the value of x and the value of y in the equation, does it satisfy the equation?
3. Look carefully at the straight line. As the “steepness” of the slope changes, does the value of the y intercept change? What changes?
4. Go to your SET UP and view the grids in the different view-window.
5. How is the gradient of the line affected when the direction of the line changes?
6. Does the y (or x) intercept change when you change the view-window?
7. What is value of y at the y intercept?
8. What do you need to know to determine the equation of the line?
9. What does m stand for in $y=mx+c$?
What does c stand for in $y=mx+c$?

TEACHING POINTS

1. Examples on calculation of positive and negative gradient:



2. If (x_1, y_1) and (x_2, y_2) are two points on the line, the gradient m is given by

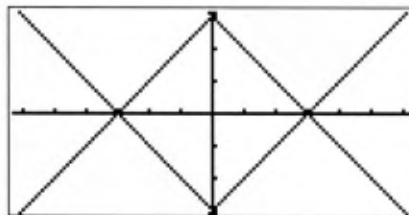
$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

LESSON 11**AIM**

- (i) Able to recognise the properties of parallel and perpendicular lines.
- (ii) Able to recognise some visual illusions (on proximity and perpendicularity).

ACTIVITY

1. Graphs that are parallel to each other have the same gradient. These pair of lines are parallel to each other: $y = -3x + 2$ and $y = -3x - 5$. Can you write another equation of the line that is parallel to these lines and pass through the point $(-4, 2)$?
2. How do the graphs of $y = x + 2$ and $y = -x + 2$ compare with each other. Try some other pairs like this, for example $y = -4x + 1$ and $y = \frac{1}{4}x + 1$. How do you draw these pair of lines? What is the parameter in the equation of the line that determines the characteristic of pairs of lines like this?
3. How would we know that two lines are perpendicular to each other from the equations of their lines? Investigate a few pairs of lines that are perpendicular to each other. Look at their gradients. How are their gradients related?
4. Change the view-window so that $x_{\min} = -8$, $x_{\max} = 8$, and $y_{\min} = -8$, $y_{\max} = 8$. What effect does this have on parallel and perpendicular pairs of graphs above?
5. Look at this quadrilateral. What is it called? Try to draw this quadrilateral on your calculator on the INIT view-window. What happens to this quadrilateral when you subsequently use the Zoom IN and Zoom OUT feature?



What happens to the shape of this quadrilateral when the view-window setting changes to $x_{\min} = -8$, $x_{\max} = 8$, and $y_{\min} = -8$, $y_{\max} = 8$?

Can you explain why?

SUGGESTED INTERVENTIONS

1. Recall the general equation of the straight line. Compare this with the equation that you have now. What do you need to know now? What is the value of x at the y intercept?

- 29 -

2. Do the graphs intersect each other?
3. If you have the gradient of the line, what else do you need to know to determine the equation of the line?
4. If you know that the line passes through a point, how can you determine the intercept of the line from the equation of the line?
5. If two lines are parallel to each other, what do they have in common?
Can their intercepts be the same?
6. If two lines are perpendicular to each other, do they have the same gradient?
7. How are the length of the scales on the x and y-axes when you draw on the INIT view-window compared to the length of the scales on the x and y-axes when you draw on the view-window with the setting $x_{\min}=-8$, $x_{\max}=8$, and $y_{\min}=-8$, $y_{\max}=8$?
How does this affect your graph?
8. How does the length of the scale on the x-axis and the y-axis change when you Zoom IN or when you Zoom OUT?
9. If you use the INIT view-window, does the grids on the screen retain as squares when you Zoom IN? Zoom OUT?
10. Which is the parameter in the equation $y=mx+c$?

TEACHING POINTS

1. *Calculator's syntax on how to type in $\frac{1}{2}x + 1$
Press $[(1 / [1]) / [a^b/c] / [4] / [D]) / [x, \theta, T] / [+] / [1]$
2. Lines that are parallel to each other have the same gradient.
3. Lines that are perpendicular to each other: product of their gradient is -1
4. When the scale of the x and y-axes are not the same length, the view-window gives the illusion that the lines are not perpendicular with each other.
5. Using the Zoom changes the scales on the x-axis and the y-axis with the same "factor". Hence if the graph is drawn on the INIT view-window or on a view-window that have equal lengths on the x-axis and the y-axis, the square grids will still be retained although the size of the graph has been diminished or enlarged. Hence the lines will still be seen as being perpendicular to one another.
6. Changing the view-window gives the "illusion" that parallel line graphs have moved closer or further away from each other.

LESSON 12**AIM**

- (i) Able to distinguish the shape of the graph of $y=x$ and $y=x^2$.
- (ii) Able to recognise some families of graphs of quadratic equations: $y=ax^2$; $y=ax^2+b$.
- (iii) Able to differentiate between variables and parameters in quadratic equations.
- (iv) Knowledge of the line of symmetry for graphs of quadratic equations.
- (v) Able to recognise the interaction between the scale and the shape of the graph.

ACTIVITY

1. An asteroid from outside the solar system goes round the sun and back out. Astronomers work out that its path as follows: $y=x^2$. Draw the path of the asteroid by drawing the graph of $y=x^2$ in the GRAPH Mode. Describe the path of the asteroid.
2. Draw the graph of $y=x$ and $y=x^2$ on the same screen. What do you observe in the shape of the graph? What other difference do you observe between the two graphs? Where is the line of symmetry for each of the graph?
3. Go to the TABLE Mode and observe the table of values for both the equations above. How are they different?
4. How does the view-window affect the shape of the graph of $y=x^2$? Draw a few graphs of $y=x^2$ on the different view-windows. Explain your observations.
5. Draw the graphs of $y=x^2$, $y=x^2+1$, $y=x^2+2$, and $y=x^2+3$ on the same screen. How are the graphs related? Why are they related in this way? Tabulate all the four equations. How do the values in the table help you to understand the graphs?

Use your observations to predict what the graph of $y=x^2-5$ will look like. How is this graph different from the graphs above? Use your calculator to check your prediction.
6. In the DYNA Mode, draw several graphs of $y=ax^2$ by varying the parameter "a". Try a few values of "a" that is negative. Observe what happens to the "steepness" of the graph. Sketch what you saw.
7. Predict how the graph of $y=-x^2+3$ will look like. Make a sketch of your prediction and check with your calculator.

SUGGESTED INTERVENTIONS

1. Is the relationship between $y=x$ and $y=x^2$ different?

- 31 -

2. How does the value of y increase with respect to x ? Is it increasing much more?
3. What makes you predict that the shape of the graph is not a straight line?
4. What is the difference between the shape of the graph for $y=x$ and $y=x^2$?
5. Do you still get a parabola when the value of the parameter “ a ” changes?
7. In your opinion, which one determines the shape of the graph, the parameter “ a ” or the variable x ?
8. What is the line that “divides” the graph of $y=x^2$ into two equal parts?
9. Does the graph of $y=x$ has a line of symmetry?

TEACHING POINTS

1. *Calculator’s syntax on how to key in x^2 :
Press $[x, \theta, T]$ (you get x)
Press $[x^2]$ OR press $[^]$ and press $[2]$

For any power of x , press $[^]$ followed by the number of the power.
2. *The line of symmetry is the line that divides the graph into two equal parts.
3. *Any equation with the variable to the power 2 is called a quadratic equation.
4. The graph of the equation $y=ax^2$ is a curve called a parabola.
5. The graph $y=ax^2$ passes through the origin.
6. The parameter “ a ” determines the “steepness” of the curve. It also determines the “direction” of parabola.
7. The viewing window affects the “steepness”/ “pointedness” of the curve.
8. Graph of $y=ax$ has a line of symmetry. The line of symmetry is the y -axis or $x=0$.
9. Graphs in the form of x^2+a have symmetry at the y -axis (or $x=0$) and cuts the y -axis at a .

LESSON 13**AIM**

- (i) Able to make the connection between the different quadratic equations and the properties of the graph
- (ii) Able to use the features of the calculator to draw graphs

ACTIVITY

1. Graphs of quadratic equations in the form of $y=x^2+a$ have a symmetry at the y-axis and the parameter a indicates where the graph cuts the y-axis. How is this different from the graphs of the quadratic equations in the form $y=(x+a)^2$? Draw a few of them with your calculator to include some values of a that is negative. Start with $a=1$. Observe what the line of symmetry is and where the graph cuts the y-axis. Use the *tracer* to help you. Does this graph cut the x-axis? Start in the GRAPH Mode and extend your work in the DYNA Mode. You may need to press the cursor keys \blacktriangle , \blacktriangledown to move the x-axis up and down or the cursor keys \blacktriangleleft , \blacktriangleright to move the y-axis to the left or right in order to view where the parabola cuts the y-axis.
2. Predict how the graphs with equation in the form $y=(x+a)^2+c$ will be like. Test your prediction by drawing a few graphs of this family for example the graph of $y=(x+1)^2+8$. Start in the GRAPH Mode and extend your work in the DYNA Mode. You may also need to change the view-window to enable you to see the whole graph. You may find the STD window very helpful OR you can use the Zoom feature of your calculator to enable you to see the whole graph OR you can use the cursor keys described above. What can you say about this family of parabolas? Is this family of parabolas the same as above? Explain.
3. Another family of parabolas has the quadratic equation in the form of $y=x^2+ax$. What are the graphs in this family have in common? Investigate with your calculator for some values of a including negative values. Start with x^2+x , x^2+2x , How are the parabolas of this family different from the parabolas of other families?
4. From your investigations, you will see that some graphs cross the horizontal axis and some do not. Give two examples each of equations of the parabola that
 - (i) cross the x-axis;
 - (ii) do not cross the x-axis;
 - (iii) cross the x-axis only once.

SUGGESTED INTERVENTIONS

1. What happens to the value of y when the graph crosses the x-axis?
2. Which parameter determines the line of symmetry?

- 33 -

3. Where would you draw a line that divides the graph/ parabola into two equal parts?
4. Is the line of symmetry a vertical or horizontal line?
5. What is the equation of the vertical line? What is the equation of the line of symmetry?
6. What does the value of “a” in $y=(x+a)^2$ tell you about the graph?
7. What does the value of “a” and “c” in $y=(x+a)^2 + c$ tell you about the graph?
8. What does the value of “a” in $y=x+ax$ tell you about the graph?

TEACHING POINTS

1. *Calculator’s syntax on how to key in equations:
Eg. $y=(x+a)^2$
Press $[(] / [x] / [+] / [a] / [)] / [x^2]$
2. All graphs of quadratic equations/ parabola has a symmetry line which passes through the lowest or highest turning point and is parallel to the y-axis.
3. Discussion on what features of the graphs are linked to the numbers/ parameters in the equation and the different forms of equation. Discussion includes the turning points, intercepts and the line of symmetry, translations

LESSON 14**AIM**

- (i) Able to make the connection between the algebraic form of the equation and the shape of the graph.
- (ii) Able to use the features of the calculator to draw graphs.
- (iii) Able to recognise the shape of graphs of cubic, quartic and reciprocal equations.

ACTIVITY

1. In the GRAPH Mode, investigate the shape of the graphs of other equations:

$$y=x^3$$

$$y=x^4$$

$$y=x^5$$

How do the shapes of the graphs compare with each other? Look at the power of the variable x and look out for lines of symmetry, turning points and “pointedness” of the graphs. Predict what the graph of $y=x^6$ would look like. Test your prediction by drawing a graph of it.

Write a report of what you discover.

2. A cubic graph is one whose equation has x^3 as the highest power of x . The simplest cubic graph is $y=x^3$. Use your calculator, to draw the graphs of:

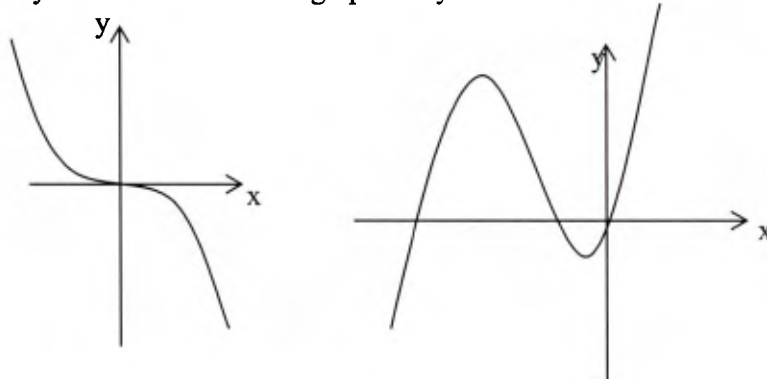
$$y=x^3$$

$$y=x(x+2)(x-2)$$

$$y=x(x+3)(x-3)$$

What do you notice about the shapes of these graphs? Do they have a line of symmetry? Where do they cut the x -axis and where do they cut the y -axis? Identify these values from the equations of their graphs.

Try to obtain these cubic graphs on your calculator:



Explain how you draw these graphs.

3. Investigate the graphs of these equations in the CONIC Mode.

$$x=y^2$$

$$x=2y^2$$

$$x=3y^2+7$$

How are they the same and how are they different from each other?

- 35 -

What are the things that you consider when you attempt to draw these graphs? Look at the general shape of these graphs with the form of equations. How are these graphs different from those of $y = x^2$? How are they similar?

4. Reciprocal graphs are obtained from equations like

$$y = \frac{1}{x}, \quad y = \frac{10}{x}, \quad y = \frac{-2}{x}, \quad \text{etc}$$

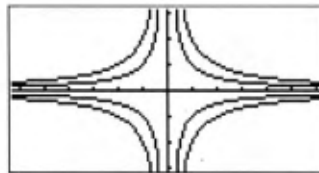
Determine the parameter in each of the equation above.

Draw these graphs on your calculator. Begin in the INIT View-window for the first graph. Trace the graph. What do you find about the values of x and y ? Explain what is happening at the ends of each graph.

Draw the second graph on the same screen but Zoom OUT after you have drawn it to enable you to see more of the graph. How does this graph compare with the first one?

Draw the third graph on the same screen. How is this graph different from the other two graphs. Observe in which quadrants they are drawn.

Then try to obtain this screen:



5. This is also an equation of a reciprocal graph:

$$y = \frac{1}{3+x}$$

Draw the graph in the GRAPH Mode and trace the graph. What happens for $x = -3$?

How does this graph compare to the graph of

$$y = \frac{1}{x}?$$

SUGGESTED INTERVENTIONS

1. How does the y values compare to the x values for some points on the graph?

- 36 -

2. What is the value of y as x increases eg. if $x = 10$, what is the value of y?
3. Is your graph a straight line or a curve? Why?
4. Is your graph continuous for all values of x?
5. How many “branches” do reciprocal graphs have?

TEACHING POINTS

1. *Calculator’s syntax on
 - to operate in the CONIC Mode:
Press CONICS from the MAIN MENU
Select $X=A(Y-K)^2 + H$
For $X=Y^2$, select $A=1$, $K=0$, $H=0$
Press [EXE]
 - how to key in algebraic expressions
Eg.
(i) x^5
Press $[x, \theta, T] / [^] / [5]$

 $\frac{1}{x}$
Press $[1] / [\div] / [x, \theta, T]$

 $\frac{1}{3+x}$
Press $[1] / [\div] / [(] / [3] / [+] / [x, \theta, T] / [)]$
2. Cubic graphs have the general shape of an “s”.
3. The graphs of reciprocal equation is a curve which approaches more closely to the x-axis as the value of x become larger and which approaches the y-axis for small values of x.
4. The parts of the lines which approach close to the x and y axis are called asymptotes.
5. The reciprocal graph has two “ branches”.
6. Reciprocal graphs have discontinuities at certain points.
7. The power of the variable determines the general shape of the graph.

LESSON 15**AIM**

- (i) Able to recognise the shapes of graphs of some equations of different forms
- (ii) Able to use “sensibly” the calculator and features of the calculator to draw and check graphs

ACTIVITY

1. Look at your calculator buttons. Explore how the graphs of these would look like:

\sqrt{x}		$\log x$	$\sin x$
$\ln x$	10^x	$ x $	$\cos x$
x^{-1}	$\sqrt[3]{x}$		$\tan x$

Observe the power of the variable and check whether your graph is continuous for all values of x .

2. Your calculator has several in-built graphs in the DYNA Mode, for example:

$y = Ax + B$
 $y = A(x + B)^2 + C$
 $y = Ax^2 + Bx + C$
 $y = Ax^3 + Bx^2 + Cx + D$
 $y = A \sin(Bx + C)$
 $y = A \cos(Bx + C)$
 $y = A \tan(Bx + C)$

You have explored some of them. Choose a few equations and explore their properties by varying the values of A , B , C and D .
How does these parameters affect your graphs?

Write a report of what you discover about the graphs you have chosen to explore.

SUGGESTED INTERVENTIONS

1. How does the y values compare to the x values for some points on the graph?
2. Is your graph a straight line or a curve?
3. Is the graph continuous for all values of x ?
4. Does your graph have any asymptotes?
5. Look at the equation of your graph. What is the highest power of the variable of your equation. How does the power of the variable influence the shape of your graph?

- 38 -

6. Look at the setting of your View-window. Do they include the values of x and y that is necessary to include the important features of the graph?
7. Would it help if you zoom OUT (or zoom in) several times to view the whole graph?

TEACHING POINTS

1. *Calculator's syntax on how to key in the algebraic expressions.
2. *Calculator's syntax on how to draw graphs from the built in functions.
Enter the DYNA Mode.
Press B-IN
Highlight the equation you want to use and press [EXE]
Continue as working in the DYNA Mode.
3. Determine whether the graph has intercepts at x and y axes.
4. Discussion on the features of the related graphs selected.

Appendix 3 Teaching Module for Interpretative Learning Model

TEACHING MODULE B

- 1 -

ABOUT THE MODULE

1. This teaching module covers the topics on Equations and their Graphs.
2. The focus is to let student use the graphing calculator as a tool to explore/investigate the graphing concepts embedded in their activities.
3. There are 15 lessons in the module.
4. Each lesson in the module is accompanied by a corresponding Student's Worksheet.
5. Each lesson in the module is divided into 4 parts which defines the aim of the lesson, the activities of the lesson, the interventions that can be used in the lesson and the teaching points or instructions that have to be carried out by the teacher.
6. The activities in each lesson are open-ended activities designed to establish an environment for exploration for students to generate meanings for themselves and with your interventions where appropriate.

HOW TO USE THE MODULE

1. Your role when using this module is more as a **facilitator** or counsellor for students to consult in their quest to make sense of their tasks.
2. The module serves only as a guide on the scope to be covered in the topic on Equations and their Graphs. Subsequent activities should be contingent on the student's current performance.
3. Student's current performance can be assessed by the Assessment tasks that are supplied with this Module.
4. Students are supplied with the corresponding Student's Worksheet for each lesson in the Module and a Student's Manual on graphing calculator use.
5. The interventions suggested in this module are in the form of goal type questions that are intended to get students to be in control of their learning. You may add your own interventions that you think are appropriate. As a guide, questions posed to student requires student to reflect on what they are doing or thinking such as:
 - What are you doing?
 - What does that mean?
 - Why are you doing it?
 - Did you find where you went wrong?
 - Is there any other method?
 - Is that all there is?
 - What have you done so far?
 - What have you tried?
6. The "Teaching Points" marked * should be introduced to the students at the start of the lesson. These are new information which is necessary to enable student to begin with the activities in the worksheets. Those unmarked are discussed at the end of the lesson so to help students clarify the mathematical concepts they generate or encounter in their learning activity.

- 2 -

LESSON 1**AIM**

- (i) To show the different ways information can be represented.
- (ii) To introduce the idea of mapping, ordered pairs/Cartesian coordinates, hence Cartesian graph.

ACTIVITY

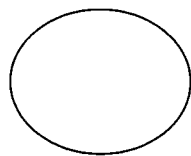
The table shows the rate of growth of a broad bean shoot.

Age in weeks	1	2	3	4	5	6
Height in cm	2	4	6	8	10	12

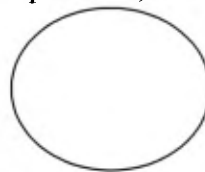
- Think of other ways to represent this information. Discuss with a friend
 **Explain to students that this information can be represented in many forms.

e.g., arrow diagram

On your first BLANK PAPER, draw two sets. Fill in the data in each set.
 Draw the arrows that show that “5 maps to 10”, etc.



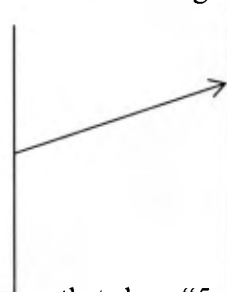
age in weeks



height in cm

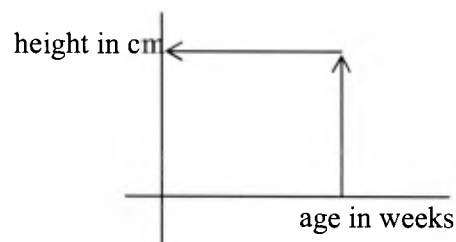
On your second BLANK PAPER, draw two parallel axes and mark them as shown:

age in weeks height in cm

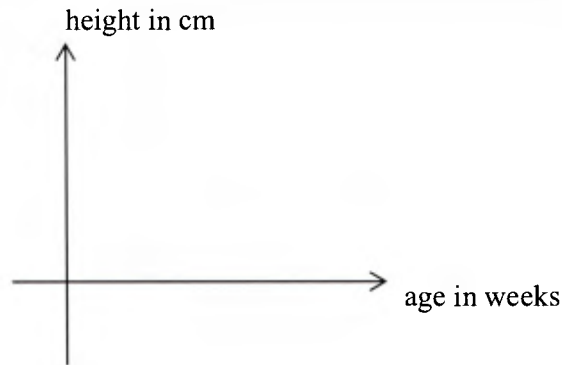


Draw the arrow that show “5 maps to 10”, etc.

Using GRAPH PAPER, draw two axes that are perpendicular to each other.
 Draw the mapping arrow with a right angled bend in it :



- 3 -



On your second GRAPH PAPER, draw two axes that are perpendicular to each other.

Mark the points where the mapping arrow “turns the corner”.

Emphasise that this point still means “5 maps to 10” or simply written as an ordered pair (5, 10).

Complete marking the points.

2. Can you tell what are the advantages and disadvantages of presenting your data in this way (by marking points on the graph paper)?
3. What can you say about the relationship between the age and the height of the broad bean?
What assumptions do you make to deduce this relationship?
4. Based on the graph, write a report on what can be deduced about the growth of the broad bean shoot.
5. If x is the age in weeks of the broad beans and y is the height in cm of the broad beans, write the relationship between x and y .

SUGGESTED INTERVENTIONS

1. Can you show me how you mark your points?
2. What do the points you marked tell you?
3. Does the broad bean grow evenly every week?
4. What assumptions do you make when you join the points?
5. Do you need to join the points? Why?

TEACHING POINTS

1. The straight line represents the graph of the relationship between x and y . This type of relationship is called a linear relationship and the graph is called a straight line graph or linear graph.

- 4 -

2. The relationship between x and y is called the mathematical equation for the line.
3. The graph drawn is the graph of the equation $y=2x$.

****Presented in the form of discussion.**

- 5 -

LESSON 2**AIM**

- (i) Familiarise with the conventions of the graphic calculator.
- (ii) Able to use the graphic calculator to plot points.
- (iii) Able to plot points and name / write them correctly as ordered pairs / use the convention that x-coordinate always comes first.
- (iv) Know that points that if you try to plot a point that does not fit on the view-window, the calculator will not plot anything.
- (v) Able to differentiate (in position) between positive and negative values for x and y.
- (vi) Knowledge of the origin (0, 0).

ACTIVITY

Use graph paper where you think appropriate.

1. Plot the point (1,2) on the graph paper and the calculator.
2. Once you have plotted the point, plot your own points. Use the cursor keys [▲], [▼], [◀], and [▶] to move the *pointer* (a flashing + sign) around the screen. Write as many things that you observe.
3. Use the SET UP command to insert grids on your screen. Write what you observe.
4. Give your friend a point to plot. Ask him why he thinks the point is positioned as such.

SUGGESTED INTERVENTIONS

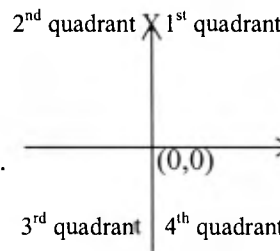
1. What are you trying to plot?
2. Where would you expect the point to be? Can you tell me why you think that the position of the point is such?
3. Is there any difference between the point you plot previously and the point you are plotting now?
4. How can you tell whether a point lie in the 1st, 2nd, 3rd or 4th quadrant (right-hand line, left-hand line)?

TEACHING POINTS

1. *Calculator's syntax on how
 - (i) to get started with the graphic calculator:
Press [AC^{ON}].
Select RUN Mode from the MAIN MENU and press [EXE].

- 6 -

- (ii) to plot points:
Press [SHIFT] / <Sketch> /> /PLOT / Plot 3,2 (eg. for the point (3,2))/
[EXE].
- (iii) to insert grid:
Press [SHIFT]/ <SET UP>.
Press cursor down (▼), select Grid :On and press EXE.
- (iv) to clear screen:
Press Cls / [EXE].
Remind students to CLEAR their graphic calculator screen each time
before starting a new task.
2. The value of the coordinate of the *pointer* are displayed at the bottom of the
calculator screen.
3. The name of the four quadrants.
4. The x and y axes.
5. The origin (0, 0) is where the x and y intersect.



- 7 -

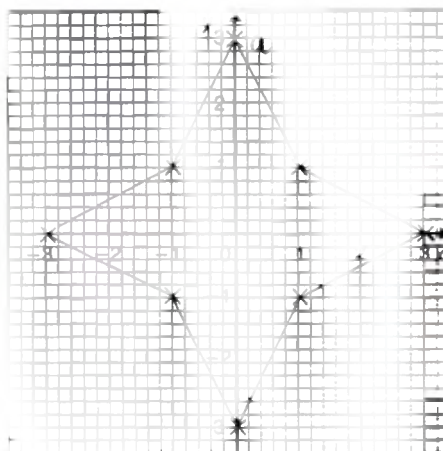
LESSON 3**AIM**

The main aim of this activity is to further induct students to using the graphic calculators and become well-verse with the conventions/ commands of the graphics calculator to plot points. Students are encouraged to explore the functions of [AC/ON], <OFF>, [EXIT], [SHIFT], [EXE] and <QUIT> commands. This activity can also be used to find out whether students are

- (i) able to name the coordinates, and
 - (ii) fluent with the conventions/ commands of the graphics calculator to plot points
- so that appropriate interventions can be given.

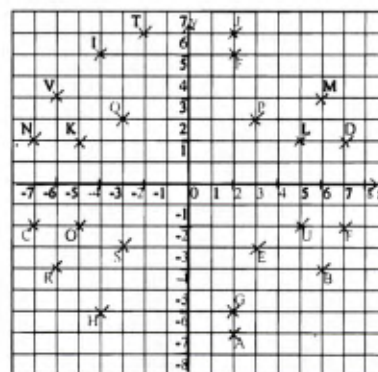
ACTIVITY

1. Use the calculator to plot points of any square. Record your results and compare with a friend.
2. List the coordinates of every corner of the figure, starting at “a” and going clockwise.



3. Which letter in the diagram opposite is at:

- (i) (5, 6)?
- (ii) (-5, 2)?
- (iii) (-2, 7)?
- (iv) (-6, -4)?
- (v) (3, -3)?



- 8 -

Choose the points that can be plotted on your graphic calculator and plot them.

Think of how you can plot the other points.

SUGGESTED INTERVENTIONS

1. Can you tell me how a square looks like?
2. What point did you begin with and what point are you going to plot next?
3. Why do you think some points do not appear on your calculator screen?

TEACHING POINTS

1. *Calculator's syntax on how to cancel a point that is wrongly plotted:
Select Pl-Off / Press [EXE].
2. Position of positive and negative values for x and y coordinates.

LESSON 4**AIM**

- (i) Recognise the effect of changing the scales/ changing the view-windows.
- (ii) Know the difference between the INIT and the STD window:
INIT window: scales are the same on both axes – square grid.
STD window: scales on both axes are not the same – rectangular grid.
- (iii) Able to relate between scales and view-windows.
- (iv) Able to move between the view-windows, store and recall the selected view-windows.

ACTIVITY

1. Plot a point in the INIT view-window. Write as many things as you can about the characteristic of this view-window. You can see the value setting by pressing [SHIFT] <V-Window> followed by selecting the INIT command. It will help if you redraw what you saw on the screen.
2. Now investigate what happens to the point that you choose if you plot it in another view-window, say, the STD view-window.
Discuss with your friend and write down what you both agreed on. It will help if you redraw what you saw on the screen.
3. Name a point that cannot be plotted on the INIT view-window. Why? How can you plot the point?
4. Plot a point where both numbers are integers that is exact on the grid/ markings (dots) of the view-window? Explain how you do this.

SUGGESTED INTERVENTIONS

1. What does the INIT view-window tell you?
2. What does the STD view-window tell you?
3. Is there any difference between the INIT and the STD view-window?
4. Recall in your previous task, did you find any points that could not be plotted or do not appear on your screen? Do you know why?
5. What can you say about the distance between two consecutive marking on the x-axis and the y-axis in different view-windows?

TEACHING POINTS

1. *Calculator's syntax on how to get into, to store and to recall the View-window.
☐ to get into view-window: press [SHIFT]/ V-Window.

- 10 -

- ☐ to store: select STO, then select the view-window you want to store in from V-W1 to V-W6.
- ☐ to recall: select RCL, then select the view-window you want to recall.

2. *The INIT view-window is the default window setting of the calculator.

3. The INIT view-window gives the actual size of the view-window that is $6.3\text{cm} \times 2$ on the y-axis and $3.1\text{cm} \times 2$ on the x-axis.

Each press on the cursor key in the INIT view-window moves the *pointer* 0.1 unit on the screen.

4. The STD view-window is the standardised view-window of the calculator with the following setting: $x_{\min}=-10$, $x_{\max}=10$ on the x-axis; and $y_{\min}=-10$, $y_{\max}=10$ on the y-axis.

5. The scale values for x and y tell what the tick marks on each axis represent. The scale values for x and y in the INIT view-window are the same: a unit on the x-axis is the same length as a unit on the y-axis. Thus the grids are squares.

6. The view-window setting may cause irregular scale spacing. The scale values for x and y may be the same but a unit on the x-axis is not the same length as a unit on the y-axis. In the STD view-window, a unit on the x-axis is not the same length as a unit on the y-axis. Thus the grids are rectangular, not squares.

LESSON 5**AIM**

- (i) Able to plot lines on the graph paper and on the calculator.
- (ii) Recognise infinite number of points passing through the line.
- (iii) Knowledge of pixels
 - (a) recognise the difference in “smoothness” between two different line segments drawn on the calculator.
 - (b) recognise the difference in “smoothness” between the line segment and the horizontal and vertical lines drawn on the calculator.
 - (c) recognise the difference in “smoothness” between the line segment drawn on the graph paper and those drawn on the calculator.
- (iv) Able to write the equations of the vertical and horizontal line.

ACTIVITY

1. Draw a line segment on your calculator and your graph paper. Write down as many things as you can about this line.
2. Draw another line segment. Write down what you see.
3. Choose any two points to draw a vertical line. Note down a few things that you observe about the vertical line.
How is this line different from the lines you have drawn above?
What do you think the equation of this vertical line would be?
4. Choose any two points to draw a horizontal line. Note down a few things that you observe about the horizontal line.
How is this line different from the other lines you have drawn?
What do you think the equation of this horizontal line would be?
5. You can draw the horizontal line and a vertical line on your calculator without plotting the points by selecting either the Vert or Hztl command or by typing in the equation of the line in the GRPH command. Try this. Then show to a friend how you draw some of these horizontal and vertical lines. Draw some of these lines on your paper.
6. Use the Trace command to move your *tracer* along the line you have drawn. Notice the x and y values as the *tracer* traces the line. What can you deduce about the points that lie on the line?
What does the equation of the horizontal or vertical line inform you about points on the graph?

SUGGESTED INTERVENTIONS

1. What do you observe about the line segment that you draw on the paper and the one you draw on the graphic calculator?

- 12 -

2. What are the two points that you choose to draw a vertical line?
Why do you choose these two points?
3. What are the two points that you choose to draw a horizontal line?
Why do you choose these two points?
4. What can you say about the horizontal line?
5. What can you say about the vertical lines?
6. What do you remember about the equation of the line in Lesson 1?

TEACHING POINTS

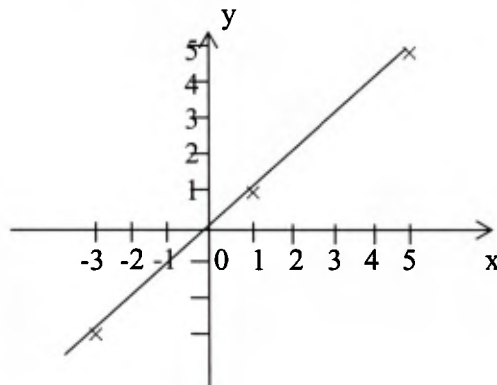
1. *Calculator's syntax on how
 - (i) to draw a line segment:
Press [SHIFT]/ <Sketch> / ▸/ LINE/ F-LINE [3] /[,]/ [2]/ [,] /[-] / [5] / [,] / [1] (for the line segment joining the points (3, 2) and (-5, 1))/ [EXE].
 - (ii) to draw a vertical and horizontal line
 - ☐ Press [SHIFT]/ <Sketch> / ▸/ select either Vert or Hztl; OR
 - ☐ Press SHIFT/ <Sketch> / GRPH/ select either Y= or X=c
 - (iv) to use Trace
Press [SHIFT]/ Trace/ and one or more of these cursor keys: [▲], [▼], [◀], [▶] until the tracer or pointer appears on the screen.
2. Recap that in the first activity the straight line graph can be “represented” by the equation of the straight line.
Equation of the horizontal line is $y = \text{number}$ and
The equation of the vertical line is $x = \text{number}$.
3. There is infinite number of points on the line.
4. The point plotted occupies one pixel. The pixel is rectangular in shape having horizontal and vertical sides.
5. The equation of the horizontal (vertical) line informs you of all the possible pairs of values on the line, however in this case the value of y (x) is unique.
6. In the Trace command, the *pointer* is also referred as the *tracer*.

LESSON 6**AIM**

- (i) Able to see the effect of view-window / scales on the pixels.
- (ii) Able to make connection between the points (ordered pairs), table of values, the graph and the algebraic expression/ equation of the line (by operating in the GRAPH Mode and TABLE Mode).
- (iii) Knowledge of straight line graph as a continuous graph.
- (iv) Knowledge of the concept variable.
- (v) Knowledge of range.
- (vi) Knowledge of the graph $y=x$ passes through the origin.
- (vii) Recognise that any pair of values that fits the relationship/ the equation of the line will plot a point somewhere along the same line.
- (viii) Able use the Dual Screen in the TABLE Mode.

ACTIVITY

1. Draw a line segment that joints any two points on the INIT view-window. Repeat drawing the same line segment on different view-windows. Discuss with a friend how the line has changed when you draw them on different view-windows.
2. Here is a straight line. Draw this straight line on your calculator



What can you say about the straight line?

Find the equation of the line (or a rule or an expression) that express the relationship between x and y .

Check your guess by “plugging” in this rule or equation into in the GRAPH Mode and obtain a graph of it. Then use the Trace command to trace the points along the line to check the your guess. What do the points tell you?

Explore in the TABLE Mode to help you compare some of the points you have traced. Select G-PLT and see what kind of graph is obtained. Observe this graph carefully and note down how the points in this graph is related to the x and y values in your table. Notice that you can draw the graph and the table side by side using the Dual Screen facility in the TABLE Mode. Press

- 14 -

RANG to change the range of your x-values to include a wider value of x.
What do you observe about the points in your graph?

Select the G-CON and observe what kind of graph is obtained. How is this graph different from the graph obtained using the G-PLT command?

Use your tracer to find out how the points on this graph is related to the x and y values in your table.

Is it possible to list down all the points on the line?

How can you determine whether a point lies on the line?

SUGGESTED INTERVENTIONS

1. What do you remember about the scales on the INIT and STD view-window?
2. In what way does drawing the line in different view-window affects the line?
3. What would you look for if you want to determine any relationship between x and y?
4. What can you say about the points that lie on a straight line?
5. Does the point (20, 20) lie on the line? Why?
6. Does the point (2, 1) lie on the line? Why?
7. What does the equation of the line tell you about the points on the line?
8. Which command do you use to get a smooth line, the G-CON and G-PLT?

TEACHING POINTS

1. *Calculator's syntax on how
 - (i) to operate in the GRAPH Mode
Select GRAPH from the MAIN MENU, press [EXE].
Type in the equation of the line at Y1=
Press DRAW
 - (ii) to operate in the TABLE mode
Select TABLE from the MAIN MENU, press [EXE].
Type in the equation of the line at Y1=
Press TABL
Press [▼] to go down the table to look at other values of x and y
Press the G-CON or G-PLT
Press RANG to look at range

To get back to the first/default screen, press the button [EXIT]

- 15 -

2. Emphasise again the rule or the algebraic expression that express the relationship between x and y is called the equation of the line.
3. The meaning of range. The points plotted on the G-PLT include only those within the specified range and the points can be viewed only if it is within the range of the view-window.
4. x and y are called variables.
5. The graph of a straight line is a continuous graph.
6. There are infinite points that lie on the line. Any points that fits (satisfies) the equation of the line will plot a point somewhere along the same straight line. Therefore to determine whether a point lies on the line, the value of x and y must satisfy the equation of the line.
7. The graph of $y=x$ passes through the origin $(0, 0)$.

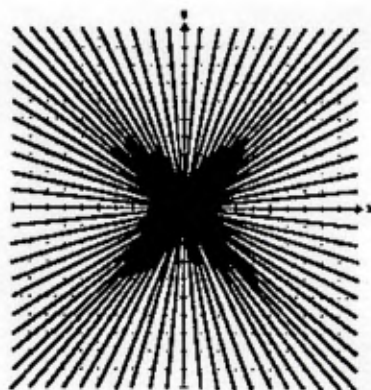
- 16 -

LESSON 7**AIM**

- (i) Recognise that the general equation of the graph that passes through the origin is $y=mx$.
- (ii) Knowledge of the concept of gradient and recognise the effect of varying the values of m in $y=mx$.
- (iii) Able to operate in the DYNA Mode.

ACTIVITY

1. Try to reproduce as close as possible the Starburst picture.
You may find it helpful to explore the DYNA Mode before you attempt to draw this figure in the GRAPH Mode
It might also be helpful if you record the lines you have used.
What do you notice about the “number” in front of the variable x ?
Write a report on how you obtain the Starburst picture.



2. What are the possible values for the “number” of a line that lies entirely in the shaded region of the figure below?



- 17 -

SUGGESTED INTERVENTIONS

1. What line did you begin with?
Why did you choose this line to begin with?
2. What is the equation of the line that you choose?
3. What is affected/ happens as the line gets steeper/ less steep?
4. How is the direction of the graph affected?
What happens to the graph when the “number” in front of the variable x takes a positive value?
What happens to the graph when the “number” in front of the variable x takes a negative value?

TEACHING POINTS

1. *Calculator’s syntax on how to operate in the DYNA mode.
 - ☐ Select DYNA in the MAIN MENU, press [EXE]
 - ☐ Press the button $\langle \text{ALPHA} \rangle$ to select the letter “A”
 - ☐ Press A.
 - ☐ Press the button $[x, \theta, T]$ to type in the variable x .
 - ☐ Press VAR and the calculator screen shows Dynamic Var : A.
 - ☐ Press DYNA

Point out: in the calculator “A” is named as the “dynamic variable”
2. *Point out the difference between the meaning of the term “dynamic variable” used in the syntax of the graphic calculator and the variables x and y .
3. The general equation of the straight line that passes through the origin is $y=mx$ m is called the “steepness” or the gradient or the slope of the line.
4. m is a “parameter” that determines the “steepness” of the line.
5. The meaning of positive and negative gradient.

- 18 -

LESSON 8

AIM

- (i) To introduce students the straight line that cuts the y-axis.
- (iii) Knowledge of the concept of the x intercept and y intercept.
- (iv) Recognise the general equation of the straight line is $y=mx+c$ where m is the gradient and c is the y intercept.

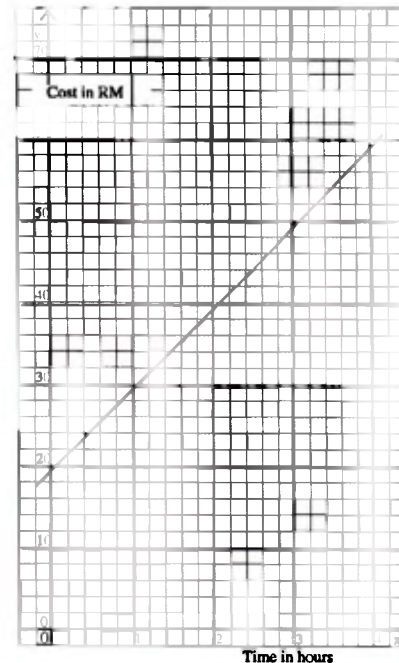
ACTIVITY

Hassan repairs domestic equipment. He charges call-out fee together with an hourly charge. Customers often ask for an estimate before work is begun. As Hassan does not know how long a job will take before he starts it. He gives this graph to customers to help them get an idea of likely charges.

1. What is Hassan's call out fee?
2. What does Hassan charge for a repair that takes $2\frac{1}{2}$ hours?

However, some information cannot be read from the graph, for example the cost of a repair taking 6 hours. We can estimate the cost by trying to judge where the line would be if we extend both axes.

We can obtain an estimate of the cost for a repair job that takes very much longer than this or any length of time by working out the relationship between the time of the repair job and its cost given by the graph. This is called **the equation of the line**.



3. What is Hassan's hourly charge rate (the cost of repair per hour)?
How do you obtain this value?
4. Write down the relationship between the cost of repair and the length of work.
5. If x is the time in hours of the repair job and y is the cost in RM, discuss with a friend how you can find the equation for this line. Write down the equation that you both agreed on.
6. From the equation of the line you obtain, determine how long Hassan has to spend on a repair to earn RM200?

- 19 -

7. Compare this graph with your previous graph of $y=x$. How are they the same? How are they different?
8. Write down the coordinate where the line cuts (intercepts) the y-axis.
9. If m is the gradient of a straight line and c is the intercept on the y-axis, write down the equation of this straight line.

SUGGESTED INTERVENTIONS

1. What do you understand by call-out fee? Does Hassan get any money for not doing any repair job?
2. Is Hassan's hourly charge the same throughout the job?
3. What do you think the graph will look like if Hassan does not charge any call-out fee?

TEACHING POINTS

1. *The term intercept: cross/ intersect/ cut
2. y-intercept ($x=0$), x-intercept ($y=0$)
3. General equation of the straight line is given by $y=mx + c$ where m is the gradient of line and c is the y intercept.
4. The meaning of y intercept
 - ☐ from the graph, the y intercept is when $x=0$
 - ☐ from the equation $y=x+\text{number}$, the number is the interceptRelate that when the y intercept is 0, the graph passes through the origin, that is when $c=0$, $y=mx$.

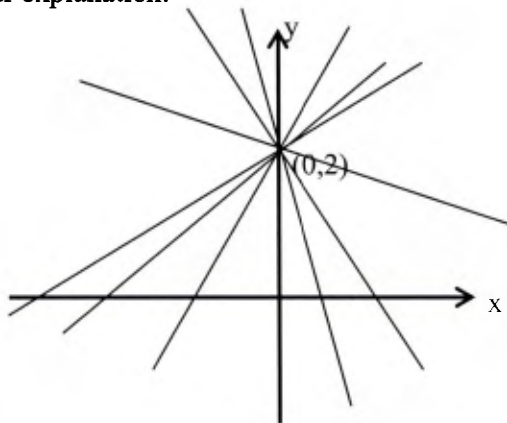
- 20 -

LESSON 9**AIM**

- (i) Recognise some properties of the straight line.
- (ii) Recognise some “families” of straight lines.
- (iii) Able to use the Dual Screen in the GRAPH Mode and the TABLE Mode.

ACTIVITY

1. Investigate the graphs of the equation $y = mx + c$ for different values of m and c . Write a short report on what you observe.
Explore the Dual Screen in the GRAPH Mode to draw two graphs side by side and the Dual Screen in the TABLE Mode to generate the table and the graph on the same screen.
2. Here is a figure that shows some straight lines that intercepts the y-axis. Write the possible equations that characterise the arbitrary chosen lines. Give your explanation.

**SUGGESTED INTERVENTIONS**

1. What are you trying to do?
2. Are you predicting that this graph will be different from the previous one that you have drawn? Which part is different?
3. Can you recall what the DYNA Mode of your graphic calculator do?
4. How many equations of straight line should you have?
5. Why do you choose that line?
6. Why do you choose that equation?
7. What does the equation of a line tell you?

- 21 -

TEACHING POINTS

1. The m and c values determine the characteristic of the graph, for example, graphs that have the same y -intercept belong to the “family” having the same y -intercept; graphs that have the same gradient belong to the “family” having the same gradient.
2. You can determine the equation of the line if you know the gradient of the line and where the line cuts/intercepts the y -axis.

- 22 -

LESSON 10**AIM**

- (i) Able to draw graphs using the paper and pencil method.
- (ii) Able to choose appropriate scales for the graph.
- (iii) Able to determine appropriate range for the graph.
- (iv) Able to determine, write and differentiate between the coordinates of the x and y intercept.
- (v) Able to determine whether a point lies on the line or not.

ACTIVITY

- The Mathematics tuition charges y by Mrs. Zaiton is related to the teaching time x by the equation $y=4x + 9$. Copy and complete the table below for the equation $y=4x + 9$

x	0	1	2	3	4	5	6	7
4x	0	4						
4x+9	9	13						
y	9	13						

Subsequently, draw the graph of this equation on your graph paper.

Mark the coordinate of the y-intercept. Identify this value from the equation of the line.

Suggest what the intercept represents, that is, why is there a charge when Mrs. Zaiton is not teaching?

Check the graph you have drawn using your calculator.

How do you check to verify the coordinate where the graph intercepts the y-axis?

- Draw the graph for the equation of $3x+5y=30$ on your graph paper. Determine your own range. You can use the calculator to help you obtain your table of values.
How many points do you need to plot a straight line graph?
What points would you choose to plot a straight line graph?

What is the difference between the intercepts at x axis and at y axis?

There is infinite number of points passing through the line. How would you determine whether a point lies on the line or not?

SUGGESTED INTERVENTIONS

- What values are you plotting for? Is it $2x$? $3y$? What do you need to know?
- How do the scales of your axes affect the graph that you are going to draw?
- Would you need to determine the range of values?

- 23 -

4. What do you know about the y intercept?
Can you show me where it is on the graph?
5. What do you know about the x intercept?
Can you show me where it is on the graph?
6. Is it necessary to rewrite the equation in the form of $y=mx+c$?
7. Can you determine the values of x and y directly from the equation $3x+5y=30$?

TEACHING POINTS

1. Any equation that can be expressed in the form $y=mx+c$ where m and c are any selected numbers will produce a straight line graph.
2. The minimum number of points necessary to fix a straight line is two, but usually at least three are plotted with the third serving as a check.
3. It is a good idea to use two extreme values of x and one other value so that we know what values need to be covered on y-axis.
4. There is no hard and fast rule for deciding the scales to be used on the axes. The aim is to choose a scale so that the graph fits the graph paper and have axes that are easy to read.
5. The coordinate on the y intercept is (0, y) and the coordinate on the x intercept is (x, 0)

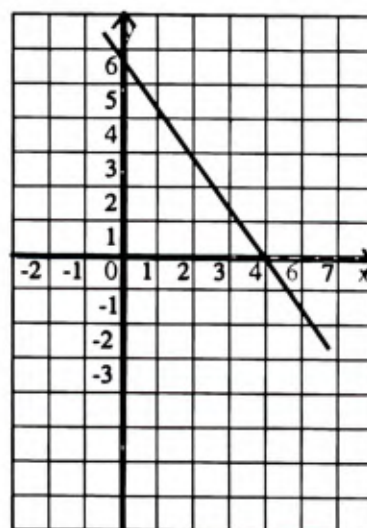
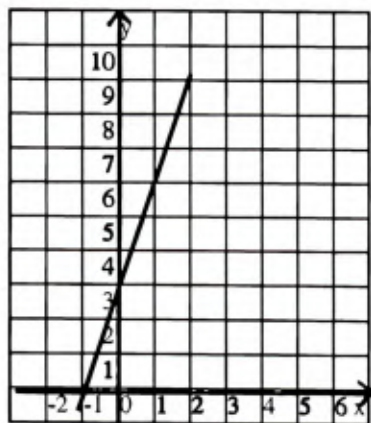
LESSON 11

AIM

- (i) Able to determine the gradient of a line.
- (ii) Able to determine the equation of a straight line that pass through any two points.
- (iii) Able to determine the equation of a straight line with given gradient and passing through a point.
- (iv) Able to recognise some effects of visual illusions (on gradients).

ACTIVITY

1. $y=x$ and $y=2x$ are two equations of straight lines with gradient 1 and 2 respectively. The gradient is a measure of the “steepness” of the line. Compare the steepness of these two lines and investigate how you can determine the gradient of each line from its graph. Compare this value with the value obtained from the equation of the line.
2. The equations of the two line segments below are not given. Extending your knowledge above, how can you determine the gradient of each of the line?



What can you say about the gradient of a straight line passing through any two points on the line?

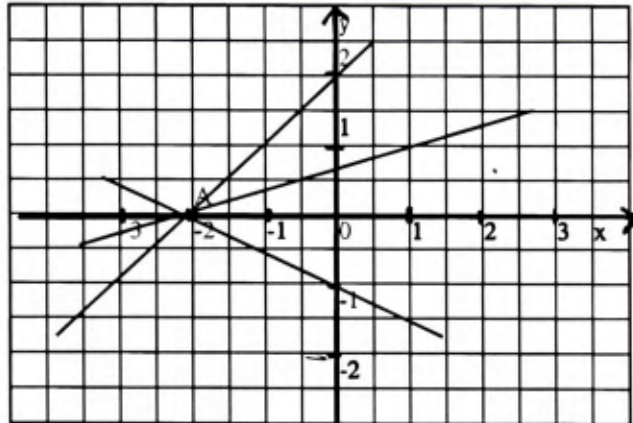
What would be the gradient of the straight line segment joining the point (5,9) and (7, 17)?

Write a mathematical formula or rule to calculate the gradient of any straight line that passes through two points.

3. Choose any straight line and investigate what happens when it is drawn on different view-windows. How does the view-window affect your straight line graphs?

- 25 -

4. Think of some straight lines passing through point A on the graph. Find their equations.
Try some different starting points.
Write a report about what you discover.



If the gradient of the line that passes through A is $-\frac{1}{4}$, what is the equation for this line?

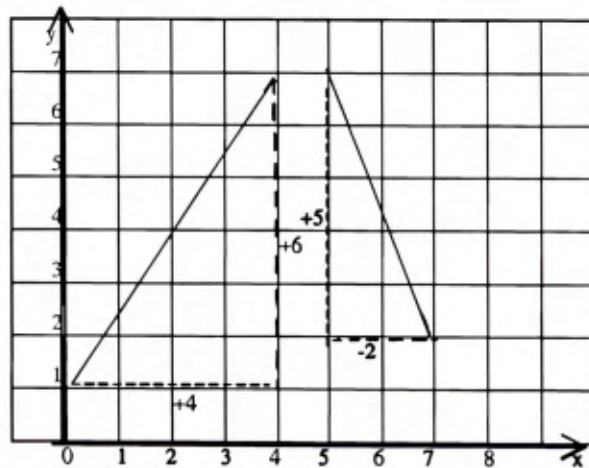
SUGGESTED INTERVENTIONS

1. What do you understand by “steepness”?
2. What would you look for to determine the gradient of the line?
3. What are affected by the gradient of the line?
4. How does the change in y with respect to x affect the gradient of the line?
5. How does the value of y change with x in each of the line?
6. Is the gradient of the line the same throughout the line?
7. What is the general equation of the straight line?
8. What do you look for to determine the equation of the straight line?
9. What other points does the line pass through? What does this tell you?
10. How do you change the view-window?
11. Why did you begin with this view-window?

- 26 -

TEACHING POINTS

1. The gradient of a straight line graph is determined by finding the value of $\frac{\text{increase in } y}{\text{increase in } x}$
2. If (x_1, y_1) and (x_2, y_2) are two points on the line, the gradient m is given by
$$m = \frac{x_2 - x_1}{y_2 - y_1}$$
3. Examples on calculation on positive and negative gradient:



- 27 -

LESSON 12**AIM**

- (i) Recognise the properties of parallel and perpendicular lines.
- (ii) Able to recognize visual illusions (on proximity and perpendicularity).
- (iii) Able to use the Dual Screen and the Zoom feature of the calculator.

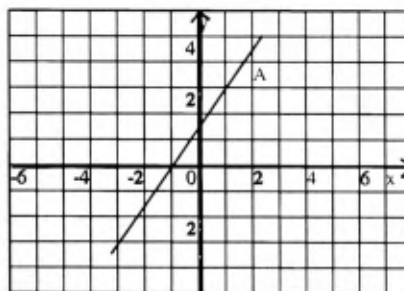
ACTIVITY

1. The graphs you drew previously are all straight-line graphs. They have equations of the form $y=mx+c$, for example in $y=2x+1$, m is 2 and c is 1. Investigate graphs that are parallel to each other and graphs that are perpendicular to each other. Write down as many things or properties about them.

You can investigate the features of the graphs using the Trace, Dual Screen and Zoom buttons of your calculator.

Investigate how you can determine the m and the c of your graph.

2. Check that the equation of line A is $2y = 3x+3$.
Explain how you determine this.
Investigate the equations of lines which are parallel to A.
What is special about their equations?



3. Complete the square.
How do you do this?
Zoom in to find the coordinates of the vertices.



What do you notice about the square when you Zoom IN? Zoom OUT?

Explain why?

Look at the square in different view-windows. What happens? Can you give an explanation?

-28 -

SUGGESTED INTERVENTIONS

1. What does the value of m in the equation of the line tells you about the graph?
2. Does the value of m change for a particular straight line?
3. What does the value of c in the equation of the line tells you about the graph?
4. What do you remember about the differences between the STD and the INIT view-window?
5. What do you do to obtain the square?
6. What is the advantage of using the Zoom to change your view-window?

TEACHING POINTS

1. *Calculator's syntax on how
 - (i) operate the DUAL SCREEN
 - ☐ Press [SHIFT]/ <SET UP>
 - ☐ Press ▼ to go down to Dual Screen
 - ☐ Select G to T to get table side by side with graph or select GRPH to get two graphs simultaneously side by side
 - (ii) use the ZOOM
 - ☐ Press [SHIFT]/ Zoom
 - ☐ Select IN to get a closer look or select OUT to get a distant view
2. Lines that are parallel to each other have the same gradient.
3. Lines that are perpendicular to each other: product of their gradient is -1
4. If the scale of the x and y -axes of a view-window are not the same length, the view-window gives the illusion that the lines are not perpendicular with each other.
5. The advantage of using the Zoom is that it will change the scales on the x and y -axes with the same "factor". Therefore, if a graph drawn starts with the INIT view-window, the square grids remains although the size of the graph has been diminished or enlarged.
6. Changing of view-windows also give the illusion that parallel lines are moving closer or further away from each other.

- 29 -

LESSON 13

AIM

- (i) Able to recognise the general shape of the graph of quadratic equation: $y=ax^2$.
- (ii) Able to recognise some families of quadratic equations: $y=(x+a)^2$; $y=ax^2$; $y=x^2+ax$ and $y=(x+a)^2+b$.
- (iii) Able to differentiate between variables and parameters in quadratic equations.
- (iv) Knowledge of line of symmetry for graphs of quadratic equations.
- (v) Able to recognise the interaction between scale and shape of the graph.

ACTIVITY

1. What do you notice about this button: x^2

Guess the shape of the graph.

Draw it using the calculator.

Why does it have this shape?

How does the view-window affect your graph?

2. Investigate the relationship between the graph of $y=x^2$ and the graphs of $y=x^2+a$ for different values of “a”, including some negative values. Describe the relationship.

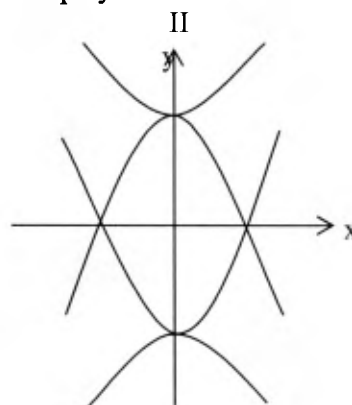
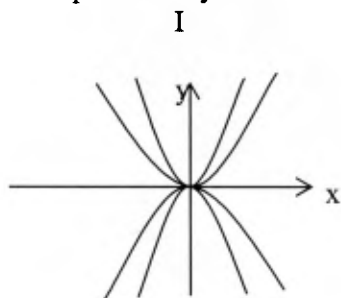
Repeat the above for the relationship between the graph of $y=x^2$ and graph of

- (i) $y=(x+a)^2$ (ii) $y=ax^2$ (iii) $y=x^2+ax$

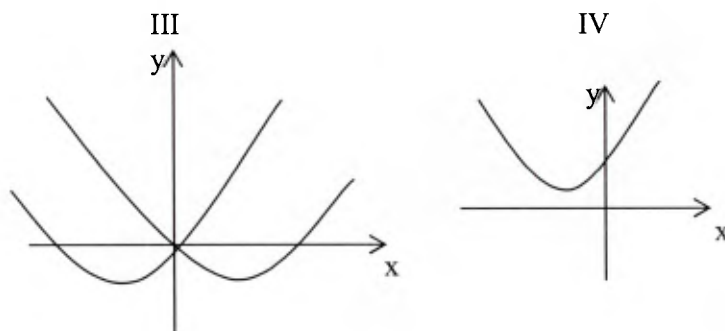
for different values of “a” including some negative values.

You can investigate graphs of the same “family” using the DYNA Mode.
Write a report of your investigation.

3. Explore the graphs of quadratic equations in the form of $y=(x+a)^2+b$. Describe what you observe.
4. Try to obtain each of these displays:
Explain how you obtain each of the display.



- 30 -

**SUGGESTED INTERVENTIONS**

1. What does the relationship $y = x^2$ inform you?
2. How does the value of x vary with respect to y ?
How does this affect the graph?
7. What happens when the value of “ a ” changes in each different form of the equations?
8. What other features of the graph do you notice?
9. What features of the graph are linked to the numbers in the equation?

TEACHING POINTS

1. *Calculator's syntax on how to key in
 - (i) x^2 :
 - ☐ Press x, θ, T (you get x)
 - ☐ Press x^2 OR press $^$ and press 2
 - ☐ (For any power of x), press $^$ followed by the number of the power.
 - (ii) $(x+a)^2$
 - ☐ Press $[(] / [x] / [+] / [a] / [)] / [x^2]$
2. *The line of symmetry is the line that divides the graph into two equal parts.
3. *Any equation with the variable to the power 2 is called a quadratic equation.
4. *In your investigation, you may need to think about the turning points, intercepts and the line of symmetry.
5. The graph of the equation $y = ax^2$ is a curve called a parabola.
6. The graph $y = ax^2$ passes through the origin.

- 31 -

7. The parameters in the quadratic equation determine the characteristic of the curve eg. a in $y=ax^2$ determines the “steepness” or the “pointedness” of the curve. It also determines the “direction” of parabola; a in $y=x^2+ax$ determines where curve cuts the x -axis
8. The view-window affects the “steepness”/ “pointedness” of the curve.
9. All quadratic graphs/ parabola has a symmetry line which passes through the lowest or highest point of the graph and is parallel to the y -axis.
10. Discussion on what features of the graphs are linked to the numbers/ parameters in the equation and the different forms of equation. Discussion includes the turning points, intercepts and the line of symmetry.

- 32 -

LESSON 14**AIM**

- (i) Able to make the connection between the algebraic form and the shape of the graph.
- (ii) Able to use the features of the calculator to draw graphs.
- (iii) Able to recognise the shape of graphs of cubic, quartic and reciprocal equations.

ACTIVITY

1. Investigate the graphs of other equations:

$$y=x^3 \qquad y=x^4 \qquad y=x^5$$

Predict what the graph of $y=x^6$ would look like? Test your prediction by drawing a graph of it.

Write a report of what you discover.

2. A cubic graph is one where x^3 is the highest power of x . The simplest cubic graph is $y=x^3$. Using your graphic calculator, draw the graphs of

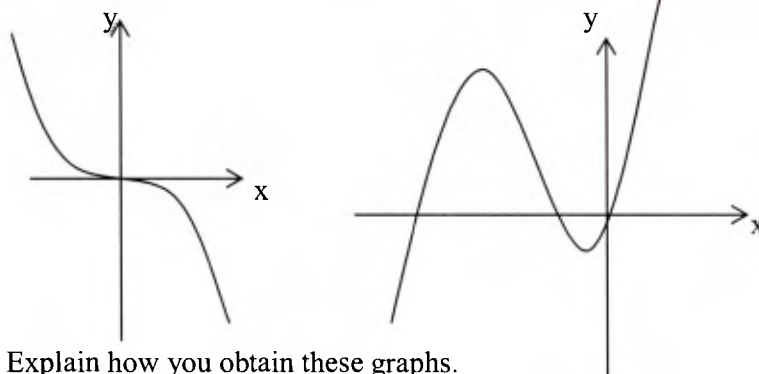
$$y=x^3$$

$$y=x(x+2)(x-2)$$

$$y=x(x+3)(x-3)$$

What do you notice about these graphs?

Try to obtain these cubic graphs on your calculator screen:



Explain how you obtain these graphs.

3. Investigate these graphs in the CONIC Mode:

$$x=y^2$$

$$x=2y^2$$

$$x=3y^2+7$$

How are they the same and how are they different from each other?

How are these graphs different from those of $y=x^2$?

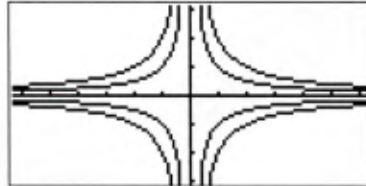
Explore in the CONIC Mode for more graphs of this family.

- 33 -

5. Reciprocal graphs are obtained from equations like

$$y = \frac{1}{x}, \quad y = \frac{10}{x}, \quad y = \frac{-2}{x}, \quad \text{etc}$$

Use your graphic calculator to help you investigate reciprocal graphs. You may start your investigation by drawing the graphs of the above reciprocal equations and note down any difference you observe. Then try to obtain this screen:



Explain what is happening at the ends of each graph.

This is the equation of the graph of the same family :

$$y = \frac{1}{3+x}$$

Trace the graph. What happens for $x = -3$?

How does this graph compare to the graph of

$$y = \frac{1}{x}?$$

SUGGESTED INTERVENTIONS

1. How does the y values compare to the x values for some points on the graph?
2. By observing the shape of the graph, what can you say about the value of y and x?
3. Why do you think the graph of this equation is not a straight line?
4. What do you remember about the View-window?
5. How can the Zoom feature of your calculator help in your investigation?

TEACHING POINTS

1. *Calculator's syntax on how

- 34 -

- to operate in the CONIC Mode:
Press CONICS from the MAIN MENU
Select $X=A(Y-K)^2 + H$
For $X=Y^2$, select $A=1$, $K=0$, $H=0$
Press [EXE]

- how to key in algebraic expressions

Eg.

- (i) x^5
Press [x,0,T] / [^] / [5]

- (ii) $\frac{1}{x}$
Press [1] / [÷] / [x,0,T]

- (iii) $\frac{1}{3+x}$
Press [1] / [÷] / [(] / [3] / [+] / [x,0,T] / [)]

2. Cubic graphs have the general shape of an “s”.
3. The graphs of reciprocal equation is a curve which approaches more closely to the x-axis as the value of x become larger and which approaches the y-axis for small values of x.
4. The lines which approach close to the x and y axis are called asymptotes.
5. The reciprocal graph has two “branches”.
6. Reciprocal graphs have discontinuities at certain points.
7. The power of the variable determines the shape of the graph.

LESSON 15

AIM

- (i) Able to recognise some of the shapes of graphs of other equations.
- (ii) Able to use the calculator and the built-in features “sensibly” to draw graphs.

ACTIVITY

1. Look at your graphic calculator buttons. Explore how the graphs of these would look like:

\sqrt{x}		$\log x$	$\sin x$
$\ln x$	10^x	$ x $	$\cos x$
x^{-1}	$\sqrt[3]{x}$		$\tan x$

2. Your graphic calculator has several in-built graphs in the DYNA Mode, for example:

$y = Ax + B$
 $y = A(x + B)^2 + C$
 $y = Ax^2 + Bx + C$
 $y = Ax^3 + Bx^2 + Cx + D$
 $y = A \sin(Bx + C)$
 $y = A \cos(Bx + C)$
 $y = A \tan(Bx + C)$

Choose any one or a few of them and investigate their graphs.

Write a report of what you discover on the graphs you have chosen to investigate.

SUGGESTED INTERVENTIONS

1. How does the y values compare to the x values for some points on the graph?
2. What would you do to enable you to see the graph globally?
3. What does the equation of the graph informs you?

TEACHING POINTS

1. *Calculator's syntax on how to key in algebraic expressions.
2. *Calculator's syntax on how to draw graphs from the built in functions.
Enter the DYNA Mode.
Press B-IN
Highlight the equation you want to use and press [EXE]
Continue as working in the DYNA Mode.

- 36 -

3. Determine whether the graph has intercepts at x and y-axes.
4. Discussion on the features of the related graphs selected.

WORKSHEET A

- 1 -

LESSON 1

The table shows the rate of growth of a broad bean shoot.



Age in weeks	1	2	3	4	5	6
Height in cm	2	4	6	8	10	12

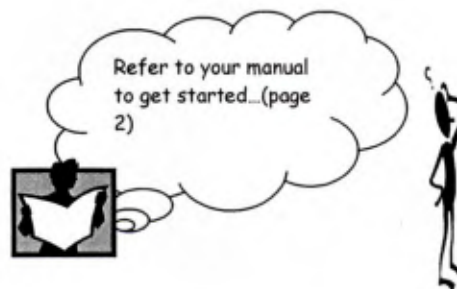
1. Think of other ways to represent this information.
Discuss with your friend.
2. What can you say about the growth of the broad bean shoot?
3. Can the height of the broad beans be the same at a particular age?
4. Can the height of the broad beans be different at one particular age?
5. How can you predict the height of the broad bean shoot when it is 10 weeks old?





6. What can you say about the relationship between the age and the height of the broad bean?
7. If x is the age in weeks of the broad bean and y is the height in cm of the broad bean, can you write the relationship in terms of x and y ?


- 2 -


LESSON 2



1. In the RUN Mode what do you observe on the screen of your calculator? 

2. Plot the following points on your graph paper as well as on your calculator in the RUN Mode:
 - (i) Plot the following point: (1, 2). What do you observe? 



 - (ii) Use the [SET UP] command to insert grids on your screen. What do you observe about these grids? 

 - (iii) Now plot the points (4, 1) and (1, 6). What do you observe? 

 - (iv) Use the *pointer* (a flashing plus sign, +) by pressing the cursor keys [▲], [▼], [◀], [▶] to move around the screen.
What do you notice about the x and y value when the *pointer* is in the
 - (a) first quadrant?
 - (b) second quadrant?
 - (c) third quadrant?
 - (d) fourth quadrant?

Plot some points and ask your friends to read these points. Make sure you deactivate (switch off) the coordinate display in the SET UP menu.

-3 -

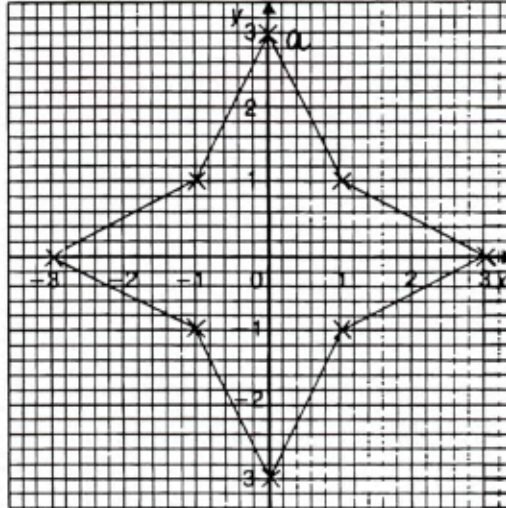
- (v) What do you observe about the movement distance of the *pointer* when you press the cursor key? 
- (vi) Plot the following points: (1, 2), (-3, 2), (-1, 2). What do you observe?
- (vii) Plot the following points: (-1, -1), (-1, 2), (-1, 3). What do you observe?
- (viii) What do observe when you move the *pointer* horizontally? Vertically?
3. What do you observe when you plot the coordinates in each of the four quadrants? 
4. What is the value of x and y where the axes intersect?

- 4 -

LESSON 3

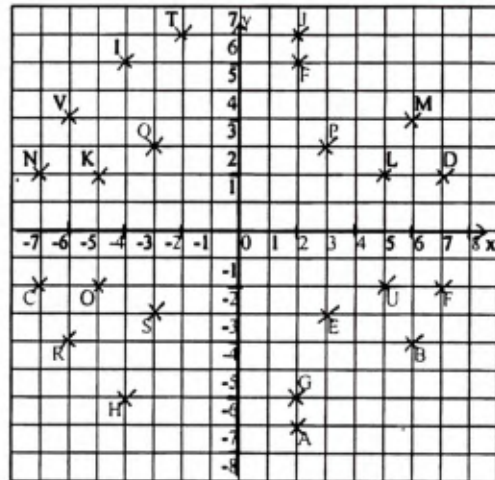
This activity is to give you practice to familiarise yourself with the conventions and commands of the calculator. Play with the following calculator commands: $[AC^{ON}]$, $[EXIT]$, $[EXE]$, $\langle OFF \rangle$, $\langle QUIT \rangle$.

1. On your calculator, plot these points which are corners of a square. Find the coordinates of the missing corner. Use the coordinates $(3, 3)$, $(1, 1)$, $(3, -1)$.
2. List the coordinates of every corner of the figure, starting at "a" and going clockwise.



3. Which letter in the diagram opposite is at:

- (i) $(-5, 2)$?
- (ii) $(-2, 7)$,?
- (iii) $(-6, -4)$?
- (iv) $(3, -3)$?



- 5 -


Choose the points that can be plotted on your calculator and plot them.
Think of how you can plot the other points.



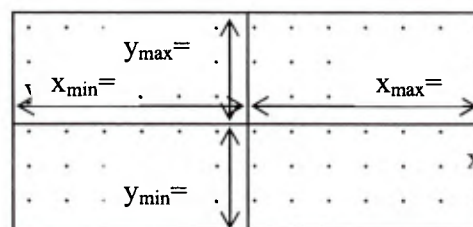
To cancel a point that has
been wrongly plotted,
press P1-Off/ [EXE]

- 6 -

LESSON 4

1. Plot the point (3, 2) on your graph paper and on your calculator in the INIT view-window. Copy the minimum and maximum x values and the minimum and maximum y values of your graph paper. With your calculator get back into the screen, move the  *pointer* along the x-axis and the minimum and maximum value it registers for the x-axis on the screen. Then move the pointer along the y-axis and write down the minimum and maximum value it registers for the y-axis. Determine the scale values for x and y in the INIT view-window setting.

Refer to your Manual to see how to operate the View-Window (page 8)



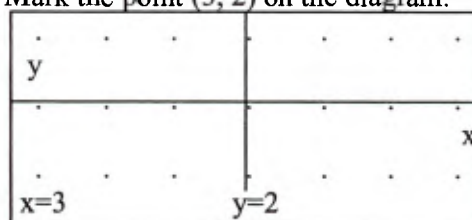
Scale for x:


Scale for y:


What happens to the tick marks on the axes when you change the scale values (for x and y), say 0.5? Look where the point (3, 2) lies.

Repeat for scale value 2. Observe the screen of your calculator. What has happened to the tick marks?

Mark the point (3, 2) on the diagram.



2. Change the view-window to show $x_{\min}=-5$, $x_{\max}=5$ and $y_{\min}=-5$ and $y_{\max}=5$ (use scale for x and y as 1). Plot the point (3, 2) on this screen. What do you notice about the grid? 

What do you notice about the x and y values displayed on the screen? Write down these values. 

x =

y =

Explain your answer.

Store this setting in view-window V-W1 by pressing:
STO/ V-W1 / [EXIT] STD.

- 7 -

3. Press [SHIFT] ⟨V-Window⟩ and choose the STD view-window. What do you observe about values set in the STD view-window?

Write down the following values: $x_{\min} =$ $y_{\min} =$

$x_{\max} =$ $y_{\max} =$

Can the point (3, 2) be plotted exactly? Why?

4. Change the view-window to $-31.5 \leq x \leq 31.5$ and $-15.5 \leq y \leq 15.5$. Press the cursor, what do you notice?

Store this view-window in V-W2.

Plot a point. Press the cursor keys, what happens?

5. Plot a point where both numbers are integers that is exact on the grid/ markings of the view-window? Explain how you do this. ✎

- 8 -

LESSON 5

1. Draw the line segment joining $(3, 2)$ and $(-5, 1)$ both on the calculator (in the RUN Mode) and on the graph paper. Compare them. What do you find?

2. Draw the line segment joining $(-3, -2)$ and $(4, 3)$ on the calculator. Compare this with the line segment you have previously drawn on the calculator. What do you find?


3. Draw the line segment joining $(2, -2)$ and $(5, -2)$ on the calculator. What can you say about this line and the points that lie on this line?
How is this line different from the lines you have drawn above?
What do you think the equation of this line would be?

4. Draw the line segment joining $(3, -2)$ and $(3, 5)$. What can you say about this line and the points that lie on this line?
How is this line different from the lines in Q.1, Q.2, and Q.3?
What do you think the equation of this line would be?

- 9 -

5. On your graph paper, draw and label the following lines:

$$x=-4, x=0, x=2, y=-1, y=0, y=2$$

Observe the similarities and the differences between these lines. 

6. You can draw the horizontal line and a vertical line on your graphic calculator without plotting the points by selecting either the Vert or Hztl command or by typing in the equation of the line in the GRPH command. Try this.

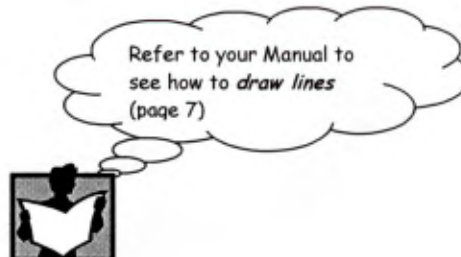
Show to a friend how you draw the lines in Q.3 and Q. 4.



Then draw the lines in Q.5.

Use the Trace command to move your *pointer* or *tracer* along the line.

Notice the x and y values as the *tracer* trace the line. What does the equation of the horizontal or vertical line inform you about points on the graph?



- 10 -

LESSON 6

The INIT V-Window is the initial window setting.
The STD V-Window is the standardised window setting.



1. Draw a line segment joining the points $(-5, -2)$ and $(6, 3)$ on the INIT view-window. Repeat drawing the same line segment on different view-windows starting with the STD view-window, and V-W1 and V-W2 that you had earlier stored. How has your line changed when you draw them on different view-windows?



What difference(s) do you notice about the pixels and the length of the line segment in each of the view-window?

2. Draw a horizontal and a vertical line on the INIT window. Then change the view-window to the STD and then the V-W1 windows. What do you notice about the position of the line?

3. Here are some points: $(-2, 2)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$
What do you notice about the value of x and the value of y ?



Make a table of values for the above information.

x						
y						

4. Plot the above points on your on your graph paper.
What do you notice about these points?

Do these points appear to form a pattern?



- 11 -

Find a mathematical relationship between x and y (or the equation of the line)?

Write down as many points as you can that passes through/ lie on the line?

Is it possible to list down all the values of x and y for this relationship?
Explain.

How is this graph different from the graph of $y=3$?

How is this graph different from the graph of $x=4$?

5. With your calculator (in the RUN Mode), draw the graph whose equation is $y=x$ using the GRPH command. Using the Trace command, write down as many points that pass through the line. Can the tracer trace all the points that lie on the line? Why?

Compare the points that you trace with those you obtain by drawing on the graph paper.

Refer to your Manual to
see how to draw graph in
the RUN Mode (page 7)



What do you find?

Does the point $(2.5, 2.5)$ lie on the line?

Does the point $(8, 8)$ lie on the line?

Does the point $(20, 20)$ lie on the line?

Does the point $(2, 1)$ lie on the line?

How can you determine whether a point lies on the line from the equation of the line? Give an example.

- 12 -

LESSON 7

1. In the RUN Mode of your calculator, plot these points: $(-1.5, -2.5)$, $(-1, -2)$, $(-0.5, -1.5)$, $(0, -1)$, $(0.5, -0.5)$, $(1, 0)$, $(1.5, 0.5)$, $(2, 1)$, $(2.5, 1.5)$, $(3, 2)$, etc. Do these points appear to form a

Another way of plotting the points efficiently is by plotting the point on the screen by moving your pointer to the desired position and press [EXE].

pattern on the screen?



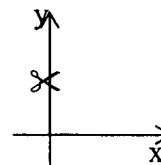
Join these points.

Find the relationship or the equation of the line of the graph.

2. Draw the graph by putting in the equation of the line you obtain above in the RUN Mode.
What do you observe?

What is the value where the line cuts (intercepts) the y-axis?

What is the coordinate where the line cuts the y-axis?



Use the *tracer* to trace the points that lie on the line.

How is this graph different from the one you drew previously that is the graph of $y=x$? In what way(s) are they the same?

- 13 -

3. Plot the graph for the above relationship in the GRAPH Mode. You can also draw the graph from in the TABLE Mode. Notice that in the TABLE Mode, the screen shows the table of values. Note them down. Observe too that you can draw the graph and the table side by side using the Dual Screen in the TABLE Mode.

Refer to your Manual to see how to draw graph in the GRAPH Mode (page 9) and how to operate in the TABLE Mode (page 13).



There are two ways to obtain graphs in the TABLE Mode, either using the G-PLT command or the G-CON command. Observe what kind of graphs you obtain when you use each of the command. What difference do you find between the graph plotted on the G-CON command and the graph plotted on the G-PLT command?

From these observations what can you deduce about the points on a straight line?

4. Determine whether these points lie on the line:


(i) $(-2, -3)$

(ii) $(4, 2)$

(iii) $(0, 0)$

Show how you do this.

- 14 -

5. Draw the graph for $y=x+1$ from the TABLE Mode. Select TABL to view the table of values. Press [EXIT] to select RANG to view the range. Try to “guess” what this range means. Draw the graph using the G-CON command. Observe carefully. 
6. Change to the STD view-window. What happens to the graph? To compare the effect of changes in the view-window, you may use the Dual Screen facility in the GRAPH Mode so that you can draw two graphs side by side by specifying different values in each view-window that is generated side by side.



Refer to your Manual to see how to draw graph in the GRAPH Mode (page 9) and how to operate in the TABLE Mode (page 13).

7. How is the graph of $y=x+1$ different from the previous two graphs of $y=x$ and $y=x-1$? How are they the same?



Play with the buttons to enable you to get the table of values simultaneously for all three graphs.

8. Predict how the graph of $y=x+3$ would look like. How would $y=x-3$ be? Test your prediction by graphing them on your calculator.

- 15 -

LESSON 8

1. Graphs of $y=x+1$; $y=x+2$; $y=x-1$ belong to the same “family”/ collection.
Can you guess why?



Write down the equations of 3 more lines which belong to the same “family”/ collection?

2. Draw graphs of relationships with $y=\text{number} \times x$. Some examples to start with are $y=2 \times x$, $y=\frac{1}{2} \times x$ and $y=(-1) \times x$. Then predict what the graph of $y=4 \times x$ will look like; graph it to check your prediction.

Begin your investigation with the GRAPH Mode.

Repeat your investigation in the DYNA Mode.

In this mode, specify the parameter

(or in the graphic calculator language,

the dynamic variable) range from -3 to 4

What can you deduce about the value of the number/ parameter in the equation?



Refer to your Manual to see how to draw graph in the GRAPH Mode (page 9) and how to operate in the DYNA Mode (page 15)

What is the common characteristic of this family of graphs?

3. Graphs of $y=2x+2$, $y=3x+2$, $y=8x+2$, $y=-x+2$, $y=-2x+2$, $y=-\frac{1}{2}x+2$ also belong to another type of family. Draw them in the GRAPH Mode.
Write down what you notice about this family of graphs.



- 16 -

Press Zoom and use IN to get a closer look where the lines seem to cut/ intersect.
Write down the coordinate of this point of intersection.



Press OUT to see the graph at a greater distance.



To use ZOOM, press [SHIFT]/ ZOOM. To get a closer look, press IN,. To get a distant view, press OUT

Write down the equations of 5 more lines which belong to the same family.

How is this family of graphs different from the family of graphs in Q.1 and Q.2?

You can change your SET UP to activate the Dual Screen feature of your graphic calculator to get the table of values side by side with your graph. Use the Dual Screen together with the Trace command to help you distinguish between the graphs.



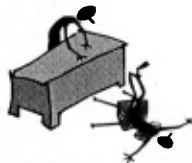
To specify the Dual Screen, press [SHIFT] and (SET UP). Then press cursor down to highlight and select Dual Screen :T+G and press [EXE].

4. What can you say about the gradient of the family of graphs that are parallel to each other?

- 17 -

LESSON 9

1. The Mathematics tuition charges, y , by Mrs. Zaiton is related to the teaching time x by the equation $y=4x + 9$. Copy and complete the table below for the equation $y=4x + 9$



X	0	1	2	3	4	5	6	7
$4x$	0	4						
$4x+9$	9	13						
Y	9	13						

Subsequently, draw the graph of this equation on your graph paper.

Note where the graph intercepts at the y-axis. Write down the coordinate of the y-intercept. Identify this value from the equation of the line.

Note where the graph intercepts the x-axis. Write down the coordinate of the x-intercept. Identify this value from the equation of the line.

Suggest what the intercept represents, that is, why is there a charge when Mrs. Zaiton is not teaching?

Check the graph you have drawn using your calculator. You can draw your graph from the RUN Mode, GRAPH Mode or TABLE Mode. Use the Trace command to check and to verify where the graph intercepts y-axis.



Refer to your Manual on how to draw graph in the RUN Mode (page 7), GRAPH Mode (page 9) and TABLE Mode (page 12)

- 18 -

2. Draw the graph for the equation of $3x+5y=30$ on your graph paper. Determine your own range. You can use the calculator to help you obtain your table of values.
How many points do you need to plot a straight line graph? Why?

What points would you choose to plot a straight line graph? Why?

What is the difference between the intercepts at the x-axis and at the y-axis?

There is infinite number of points passing through the line. Without using your tracer, how would you determine whether a point lies on the line or not?

LESSON 10

1. The equations of several lines are given below. For each one, write down the gradient of the line and give at least two points that pass through each line:

(i) $y=2x$

(ii) $2y=6x + 5$

(iii) $y=-4x + 3$

(iv) $x+y=4$

2. Choose any one graph above and investigate what happens to the graph when you look in different view-windows. Use the Dual Screen facility in the GRAPH Mode to help you. Write a short report on what you observe. Use the Trace to examine the value of the y-intercept in each of the view-window.

3. The gradient of a line is a measure of how steep it is.

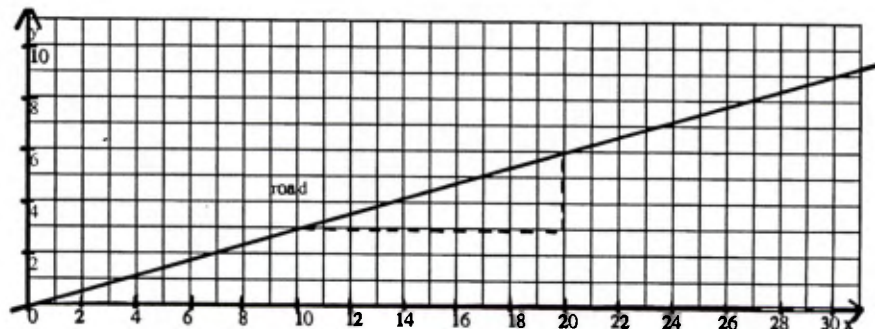
The next activity is to explore how to determine



the gradient of a line.

One way of determining the gradient of the line is from the equation of the line.

A civil engineer makes a survey of a new road to be built from Kelana Road to Subang Road up a hill. She then makes a scale drawing on squared paper to show the road builders how steep to build the road:



The road is represented by a graph of straight line.

- 20 -

What is the equation of the road, that is, the equation of the line y in terms of x ? Show did you determine this equation?

From the equation, what is the gradient of the line?

Choose any two points on the line.

Calculate the increase in y (vertical distance) and calculate the increase in x (horizontal distance)

Then find the value of $\frac{\text{increase in } y}{\text{increase in } x}$

Compare this value to the gradient of the line from the equation of the line you obtain previously. What can you say about this value?

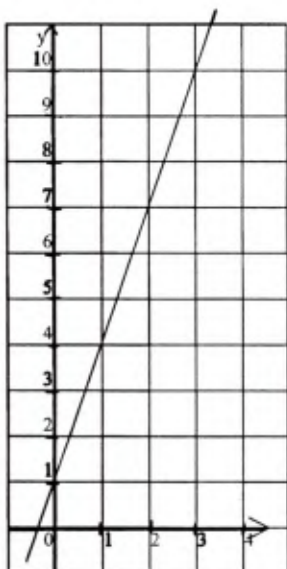
Repeat by choosing any two other points on the line. What do you discover?

How do you determine the gradient of a line that passes through any two points on the line? Explain.

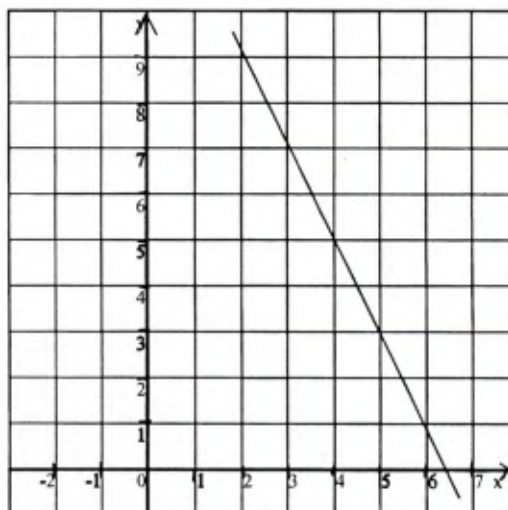
- 21 -

4. Determine the gradient of each of the line given below:

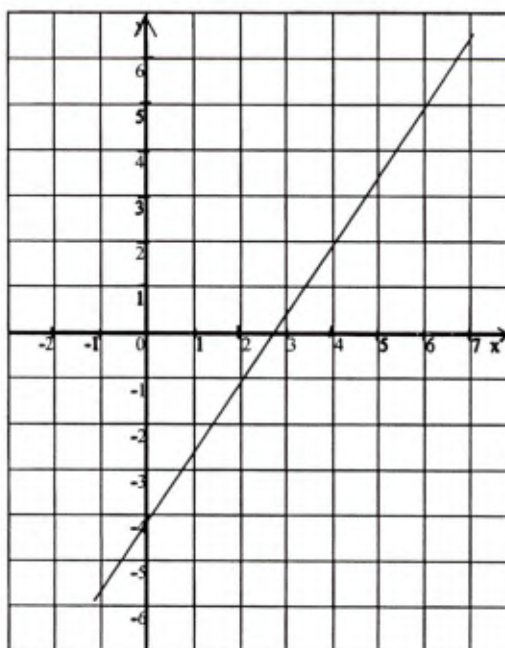
(i)



(ii)



(iii)

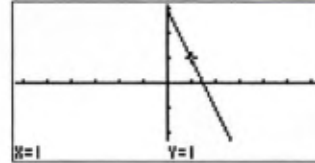
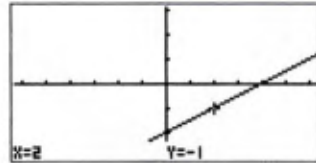


Determine the y intercept of each line.

- 22 -

What is the equation of each of the line?

5. Write the equations of the graphs that have been graphed here on the INIT screen?



6. Find the gradient of the straight line segments joining each pair of given points:

(i) (5, 9) and (7, 17)

(ii) (-1, 5) and (-3, 19)

(iii) (-3, 7) and (2, 2)

(iv) (-2, -2) and (2, 0)


Subsequently, determine the equation of the straight line in each case.

7. A straight line of gradient -5 passes through the point (1, 2). Determine the equation of this straight line.

- 23 -


LESSON 11

1. These pair of lines are parallel to each other: $y = -3x + 2$ and $y = -3x - 5$. Write another equation of the line that is parallel to these lines and pass through the point $(-4, 2)$?



Lines that are parallel to each other have the same gradient. They do not intersect each other.

2. How do the graphs of $y = x + 2$ and $y = -x + 3$ compare with each other? Try some other pairs like this, for example $y = -4x + 1$ and $y = \frac{1}{4}x + 1$. How do you draw these pair of lines? What is the parameter in the equation of the line that determines the characteristic of pairs of lines like this?



m is one of the parameter of the equation of the straight line.

3. How would we know that two lines are perpendicular to each other from the equations of their lines? Investigate a few pairs of lines that are perpendicular to each other. Look at their gradients. How are their gradients related?

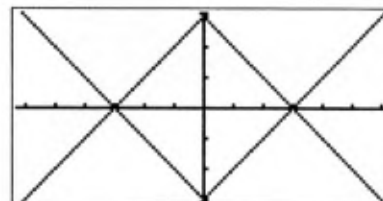
- 24 -

4. Change the view-window so that $x_{\min}=-8$ and $x_{\max}=8$. What effect does this have on parallel and perpendicular pairs of graphs above?



Refer to your Manual
on how to change the
view-window setting
(page 8)

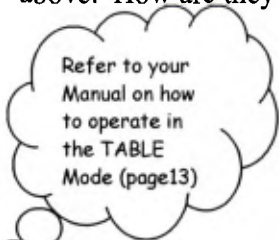

5. Look at this figure. What is it called? Try to draw this quadrilateral on your calculator in the INIT view-window.
What happens to the shape of this quadrilateral when you use the Zoom IN and Zoom OUT command?



What happens when the shape when the view-window changes from $x_{\min}=-8$, $x_{\max}=8$ and $y_{\min}=-8$, $y_{\max}=8$? Explain why.

- 25 -

LESSON 12

1. An asteroid from outside the solar system goes round the sun and back out. Astronomers work out that its path as follows: $y=x^2$. Draw the path of the asteroid by drawing the graph of $y=x^2$ in the GRAPH Mode. Describe the path of the asteroid.
2. Draw the graph of $y=x$ and $y=x^2$ on the same screen. What do you observe about the shape of the graph? What other difference do you observe between the two graphs? Where is the line of symmetry of each graph?
3. Go to the TABLE Mode and observe the table of values for both the equations above. How are they different?


4. How does the view-window affect the shape of the graph of $y=x^2$? Draw a few sketches of the graphs on the different view-windows.

- 26 -

5. Draw the graphs of $y=x^2$, $y=x^2 + 1$, $y=x^2 + 2$, and $y=x^2 + 3$ on the same screen. How are the graphs related? Why are they related in this way? Tabulate all the four equations. How do the values in the table help you to understand the graphs?

Use your observations to predict what the graph of $y=x^2 - 5$ will look like. How is this graph different from the graphs above? Use your calculator to check your prediction.

6. In the DYNA Mode, draw several graphs of $y=ax^2$ by varying the parameter "a". Try a few values of "a" that is negative. Observe what happens to the "steepness" of the graph.

Refer to your Manual on
how to operate in the
DYNA Mode (page 15)



7. Predict how the graph of $y=-x^2 + 3$ will look like. Make a sketch of your prediction and check with your calculator.

- 27 -

LESSON 13



Graphs of quadratic equations in the form of $y=x^2+a$ have a symmetry at the y-axis and the parameter a indicates where the graph cuts the y-axis.

1. How is this different from the graphs of the quadratic equations in the form $y=(x+a)^2$? Draw a few of them with your calculator to include some values of a that is negative. Start with $a=1$. Observe what the line of symmetry is and where the graph cuts the y-axis. Use the tracer to help you. Does this graph cut the x-axis? Start in the GRAPH Mode and extend your work in the DYNA Mode.

You may need to press the cursor keys [▲], [▼] to move the x-axis up and down or the cursor keys [◀], [▶] to move the y-axis to the left or right in order to view where the parabola cuts the y-axis.



2. Predict how the graphs with equation in the form $y=(x+a)^2+c$ will be like. Test your prediction by drawing a few graphs of this family for example the graph of $y=-(x+1)^2+8$. Start in the GRAPH Mode and extend your work in the DYNA Mode. What can you say about this family of parabolas? Is this family of parabolas the same as above? Explain.

You may also need to change the view-window to enable you to see the whole graph. You may find the STD View-window very helpful OR you can use the Zoom feature of your calculator to enable you to see the whole graph.



- 28 -

3. Another family of parabolas has the quadratic equation in the form of $y = x^2 + ax$. What are the graphs in this family have in common? Investigate with your calculator for some values of a including negative values. Start with $x^2 + x$, $x^2 + 2x$, Look at where the parabola cuts the x and y -axes. How are the parabolas of this family different from the parabolas of other families?

4.

Some graphs cross the horizontal axis and some do not.



Give two examples each of equations of the parabola that

- (i) cross the x -axis;
- (ii) do not cross the x -axis;
- (iii) cross the x -axis only once.

- 29 -

LESSON 14

1. In the GRAPH Mode, investigate the shape of the graphs of other equations:

$$y=x^3$$

$$y=x^4$$

$$y=x^5$$

How do the shapes of the graphs compare with each other? Look at the power of the variable x and look out for the line of symmetry, turning points and “pointedness” of the graphs. Predict what the graph of $y=x^6$ would look like. Test your prediction by drawing a graph of it.

Write a report of what you discover.

2. The simplest cubic graph is $y=x^3$.
Use your calculator to draw the following cubic graphs:

$$y=x^3$$

$$y=x(x+2)(x-2)$$

$$y=x(x+3)(x-3)$$

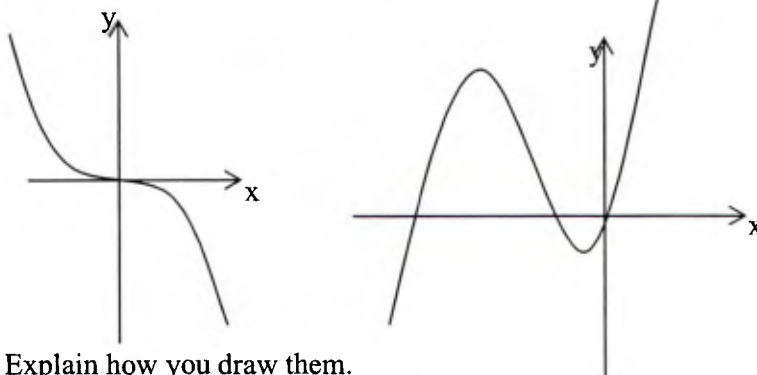


A cubic graph is one whose equation has x^3 as the highest power of x .

What do you notice about the shapes of these graphs? Do they have a line of symmetry? Where do they cut the x -axis and where do they cut the y -axis? Identify these values from the equations of their graphs.

- 30 -

Try to obtain these cubic graphs:



Explain how you draw them.

3. Investigate these graphs in the CONIC Mode.

$$x=y^2 \quad x=2y^2 \quad x=3y^2+7$$

How are they the same and how are they different from each other?



Refer to your Manual
on how to operate in
the CONIC Mode
(page 12)

Look at the general shape of these graphs. How are these graphs different from those of $y=x^2$? How are they similar?

4. Reciprocal graphs are obtained from equations like

$$y = \frac{1}{x}, \quad y = \frac{10}{x}, \quad y = \frac{-2}{x}, \quad \text{etc}$$

Can you determine what is the parameter in each of the equation above?

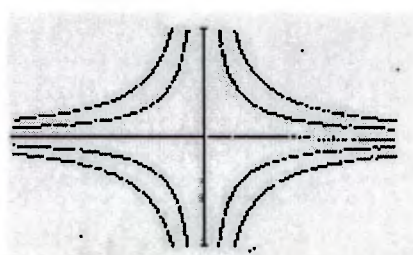
- 31 -

Draw these graphs on your calculator. Begin in the INIT view-window for the first graph. Trace the graph. What do you find about the values of x and y ? Explain what is happening at the ends of each graph.

Draw the second graph on the same screen but Zoom OUT after you have drawn it to enable you to see more of the graph. How does this graph compare with the first one?

Draw the third graph on the same screen. How is this graph different from the other two graphs? Observe in which quadrants they are drawn.

Then try to obtain this screen:



5. This is the equation of the graph of the same family of reciprocal graphs:

$$y = \frac{1}{3+x}$$

Draw this graph in the GRAPH Mode and trace the graph. What happens for $x=-3$?

How does this graph compare to the graph of

$$y = \frac{1}{x}?$$

To key in $y = \frac{1}{3+x}$
Press [1] / [÷] / [(] [x, 0,T] /
[+] / [3] / [)]



LESSON 15

1. Look at your calculator buttons. Explore how the graphs of these would look like:

\sqrt{x}

$\ln x$

x^{-1}

10^x

$\sqrt[3]{x}$

$\log x$

$|x|$

$\sin x$

$\cos x$

$\tan x$



2. Your calculator has several in-built graphs in the DYNA Mode, for example:

$y = Ax + B$

$y = A(x+B)^2 + C$

$y = Ax^2 + Bx + C$

$y = Ax^3 + Bx^2 + Cx + D$

$y = A \sin(Bx + C)$

$y = A \cos(Bx + C)$

$y = A \tan(Bx + C)$

You have explored some of them. Choose a few of them and explore their properties by varying the values of A, B, C and D?

How do these parameters affect your graphs?

Write a report of what you discover about the graphs of the equations you have chosen to explore.

WORKSHEET B

- 1 -

LESSON 1

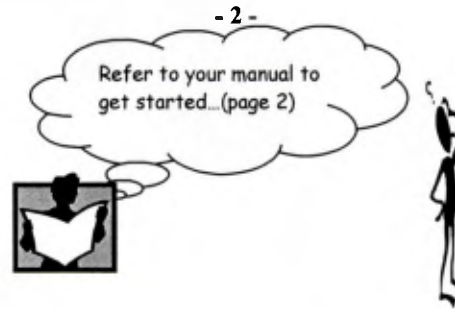
The table shows the rate of growth of a broad bean shoot.



Age in weeks	1	2	3	4	5	6
Height in cm	2	4	6	8	10	12

1. Think of other ways to represent this information.
Discuss with your friend.
2. What are the advantages and disadvantages of presenting your data as points?
3. What can you say about the relationship between the age and the height of the broad bean?
What assumptions do you make to deduce this relationship?
4. Based on the graph, write a report on what can be deduced about the growth of the broad bean shoot.
5. If x is the age in weeks of the broad beans and y is the height in cm of the broad beans, write the relationship between x and y .

LESSON 2



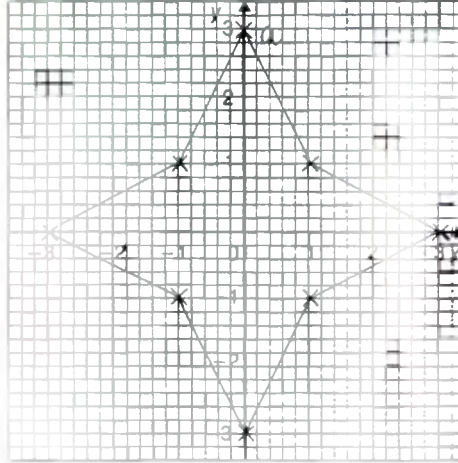
Use graph paper where you think appropriate.

1. Plot the point (1, 2) on the graph paper and the calculator (in the RUN Mode).
2. Once you have plotted the point, plot your own points. Use the cursor keys [▲], [▼], [◀], and [▶] to move the *pointer* (a flashing + sign) around the screen. What do you observe? Write as many things that you observe.
3. Use the SET UP command to insert grids on your screen. Write what you observe.
4. Give your friend a point to plot. Ask him why he thinks the point is positioned as such.

- 3 -

LESSON 3

1. Use the calculator to plot points of any square.
Record your results and compare with a friend.
2. List the coordinates of every corner of the figure, starting at "a" and going clockwise.



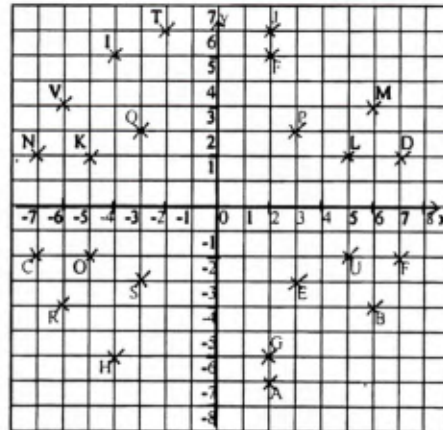
2. Which letter in the diagram opposite is at:
(i) (5, 6)?

(ii) (-5, 2)?

(iii) (-2, 7)?

(iv) (-6, -4)?

(v) (3, -3)?



- 4 -

Choose the points that can be plotted on your graphing calculator and plot them.

Think of how you can plot the other points.




To cancel a point that has been wrongly plotted, press PI-Off/ [EXE]

- 5 -

LESSON 4

1. Plot a point in the INIT view-window. Write as many things as you can about the characteristics of this view-window. You can see the value setting by pressing [SHIFT] (V-Window) followed by selecting the INIT command. It will help if you redraw what you saw on the screen.



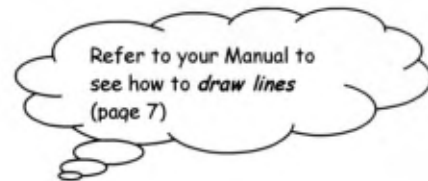
Refer to your Manual to see how to operate the view-window (page 8)
2. Now investigate what happens to the point that you choose if you plot it in another view-window, say, the STD view-window. Discuss with your friend and write down what you both agreed on. It will help if you redraw what you saw on the screen.

- 6 -

3. Name a point that cannot be plotted on the INIT view-window. Why? How can you plot the point?
4. Plot a point where both numbers are integers that is exact on the grid/ markings (dots) of the view-window? Explain how you do this.

- 7 -

LESSON 5



1. Draw a line segment on your calculator and your graph paper. Write down as many things as you can about these lines.




2. Draw another line segment. Write down what you see.

3. Choose any two points to draw a vertical line. Note down a few things that you observe about the vertical line.
How is this line different from the lines you have drawn above?
What do you think the equation of this line would be?

4. Choose any two points to draw a horizontal line. Note down a few things that you observe about the horizontal line.
How is this line different from the other lines you have drawn?
What do you think the equation of this line would be?

- 8 -

5. You can draw the horizontal line and a vertical line on your calculator without plotting the points by selecting either the Vert or Hztl command or by typing in the equation of the line in the GRPH command. Try this. Then show to a friend how you draw some these horizontal and vertical lines. Draw some of these lines on your paper.
6. Use the Trace command to move your *pointer* or *tracer* along the line you have drawn. Notice the x and y values as the *tracer* traces the line. What can you deduce about the points that lie on the line?
What does the equation of the horizontal or vertical line inform you about points on the graph?



In the Trace command, the *pointer* is also known as the *tracer*.

- 9 -

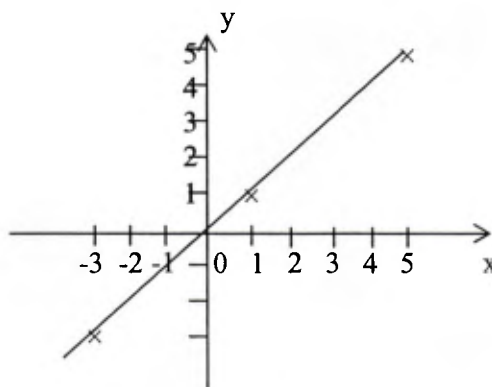
LESSON 6



The INIT V-Window is the initial window setting.
The STD V-Window is the standardised window setting.

1. Draw a line segment that joints any two points on the INIT view-window. Repeat drawing the same line segment on different view-windows. Discuss with a friend how the line has changed when you draw them on different view-windows.

2. Here is a straight line. Try to draw this straight line on your calculator



What can you say about the straight line?

Write a rule or an expression or the equation of the line that expresses the relationship between x and y .

Check your guess by “plugging” in this rule or equation into your calculator in the GRAPH Mode to obtain a graph of it. Then use the Trace command to trace the points along the line to check the your guess. What do the points tell you?



Refer to your Manual to see how to operate in the GRAPH Mode (page 9)

- 10 -

Explore in the TABLE Mode to help you compare some of the points you have traced. Select G-PLT and see what kind of graph is obtained. Observe this graph carefully and note down how the points in this graph is related to the x and y values in your table. Notice that you can draw the graph and the table side by side using the Dual Screen facility in the TABLE Mode. Press RANG to change the range of your x-values to include a wider value of x.

Refer to your Manual to
see how to operate in the
DYNA Mode (page 15)

What do you observe about the points in your graph?



Select the G-CON and observe what kind of graph is obtained. How is this graph different from the graph obtained using the G-PLT command?
Use your *tracer* to find out how the points on this graph is related to the x and y values in your table.
Is it possible to list down all the points on the line?
How can you determine whether a point lies on the line?

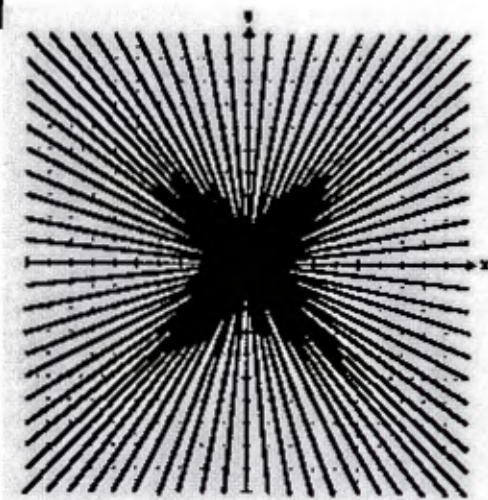
- 11 -

LESSON 7

1. Try to reproduce as close as possible the Starburst picture. You may find it helpful to explore the DYNA Mode before you attempt to draw this figure in the GRAPH Mode. It might also be helpful if you record the lines you have used. What do you notice about the “number” in front of x ? Obtain the Starburst picture.

Refer to your Manual to see how to operate in the DYNA Mode (page 10)

Write a report on how you



2. What are the possible values for the “number” of a line that lies entirely in the shaded region of the figure below? Explain.



LESSON 8

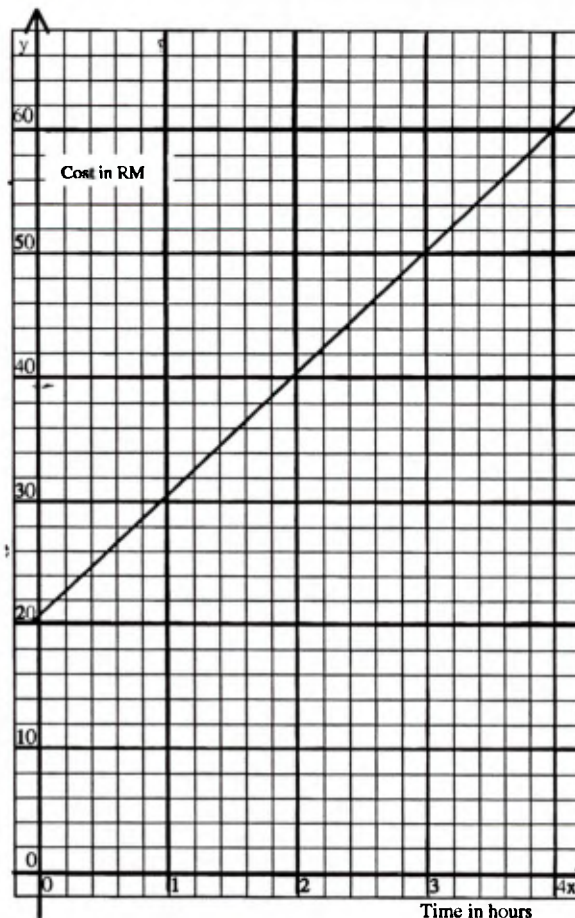


- 12 -

Hassan repairs domestic equipment. He charges call-out fee together with an hourly charge. Customers often ask for an estimate before work is begun. As Hassan does not know how long a job will take before he starts it, he gives this graph to customers to help them get an idea of likely charges.

1. What is Hassan's call-out fee?
2. What does Hassan charge for a repair that takes $2\frac{1}{2}$ hours?

However, some information cannot be read from the graph, for example the cost of a repair taking 6 hours. We can estimate the cost by trying to judge where the line would be if we extend both axes.



We can obtain an estimate of the cost for a repair job that takes very much longer than this or any length of time by working out the relationship between the time of the repair job and its cost given by the graph. This is called **the equation of the line**.

3. What is Hassan's hourly charge rate (the cost of repair per hour)?
How do you obtain this value?
4. Write down the relationship between the cost of repair and the length of work.

- 13 -

5. If x is the time in hours of the repair job and y is the cost in RM, discuss with a friend how you can find the equation for this line. Write down the equation that you both agreed on.
6. From the equation of the line you obtain, determine how long Hassan has to spend on a repair to earn RM200.
7. Compare this graph with your previous graph of $y=x$. How are they the same? How are they different?
8. Write down the coordinate where the line cuts (intercepts) the y -axis.
9. If m is the gradient of a straight line and c is the intercept on the y -axis, write down the equation of this straight line.

- 14 -

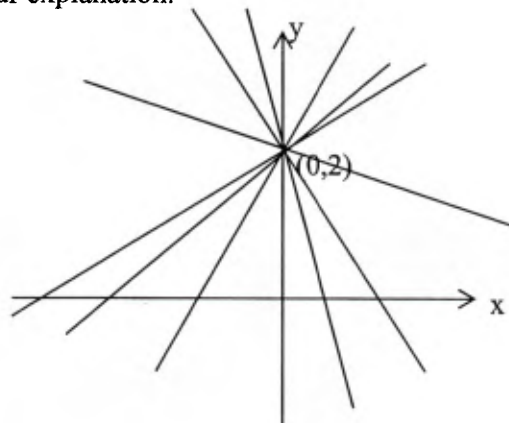
In the equation of the straight line, $y=mx + c$, m is the gradient and c is the y intercept of the line.

LESSON 9

1. Investigate the values of m and c . Write a short report on your investigation. Explore the Dual Screen in the GRAPH Mode to draw two graphs side by side and the Dual Screen in the TABLE Mode to generate the table and the graph on the same screen.



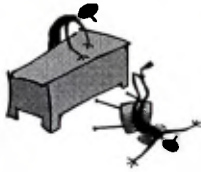
2. Here is a figure that shows some straight lines that intercepts the y -axis. Write the possible equations that characterise the arbitrary chosen lines. Give your explanation.



- 15 -

LESSON 10

1. The Mathematics tuition charges, y , by Mrs. Zaiton is related to the teaching time x by the equation $y=4x + 9$. Copy and complete the table below for the equation $y=4x + 9$



x	0	1	2	3	4	5	6	7
$4x$	0	4						
$4x+9$	9	13						
y	9	13						

Subsequently, draw the graph of this equation on your graph paper.

Mark the coordinate of the y -intercept. Identify this value from the equation of the line.

Suggest what the intercept represents, that is, why is there a charge when Mrs. Zaiton is not teaching?

Check the graph you have drawn using your calculator.

How do you check to verify the coordinate where the graph intercepts the y -axis?

- 16 -

2. Draw the graph for the equation of $3x+5y=30$ on your graph paper. Determine your own range. You can use the calculator to help you obtain your table of values.

How many points do you need to plot a straight line graph?

What points would you choose to plot a straight line graph?

What is the difference between the intercepts at x axis and at y axis?

There is infinite number of points passing through the line. Without using your *tracer*, how would you determine whether a point lies on the line or not?

- 17 -

LESSON 11

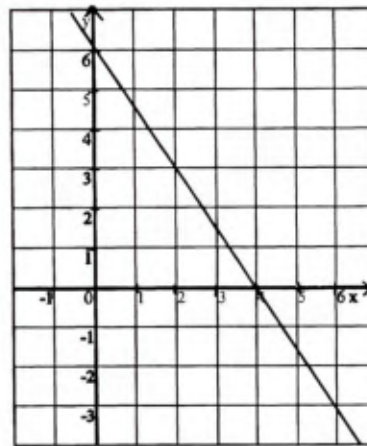
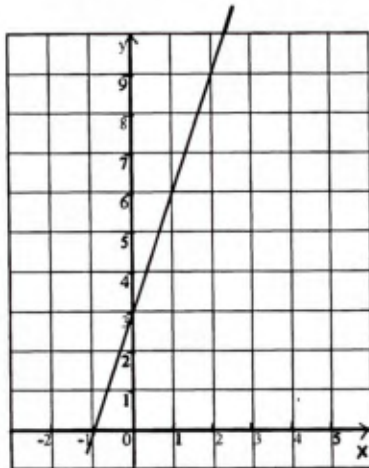
1. $y=x$ and $y=2x$ are two equations of 1 and 2 respectively. Compare the and investigate how you can determine from its graph. Compare this value from the equation of the line.



The gradient is a measure of the "steepness" of the line.

straight lines with gradient steepness of these two lines the gradient of each line with the value obtained

2. The equations of the two line segments below are not given. Extending your knowledge above, how can you determine the gradient of each of the line?



What can you say about the gradient of a straight line passing through any two points on the line?

What would be the gradient of the straight line segment joining the point (5, 9) and (7, 17)?

- 18 -

Can you write a mathematical formula or rule to calculate the gradient of any straight line that pass through two points (a, b) and (c, d)?

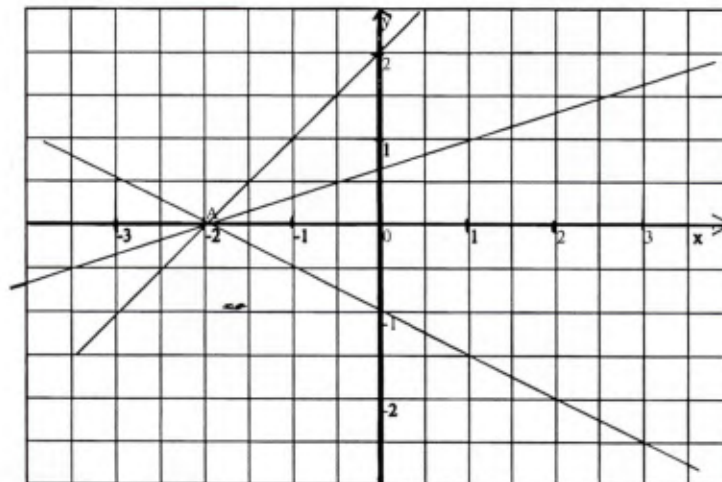
3. Choose any straight line and investigate what happens when it is drawn on different view-windows.

4. Think of some straight lines passing through point A on the graph. Find their equations.



Try some different starting points.

Write a report about what you discover.



If the gradient of the line that passes through A is $-\frac{1}{4}$, what is the equation for this line?

LESSON 12

Which lines are perpendicular to each other?



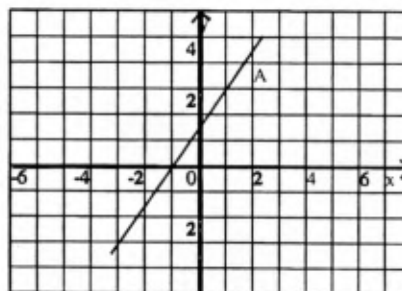
- 19 -



Can you identify a pair of parallel lines?

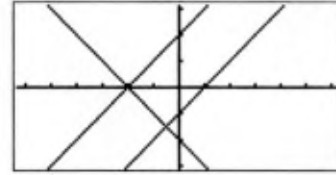
- The graphs you drew previously are all straight-line graphs. They have equations of the form $y=mx+c$, for example in $y=2x+1$, m is 2 and c is 1. Investigate graphs that are parallel to each other and graphs that are perpendicular to each other. Try to find as many “things” or properties about them. You can investigate the features of the graphs using the Trace, Dual Screen and Zoom buttons of your calculator. Investigate how you can determine the m and the c of your graph.

- Check that the equation of line A is $2y = 3x+3$. Explain how you determine this. Investigate the equations of lines which are parallel to A. What is special about their equations?



- 20 -

3. Complete the square. How do you do this?
Zoom in to find the coordinates of the vertices.



What do you notice about the square when
you Zoom IN? Zoom OUT?
Can you explain why?




Refer to your Manual
on how to use the
Zoom (page 18)

Look at the square in different view-windows. What happens? Can you give
an explanation?

- 21 -

LESSON 13

1. What do you notice about this button: 

Guess the shape of the graph.

Draw it using the calculator.

Why does it have this shape?

How does the view-window affect your graph?



2. Investigate the relationship between the graph of $y = x^2$ and the graphs of $y = x^2 + a$ for different values of “a”, including some negative values. Describe the relationship.

Repeat the above for the relationship between the graph of $y = x^2$ and graph of

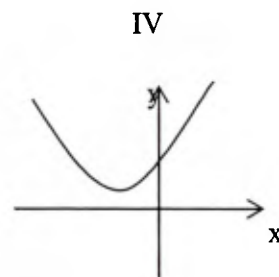
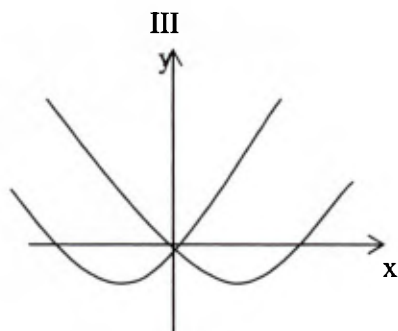
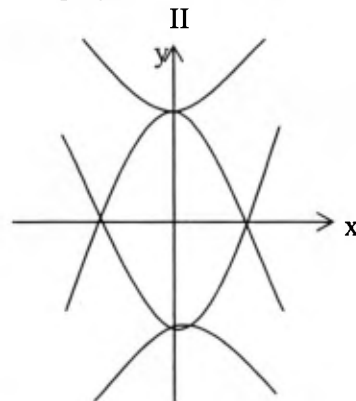
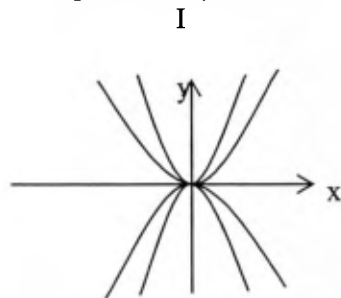
(i) $y = (x+a)^2$ (ii) $y = ax^2$ (iii) $y = x^2 + ax$
for different values of “a” including some negative values.

You can investigate graphs of the same “family” using the DYNA Mode.
Write a report of your investigation.

- 22 -

3. Explore the graphs of quadratic equations in the form of $y = (x+a)^2 + b$. Describe what you observe.

4. Try to obtain each of these displays:
Explain how you obtain each of the display.



- 23 -

LESSON 14

1. Investigate the graphs of other equations:

$$y=x^3$$

$$y=x^4$$

$$y=x^5$$

Predict what the graph of $y=x^6$ would look like. Test your prediction by drawing a graph of it.

Write a report of what you discover.

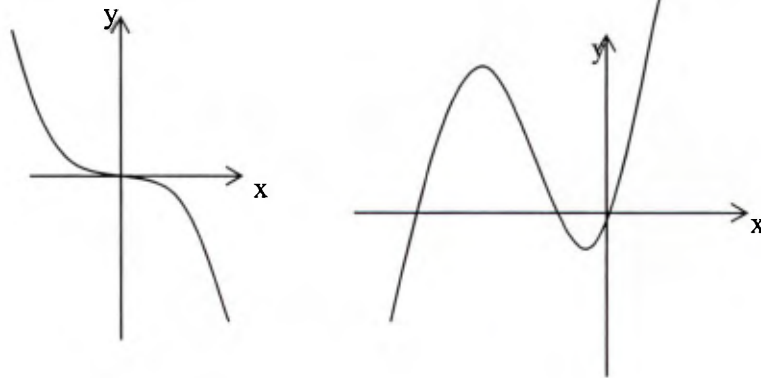
2. The simplest cubic graph is $y=x^3$.
Using your graphing calculator, draw the graphs of
 $y=x^3$
 $y=x(x+2)(x-2)$
 $y=x(x+3)(x-3)$
What do you notice?



A cubic graph is one where x^3 is the highest power of x

- 24 -

Try to obtain these cubic graphs:



Explain how you obtain these graphs.

3. Investigate these graphs in the CONIC Mode.

$$x=y^2 \quad x=2y^2 \quad x=3y^2+7$$

How are they the same and how are they different from each other?

How are these graphs different from those of $y=x^2$?

Explore in the CONIC Mode for more graphs of this family.

Refer to your Manual
on how to operate in
the CONIC Mode
(page 20)

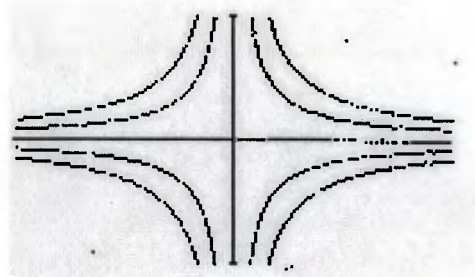


- 25 -

5. Reciprocal graphs are obtained from equations like

$$y = \frac{1}{x}, \quad y = \frac{10}{x}, \quad y = \frac{-2}{x}, \quad \text{etc}$$

Use your graphing calculator to help you investigate reciprocal graphs.
Then try to obtain this screen:

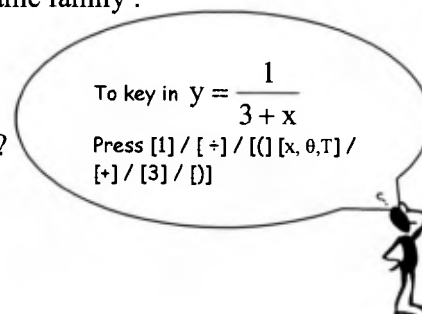


Explain what is happening at the ends of each graph.

This is the equation of the graph of the same family :

$$y = \frac{1}{3+x}$$

Trace the graph. What happens for $x = -3$?



How does this graph compare to the graph of

$$y = \frac{1}{x}?$$

- 26 -

LESSON 15

1. Look at your graphing calculator buttons. Explore how the graphs of these would look like:

\sqrt{x}

$\ln x$

x^{-1}

10^x

$\sqrt[3]{x}$

$\log x$

$|x|$

$\sin x$

$\cos x$

$\tan x$



2. Your graphing calculator has several in-built graphs, for example:

$y = Ax + B$

$y = A(x+B)^2 + C$

$y = Ax^2 + Bx + C$

$y = Ax^3 + Bx^2 + Cx + D$

$y = A \sin(Bx + C)$

$y = A \cos(Bx + C)$

$y = A \tan(Bx + C)$

Investigate their graphs. Choose any one or a few of them.

Write a report of what you discover

Appendix 6 Graphing Calculator Manual

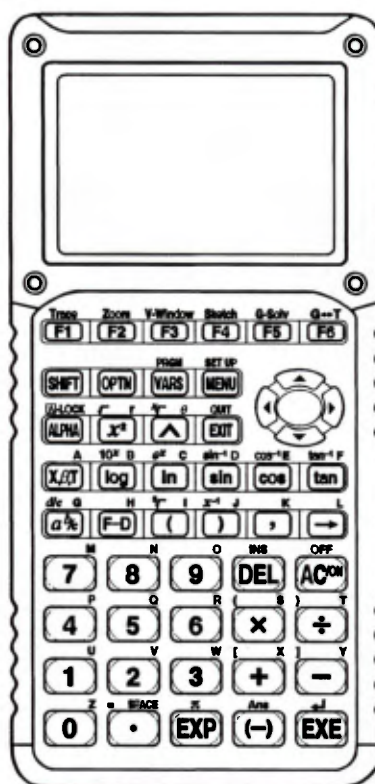
★★★★★★★★★★★★★★★★★★
 ★★★★★★★★★★★★★★★★★★
 ★★★★★★★★★★★★★★★★★★
 ★★★★★★★★★★★★★★★★★★
 ★★★★★★★★★★★★★★★★★★

COLOUR POWER GRAPHIC CALCULATOR

CASIO CFX-9850G/ CFX-9850G PLUS

*Student's Manual*

★★★★★★★★★★★★★★★★★★
 ★★★★★★★★★★★★★★★★★★
 ★★★★★★★★★★★★★★★★★★
 ★★★★★★★★★★★★★★★★★★
 ★★★★★★★★★★★★★★★★★★





-1-
CONTENTS

Topic	Page
GETTING STARTED...	2
EDITING AND ERRORS	2
SYMBOLS	3
SET UP	3
RUN MODE	6
Plotting points	6
Drawing lines	7
Drawing graphs	7
VIEW-WINDOWS	8
View-window memory	8
Storing view-window settings	8
GRAPH MODE	9
Drawing graphs	9
Storing an equation	10
Recalling an equation	10
Deleting an equation	10
Selecting a graph	10
Viewing the graphing region	10
Using the Graph-to-Table facility	11
Using the Dual Graph facility	12
TABLE MODE	13
Generating a table	13
Drawing a graph	13
Generating a table and a graph using the Dual Screen	13
Changing the range values	14
DYNA MODE	15
Drawing a dynamic graph	15
Adjusting the dynamic graph speed	15
Adjusting the animation	17
TRACE AND ZOOM	18
Using the zoom box	19
Using the zoom factor	19
Using the AUTO command	19
CONIC MODE	20



GETTING STARTED...

The following signs are used in this manual:

Commands or symbols found on the calculator keys are put in this bracket : []

Commands or symbols in yellow and red above the calculator keys are in these brackets < >.

Commands that appear on calculator screen are simply written without any brackets or boxes.
The corresponding calculator keys: F1, F2, F3, F4, F5 or F6 are used to execute these commands.

The / is used to indicate to continue the press instruction.

To turn the calculator on: Press [AC^{ON}]

AC stands for All Clear.

To turn the calculator off: Press [SHIFT] / <OFF>

When you turn the calculator on, the screen showing the MAIN MENU appears as shown in the figure.



Figure showing the
RUN Mode is selected

Each small picture or **icon** on the MAIN MENU leads to different calculator capabilities or **mode** and you can select each by using the cursor keys: [▲] [▼] [◀] [▶]

After selection, press the blue key: [EXE]
This blue key is used to execute any command.



Figure of cursor keys

To get back to the MAIN MENU press [MENU]

EDITING AND ERRORS

To correct any typing errors, use the cursor keys [◀] [▶] and press [DEL] to delete, press [SHIFT] / <INS> to insert before pressing [EXE]

To correct any key pressing errors immediately after [EXE] has been pressed, press [◀] or [▶] which will recall the previous command line so that you can modify it.

To clear the entire screen, press the clear key CIs and [AC^{ON}]

If your calculator screen shows **Syn ERROR** this means you have not followed the calculator's syntax or its rules of entering expressions.

To find the syntax error, just press [◀] and a flash will appear on the screen to give hint where the problem lies.



SYMBOLS

These are a few examples of the symbols found in the calculator keys and the calculator's syntax on how to use them:

Calculator Key	Key for	Example	Key to Press	Display on Screen
[X,θ,T]	X	2x	[2] [X,θ,T]	2x
[^]	“power of”	x^3	[X,θ,T] [^] [3]	x^3
[x ²]	“squared”	x^2	[X,θ,T] [x ²]	x^2
[(-)]	Negative sign	-3	[(-)] [3]	-3
[a ^b /c]	Fraction	$\frac{3}{8}$ $2\frac{1}{4}$	[3] [a ^b /c] [8] [2] [a ^b /c] [1] [a ^b /c] [4]	3J8 2 J1 J4
		x	[OPTN] / NUM/ Abs	Abs x
		0.002	[0] [.] [0] [0] [2]	2. E-03

SET UP

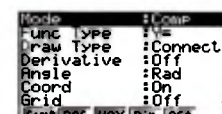
Many calculator commands are not shown on the keyboard. Most of them are contained in **menus**.

In each mode of operation, the calculator can be set up to work in different way.

The status of the setting of the mode you enter is in the set up screen.

You can adjust the setting of each mode you are working in.

Press [SHIFT] / <(SET UP) to display the mode “SET UP” screen. The figure opposite shows part of the SET UP for the RUN Mode. Use the [▼] and [▲] cursor keys to move the highlighting to the item whose setting you want to see or change. Press the corresponding function key [F1], [F2], [F3], [F4] or [F5] at the bottom of the calculator screen. Then press [EXIT] to return to the default screen.



[F1][F2][F3][F4][F5]

Figure of SET UP screen in the RUN Mode

The items in the set up may vary in different modes. Below are some of the items found in the set up and some of the corresponding options on what can be changed in the setting which is displayed at the bottom of the screen:

Func Type

Y= Rectangular Coordinate Graphs

X=C Graphs in which value of X is constant

Draw Type

Con Connection of points plotted on graph

Plot Plotting of points of graphs without connection



- 4 -

Coord

- On Turns on display of coordinates of current graph screen pointer location
- Off Turns Off display of coordinates of current graph screen pointer location

Axes

- On Turns on display of graph screen axes
- Off Turns off display of graph screen axes

Label

- On Turns on display of graph screen axis labels (of x and y)
- Off Turns off display of graph screen axis labels

Graph Func

- On Turns on display of function (equation) during graph drawing and trace
- Off Turns off display of function during graph drawing and trace

Dual Screen

This differs depending upon whether you are using the GRAPH Mode set up or the TABLE Mode set up.

GRAPH Mode

- Grph Divides screen into two parts, each of which can be used for graphing
- G to T Divides screen into two parts for generation table from graph
- Off Dual Screen off

TABLE Mode

- T+G Divides screen into two parts, one for graphing and one for table
- Off Dual Screen off

Simul Graph (Simultaneous Graph)

- On Turns on simultaneous graphing of all functions (equations) in memory
- Off Simultaneous graphing off (graphs drawn one by one)

Dynamic Type

- Cnt Continuous drawing of Dynamic Graphs
- Stop Automatic stopping of Dynamic Graph drawing after 10 draws

Variable

- Rang Table generation and graph drawing using table range

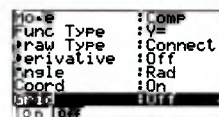


- 5 -

Example: To change setting in the SET UP

To Insert Grid

Press [SHIFT] / <SET UP>
 Press cursor key down [▼] until you see Grid : Off
 Press “On” (function key [F1]) if you want the grids
 (with dots corresponding to tick marks) to be placed
 on the screen.



[F1]
 (SET UP in RUN Mode)

To Delete Label

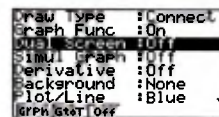
Press [SHIFT] / <SET UP>.
 Press cursor key down [▼] until you see Label : Off
 Press “Off” (function key [F2]) if you want the axes labels
 (i.e., x and y) not to be shown.



[F2]
 (SET UP in GRAPH Mode)

To Insert Dual Screen

Press [SHIFT] / <SET UP>.
 Press cursor key down [▼] until you see
 Dual Screen : Off
 Press GRPH Or GtoT if you want the screen to be divided
 in half with either a pair of graphs or a graph and a table
 showing side by side respectively.



(SET UP in GRAPH Mode)



- 6 -

RUN MODE

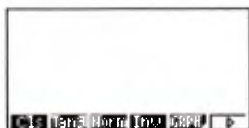


On the MAIN MENU, select the RUN icon and enter the RUN Mode by pressing [EXE].

Many calculator commands are not shown on the keyboard, they are contained in **menus**.

Press [SHIFT] / <Sketch> to get into the sketching menu of the calculator.

Press \triangleright (function key [F6]) to see more of the menu.



[F6]

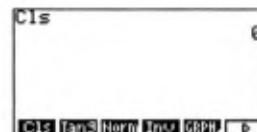
Among the menus in the Sketch menu that you might use:

Cls	Clears drawn line and screen
GRPH	Graph command menu
\triangleright	Next menu or Previous menu
LINE	Line menu
PLOT	Plot menu
Vert	Vertical line
Hztl	Horizontal line

To **clear screen**, press Cls and [EXE].

The screen will display a \emptyset to tell that it has finished.

Press [EXIT] to **return to the screen** you are working in.



Plotting Points

Press [SHIFT].

Press <Sketch> / \triangleright / PLOT (F1).

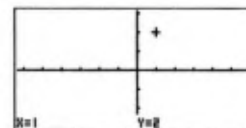
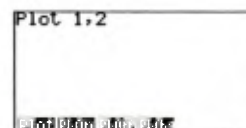
Press Plot again and the word Plot appears on the screen.

Press [2] [,] [1] to plot the points (2,1).

Note that the calculator does not use brackets.

Press [EXE] to finish plotting the point. The cursor, a plus sign + is shown on the screen or as close as it can get to it.

The coordinates are shown at the bottom of the screen.



To **delete a point** that has been mistakenly plotted, for example the point (2,1), press Pl-Off followed with [2] [,] [1] [EXE].

To **return to the screen**, press [SHIFT] <QUIT> **OR** **clear the screen** by pressing [AC^{ON}].



- 7 -

Drawing Lines

The calculator allows you to join points to draw horizontal lines, vertical lines and line segments.

To Draw Line Segment:

Press [SHIFT] / <Sketch> / ▸ / PLOT / Plot .

Press [3] [,] [2] to plot the points (3,2).

Press [EXE].

Press G⁴T to get back to the “Plot” screen.

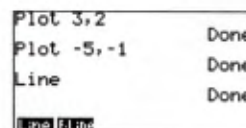
Press Plot.

Press [-5] [,] [1] to plot the points (-5,1).

Press [EXE] / G⁴T / [SHIFT] [Sketch] / ▸ / Line.

Press Line again.

Press [EXE].



OR ANOTHER WAY of drawing line is to use the F-Line command:

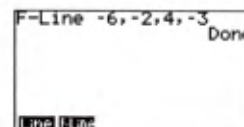
Press Line and F-Line.

Press [-] [6] [,] [-] [2] [,] [-5] [,] [1]

(to draw line segment joining the points (-6,-2) and (-5,1)).

(Note that this command specifies the four coordinates of the two end points in order separated by commas).

Press [EXE].



To Draw Horizontal Line and Vertical Line

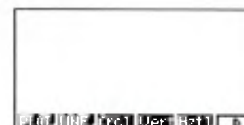
Press [SHIFT] <Sketch>

Press ▸

Press Vert [F4] for Vertical line (OR Press Hztl [F5] for Horizontal line)

Press 3 for a vertical line drawn at x=3

Press [EXE]



[F4] [F5]

To Draw Graph

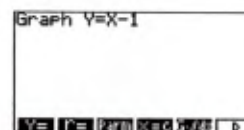
Press [SHIFT] <Sketch> / GRPH

Press “Y =” [F1] and the expression

“Graph Y=” appears on the screen

To graph $y=x-1$: Press [X,θ,T] (to key in x) [-] [1]

Press [EXE]



[F1]



VIEW-WINDOW

The graphic screen shows only part of the coordinate plane. You need to tell the calculator which part of the coordinate plane to use called the view-window. Use the view-window specify the range of x and y-axes and to set the spacing between the increments on each axis.

To **see or alter the current view-window setting**, press [SHIFT] <V-Window>. Then use the [▼] and [▲] cursor keys to highlight the setting that you want to alter.

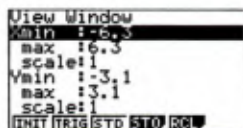
The settings will be whatever they last were as the calculator keeps them the same until they are changed.

At the bottom of the view-window you will find some of the following menu:

- INIT View-window initialized setting (default window setting)
- STD Standardized view-window setting
- STO Store view-window settings to view-window Memory
- RCL Recall view-window settings from view-window Memory

The INIT View-window automatically chooses the initial settings:

Xmin : -6.3
 max : 6.3
 scale : 1
 Ymin : -3.1
 max : 3.1
 scale : 1

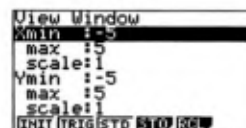


View-window Memory

You can store up to **six** sets of view-window settings in view-window memory for recall when you need them.

To **save or store view-window settings**

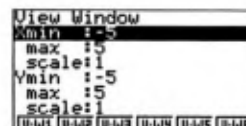
Example Xmin = -5 Ymin = -5
 max = 5 max = 5
 scale = 1 scale = 1



After keying in the desired setting, press STO / V-W1 / [EXIT]

This view-window setting is stored in V-W1.

To **recall this view-window**, press RCL





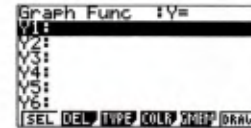
- 9 -

GRAPH MODE

Use this mode to store the equation of graph and to draw the graph.

On the MAIN MENU, select the GRAPH icon and press [EXE] to enter the Graph Mode. The screen will show the expression “Graph Func : Y=” and at the bottom of the screen it will show some of the following menu:

SEL	Draw/ non draw status
DEL	Graph delete
TYPE	Graph Type menu
COLR	Graph colour
GMEM	Graph memory save/ recall
DRAW	Draws graph



[F1] [F2] [F3] [F4] [F5] [F6]

You can store up to 20 “functions” (equations) in memory. They can be edited, recalled and graphed.

In the Graph Type menu, several types of “functions” (equations) can be graphed. In this lesson, we are concerned only with “Y=” (rectangular coordinate graph) and “X=c”(constant graph) types.

To Draw Graph

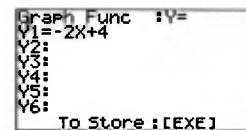
Example:

To draw graph of $y=-2x+4$

Type in the equation of the graph at Y1=

Press [(-)] [2] [X,θ,T] [+] [4]

Press DRAW



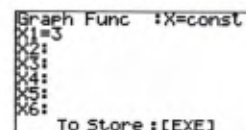
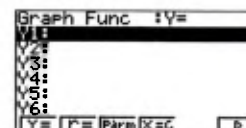
To draw the graph of $x=3$

Press TYPE on the default screen.

Press X=C ([F4])

At “X=1” press [3]

Press [EXE]

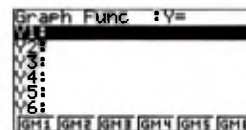
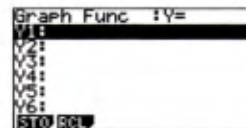




- 10 -

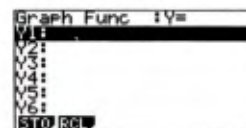
To Store Equations

As soon as you key in an equation on one of the “Y”, and press [EXE], the equation will be stored on the screen area. To store it in the memory area, after pressing [EXE]: press GMEM and press STO. Choose one of the six memory areas: GM1, GM2, GM3, GM4, GM5, or GM6. (Storing an equation in a memory area that already contains an equation replaces the existing equation with the new one) Press the corresponding key of your choice. Press [EXIT] to return to the default screen.



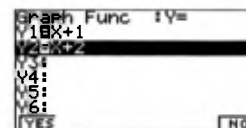
To Recall Equations

To recall the equation kept in the memory:
Press GMEM.
Press RCL.
Press the memory are where the equation is kept.



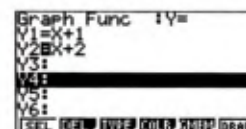
To Delete Equations

On the “Graph Func” screen, choose the equation to be deleted and highlight the equation.
Press DEL.
Press YES.
(If you change your mind and still require the equation, press NO.)



To Select the Graph to draw

If you want only a specific graph/graphs to be displayed on the screen, press the SEL command to choose the graphs. Highlight the equation and press SEL again. The highlight is shown on the “=” sign of the equation indicating that the corresponding graph will be displayed on the screen. The unhighlighted “=” will not display the corresponding graph. You sometimes have to press the SEL command once or twice to obtain the desired output.



To see the Graphing Region

Pressing any of these cursor keys [▲], [▼], [◀] or [▶] allows you to scroll up or bottom and to the left or right of your screen to see different sections of your graph.



- 11 -

Using Graph-to-Table facility

You can split the screen to two sections, the left displaying the graph and the right displaying the table side by side by using the Dual Screen facility and selecting the Graph-to-table (GtoT) command in the SET UP. You can also move the tracer in the graph and store the coordinate value you want in your table.

Press [SHIFT] <[SET UP].

Change the setting in the Dual Screen.

Choose the command GtoT by pressing the function key [F2].

Press [EXIT] and the following screen will be shown.



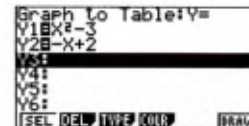
[F2]



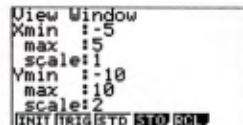
[F1] [F2] [F3] [F4] [F6]

Example:

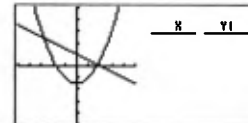
Key the two equations: $Y1=x^2-3$
 $Y2=-x+2$



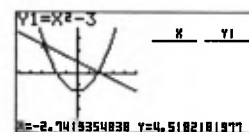
Choose the following view-window settings:



Press DRAW and the following graphs will be displayed.

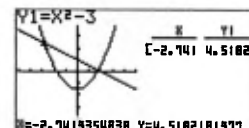


Press [SHIFT] <[TRACE] to trace the points of intersection. The points of intersection is displayed at the bottom of the screen.



Press [EXE] to store the coordinates.

The coordinate values are stored in the table.





Using the Dual Graph facility

You can split the screen to two sections with (i) two of the same graph displayed side by side in different view-windows or (ii) two or more graphs displayed side by side in the same view-window by using the Dual screen facility and selecting the Graph (GrPh) command in the SET UP. The left screen is the active screen and the graphs generated are active graphs. The right screen is the **inactive screen** and the graphs generated are inactive graphs.

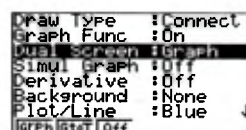
Drawing two graphs side by side in different view-windows

Press [SHIFT] <SET UP>.

Change the setting in the Dual Screen.

Choose the command GrPh by pressing the function key [F1].

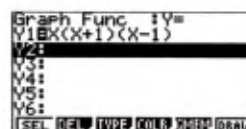
Press [EXIT] to return to the default screen.



[F1]

Example: Drawing the graph of $x(x+1)(x-1)$

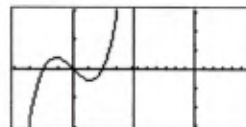
Key in the equation for the graph.



[F6]

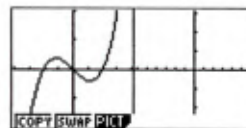
Use the following view-window settings (see below).

Press the DRAW command and the following graph is generated in the active screen (left).



Press the [OPTN] key and the following menu will be displayed on the screen:

COPY copies active graph to inactive screen
SWAP switches active screen and inactive screen
PICT picture function

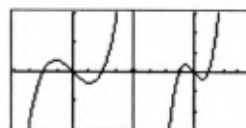


[F1] [F2] [F3]

To copy the active graph to the inactive screen:

Press COPY. The same graph is generated on the left.

Press [EXIT] and the default screen will show B on the left of the equation to show that the same graph is drawn on both screens.



To draw the graph on the left screen only:

Press SWAP. The graph is shifted to the left screen.

Press [EXIT] and the default screen will show R on the left of the equation to show that the same graph is drawn on the screen on the right

To change the view-window setting

You can set different view-window parameters on both screens:

press [SHIFT] <V-WINDOW>.

Use these parameters for the above examples.

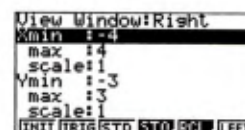
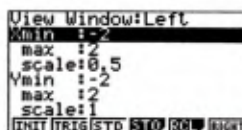


TABLE MODE

On the MAIN MENU, select the TABLE icon and press [EXE] to enter the TABLE Mode. The screen will show the expression "Table Func : Y=" and at the bottom of the screen it will show some of the following menu:

SEL Table generation/ non-generation status
DEL "Function" (equation) delete
TYPE "Function" (equation) type specification
COLR Graph colour specification
RANGE Table range specification screen
TABL Start table generation



To Generate a Table

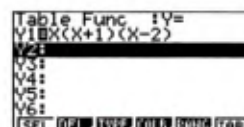
Example: $y=x(x+1)(x-2)$

Key in the equation at Y1:

press $[X,0,T] [() [X,0,T] [+] [1] [)] [X,0,T] [-] [2]$

Press [EXE] to store the equation.

Press [TABL] to view the table of values of x and y.



X	Y1
1	-2
2	0
3	12
4	40

To Draw the Graph

Press [G-CON] for a continuous graph or [G-PLT] for a discrete plot of the table values.

Press [EXIT] twice to return to the main screen.

To generate Table and Graph using the Dual Screen

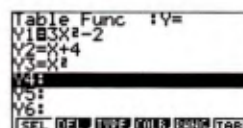
Press [SHIFT] <[SET UP].

Choose Dual Screen: T+G.

Press [EXIT] to return to the main screen.



Press SEL to choose the equation for the table and graph to be generated. In this example the equation chosen is $y=3x^2-2$.

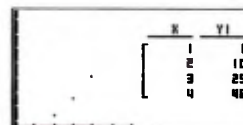


Press TABL and the table is generated.

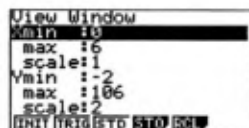
X	Y1
1	1
2	10
3	25
4	46

Note: The table is generated for the range values of x from 1 to 4.

Press G-PLOT (or G-CON according to the type of graph you want) and the following graph is produced.



Note: The graph generated for the range values of x from 1 to 4 with the following view-window parameters:



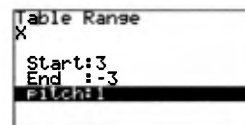


- 14 -

To change the Range values

Press RANG to change the values that are tabulated.

Change the first (Start) and last (End) values for x and the increment (Pitch), for example, to generate a table with value of x changing from -3 to 3 in increments of 0.5, at



Start: Press [(-)] [3]

End: press [3]

Pitch: press [0] / [.] / [5]

Press TABL or [EXE] to generate the table.

If you change the pitch to 0.5, look what happens to the values in your table!

DYNA MODE

The DYNA Mode is used to for dynamic graphing, that is to display graphs in quick succession giving an effect of animation. Only one “family” of equations/ graphs can be defined at any one time.

To enter the dynamic graphing mode, select the DYNA icon and press [EXE] after selection. The screen will immediately show “Dynamic Function: Y= “ and several menus. Among the menus are:

SEL	Graph draw/ non draw status
DEL	“Function” (equation) delete
TYPE	“Function” type specification
VAR	Dynamic variable menu
B.IN	Menu of built-in “functions”
RCL	Recall and execution of Dynamic Graph conditions and screen data



[F1] [F2] [F3] [F4] [F5] [F6]

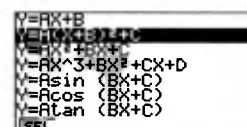
To Draw Dynamic Graph

Example 1: To draw the dynamic graph with the equation $y=A(X-1)^2-1$

Press the B-IN command on the main screen and highlight the equation $Y=A(X+B)^2+C$.

Press SEL.

Press [EXE] or press [VAR].



[F1]



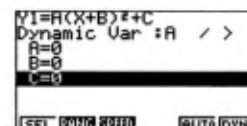
[F4]

The next screen will display Dynamic Var :A /> with the following menu at the bottom of the screen:

SEL	Selects dynamic variable
RANG	Dynamic variable (parameter) range Setting
SPEED	Dynamic graph drawing speed (indicated by > , >, >, »)
AUTO	Automatic setting
DYNA	Dynamic graph draw operation

The calculator calls “A” in the above example a dynamic variable.

“A” is the parameter that will control the **animation**.

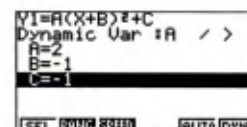


[F1] [F2] [F3] [F5] [F6]

Specify the value of the dynamic variable.

Press [EXE] after keying in the value.

In this example, press at A: [2] [EXE]
B: [(-)] [1] [EXE]
C: [(-)] [1] [EXE]



[F2]



- 16 -

If you want to change the value of the dynamic variable, highlight the corresponding row of the dynamic variable that you want to change in the VAR menu and press SEL. In the example above, the dynamic variable is A.

Specify the range values for the dynamic variable by pressing RANG.

For the start value, Start: press [2] and [EXE].

For the end value, End: press [5] and [EXE].

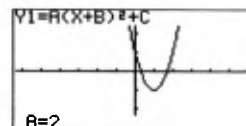
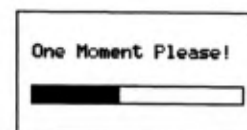
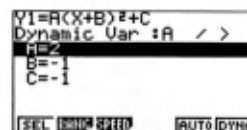
For the increment value, Pitch: press [1] and [EXE].

(The range value that is chosen above will generate dynamic graphs of $y=A(x-1)^2-1$ for $A=2, 3, 4$, and 5 . Note that the pitch value is 1 . If the pitch value is 0.5 , the dynamic graphs that are generated are for $A=2, 2.5, 3, 3.5, 4, 4.5$, and 5)

The screen will return to the previous screen with the VAR parameters.

To generate the graph, press [DYNA]. The calculator will ask you to wait while it constructs all the graphs and then display them in the way you want.

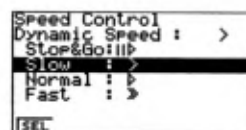
The graphs that generated will begin with $A=2$, $A=3$, $A=4$, $A=5$ and repeated with $A=4$, $A=3$, $A=2$ and $A=2$ and continues.



To Control the Dynamic Graph Speed

If you want to change the Dynamic Graph speed, press the command SPEED. The screen for the speed menu is outlined as follows:

Stop&Go:	Each step of the Dynamic Graph draw operation is performed each time you press [EXE]
Slow:	Half of the normal speed
Normal:	Default speed
Fast:	Twice the normal speed



[F1]

Use the cursor keys, [▲] or [▼] to move the highlighting to the speed you want to use.

Then, press SEL to set the highlighted speed.

Press [EXIT] to return to the "Dynamic Variable" screen.

Press DYNA to generate the graph.

To Return to the Previous Screen

Press the [EXIT] to return to the previous screen. You may need to press the [EXIT] key a few times to obtain the "Dynamic Variable" screen to generate the dynamic graph.

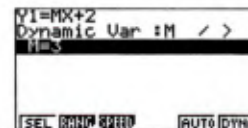


Example 2: To draw the dynamic graph for the equation $y=Mx+2$

Key in the equation at $Y1=$: Press [ALPHA] [M][x,θ,T] [+] [2]

Press [EXE] / VAR

The screen following screen will appear.



The calculator names “M” as the “dynamic variable”.

“M” is the parameter that controls the animation.

Press RANG to specify the range values for M.

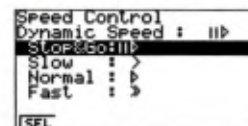
Press [EXIT].

Press either AUTO or DYNA or choose the type of animation that you want.

OR another way to draw the graph of $Y=Mx+2$ is to press the B-IN command and choose $Y=AX+B$ by pressing SEL. Key in the value $B=2$ in the “Dynamic Variable” screen.

To Control the Animation

1. To draw the dynamic graph one by one,
Choose the Stop&Go command from the
SPEED menu.
Press SEL / [EXE] / DYNA.
Each time you press [EXE], a graph will be
generated.
2. To draw only 10 dynamic graphs, change the
setting in the SET UP.
Press [SHIFT] <[SET UP]>.
Choose the StoP command.
Only 10 versions of the graph are drawn and
then the draw operation stops automatically.
3. To continuously draw the graph, change the
setting in the SET UP.
Press [SHIFT] <[SET UP]>.
Choose the Cnt command.
The graphing operation is repeated until you
press the [AC] key.



[F1]



[F2]



Be sure that you do not forget to stop the dynamic operation after you are finished. Allowing it to continue will run down the batteries!

4. Pressing the AUTO command draws up to 11 versions of the dynamic graph,
starting from the start value of the dynamic coefficient.



TRACE AND ZOOM

Tracing a graph is like running your finger along it and seeing the coordinates of each point that has been plotted.

To **activate** the trace function press [SHIFT] and <Trace>.

Note that the trace function can be activated only after a point or line or graph has been plotted or drawn.

To **trace the graph**, use the cursor keys: [◀] or [▶].

To **shift the tracer** between the graphs, use the cursor keys: [▲] or [▼].

Points traced are shown on a graph with a flashing plus sign (+). This is usually referred as a *pointer* or a *tracer*.

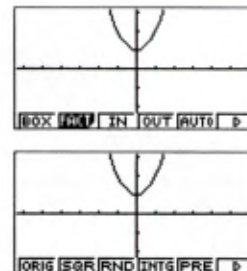
To deactivate the trace function, prece [SHIFT] and <Trace>.

Do not press the [AC^{ON}] key when you are operating the trace function.

Zooming allows you to change the view-window very quickly with having to change each of the Xmin, Xmax, Ymin and Ymax values seperately.

Press [SHIFT] <Zoom> to access the zoom menu. Some of this menu appears at the bottom of the screen:

BOX	To define a new screen by making a rectangular box
FACT	Factors for zooming in and out
IN	To zoom in
OUT	To zoom out
AUTO	Automatic scaling zoom
ORIG	To zoom back to the original viewing window
SQR	“Squares up” the axes by giving the same scale to each
INTG	Converts view-window x-axis and y-axis values to integers
PRE	After zoom operation, returns view-window parameters to previous settings



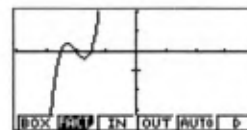
Press IN to zoom in from the current cursor location. Zooming in is a bit like using a magnifying glass to look more closely at some parts of the graph. Note that the “thickness” of the graphs and the axes do not change!

Press OUT to zoom out from the current location. Zooming out is looking at graphs from further away so that you can see more of them. This is helpful if you want to get a better idea of the overall shape of the graph



To Use Box Zoom

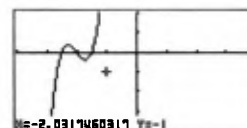
After graphing the equation, press [SHIFT] <Zoom> to access the zoom menu.



[F1]

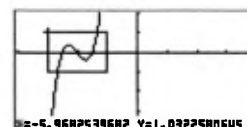
Press BOX and use the cursor key to move the pointer to the location of one of the corners of the box you want to draw on the screen.

Press [EXE] to specify the location of the corner.

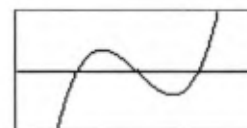


Use the cursor keys to move the pointer to the location of the corner that is diagonally across from the first corner.

Press [EXE] to specify the location of the second corner. When you do, the part of the graph inside the box is immediately enlarged so it fills the entire screen.

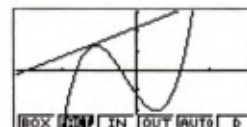


To return to the original graph, press [SHIFT] <Zoom> ▷ ORIG.



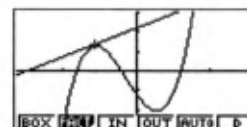
To Use Factor Zoom

With factor zoom, you can zoom in or zoom out on the display, with the current pointer location being at the centre of the new display. The example shown illustrates the use of the factor zoom to determine whether or not the two graphs are tangential.



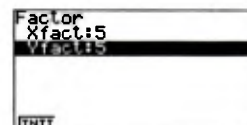
After graphing the equations, press [SHIFT] <Zoom>.

Use the cursor keys to move the pointer to the location that you want to be the centre of the new display.



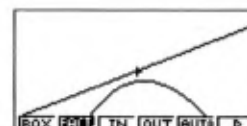
Press the FACT command.

If you want the screen to enlarge 5 times at the x-axis, and 5 times at the y-axis, press [5] [EXE] [5] [EXE].



Press [EXIT] to return to the graphs.

Press the IN command to enlarge the graphs or press the OUT command to reduce the size of the graphs. You can repeat the factor zoom procedure more than once to further enlarge or reduce the graph.



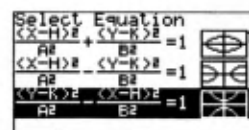
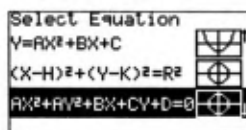
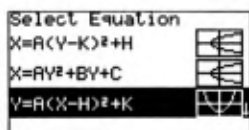
To Use the AUTO Command

The AUTO command automatically adjusts y-range view-window values so that the graph fills the screen along the y-axis. After graphing the equations, press [SHIFT] <Zoom>. Then, press the AUTO command.

CONIC MODE

On the MAIN MENU, select the CONIC icon and enter the CONIC Mode by pressing [EXE].

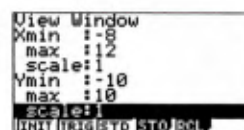
The screen on the right shows the general shape of the graph for the equation. Select one of the nine equations by pressing [EXE] when it is highlighted.



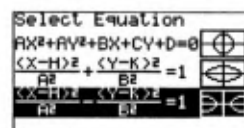
Example:

To graph of $\frac{(X-3)^2}{2^2} - \frac{(Y-1)^2}{2^2} = 1$

Use the following view-window parameters.



Select the equation whose graph you want to draw.



Press [EXE] and the “variable” input screen appears.

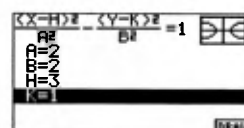
Assign values to each of the “variable” by pressing

at A: [2] [EXE]

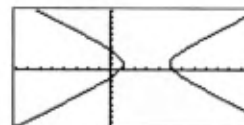
B: [2] [EXE]

H: [3] [EXE]

K: [1] [EXE]



Press the DRAW command to draw the graph..





- 21 -

To graph $x=y^2$

Select the equation $X=A(Y-K)^2 + H$.

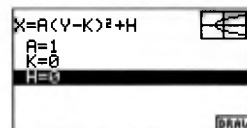
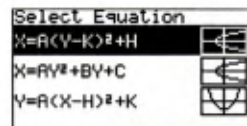
Press [EXE].

Enter the values of the parameters, in this case $A=1$, $K=0$ and $H=0$ so that the equation is $x=y^2$.

Press at A: [1] [EXE]

 K: [0] [EXE]

 H: [0] [EXE]



Press DRAW to draw the graph of $x=y^2$.

To graph $y=x^2 + 2$.

Select either the equation $Y=A(X-H)^2 + K$ or $Y=AX^2 + BX + C$ and enter the appropriate values of the parameters. If you select $Y=A(X-H)^2 + K$, then $A=1$, $H=-2$ and $K=0$.

Press at A: [1] [EXE]

 K: [0] [EXE]

 H: [(-)] [2] [EXE]

Press DRAW to draw the graph of $y=x^2 + 2$.

Appendix 7 Assessment 1

Aim:

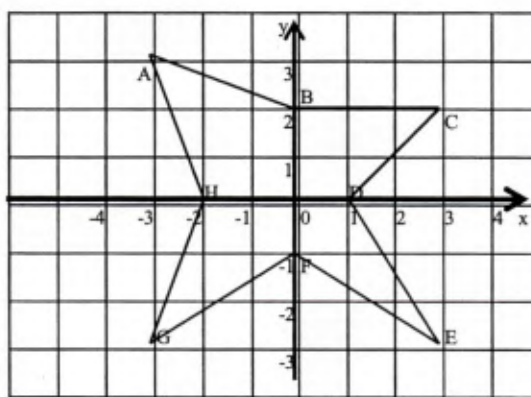
To find out whether student

- (i) can plot points correctly on the graphic calculator including choosing the appropriate view-window.
- (ii) can name given points.
- (iii) can plot points on the graph paper.
- (iv) can name the x and y axes.
- (v) recognise the origin.
- (vi) understand the concept of horizontal and vertical line.

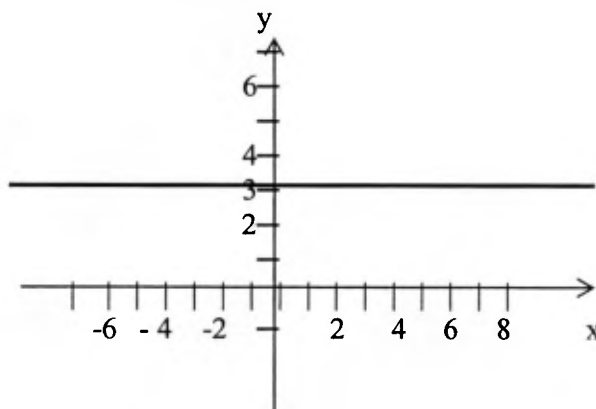
1. Plot the following points on your graphing calculator :

$(-1, 2)$
 $(-8, -3)$

2. Name the coordinates of the each of the vertex of figure below.



3. Draw the triangle with the following points on your graph paper: $(3, 6)$, $(-1, -1)$, $(6, -2)$. Name the axes and mark the origin. Draw the triangle on your calculator.
4. Write down any two coordinates that lie on the line.



What do you think is the equation of the line?

5. Give at least 3 equations of straight lines that are parallel to y axis.

Appendix 8 Assessment 2

Aim:

To find out whether students

- (i) understand the concept of $y=mx$.
- (ii) able to make the connection between the value of m and the steepness of the graph.

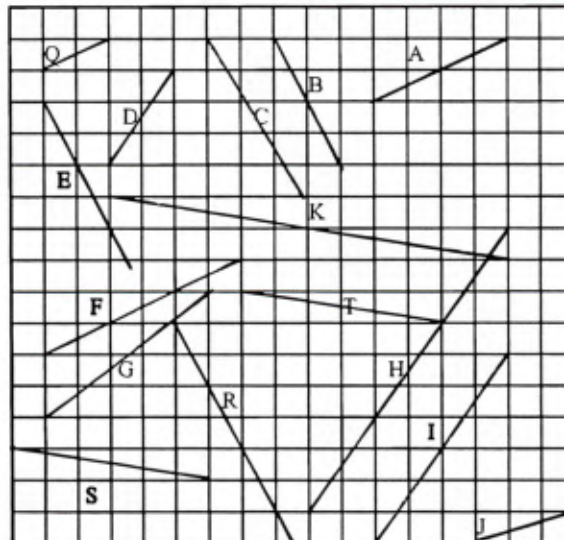
1. Write down a few equations of straight lines that pass through the origin?
2. Here are three lines with the equations $y=5x$, $y=3x$ and $y=x$. If you want to draw the line $y=2.5x$, between which lines would it go? Illustrate your answer with a sketch and give an explanation.
3. Which of the lines have the same gradient as

(i) line Q?

(ii) line R?

(iii) line T?

(iv) line S?



Appendix 9 Assessment 3

- 1 -

Aim: To find out whether students understand the concept of $y = mx + c$

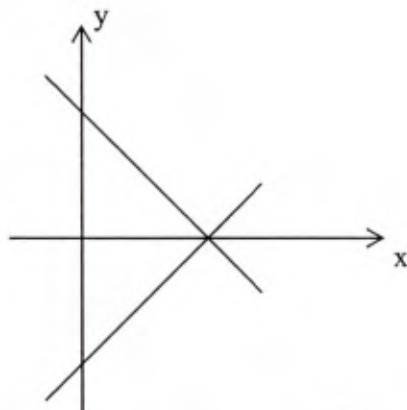
1. The equations of several straight line graphs are given. For each one, find the gradient of the line and where the line cuts the y-axis.

$$y = 6 + 3x$$

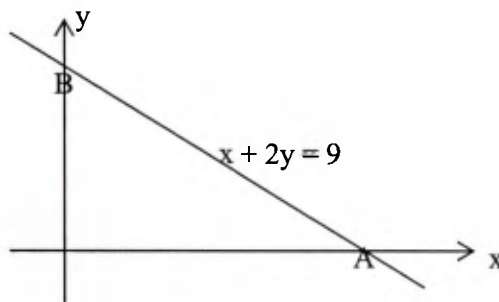
$$y = 4 - x$$

$$2x + y = 8$$

2. The graphs of $y = x - 6$ and $y = 6 - x$ are shown in the diagram. Which is which? Write the equations on the lines.



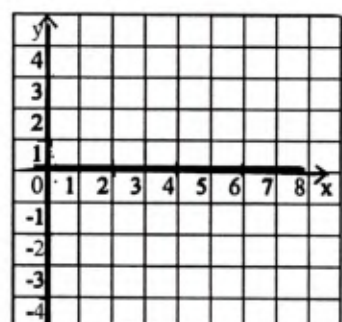
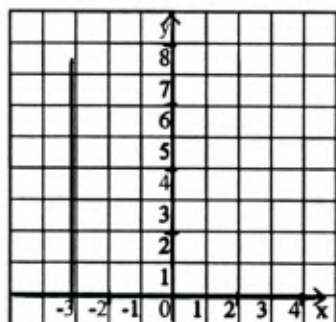
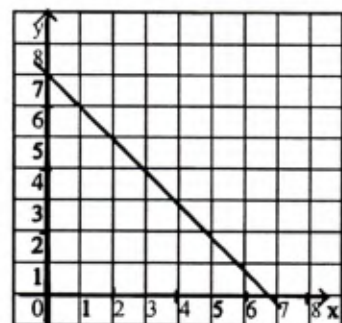
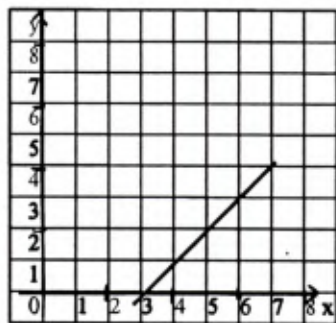
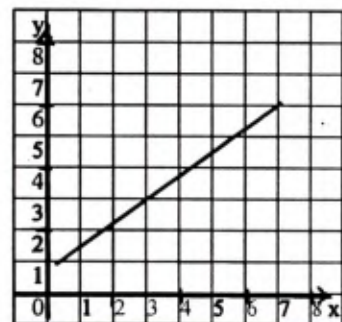
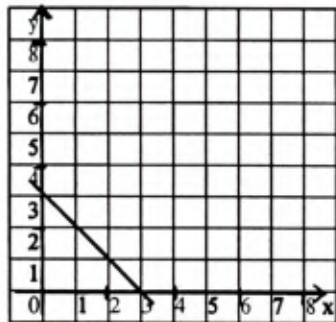
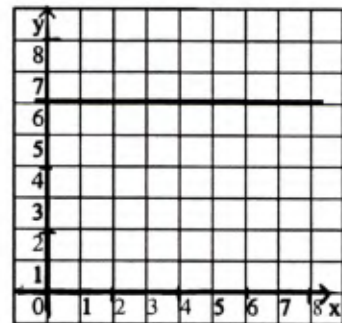
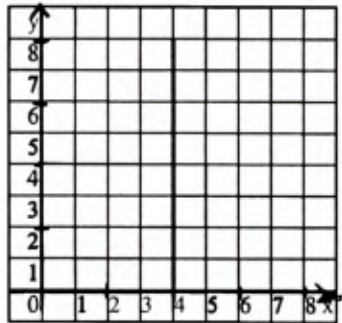
3. Give two points that lie on the graph of $y = 5x - 2$?
4. Does $(-2, 3)$ lie on the graph of $y = 2x + 7$? Explain your answer.
5. The diagram shows a sketch of the graph $x + 2y = 9$. It crosses the x-axis at A and the y-axis at B. What are the coordinates of A and B?



Assessment 3

- 2 -

6. Write down the equation of each of the straight line graph below:



Appendix 10 Assessment 4

Aim: To find out whether students can

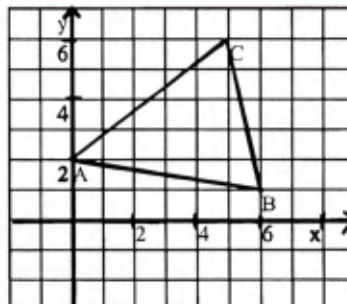
- (i) determine the gradient of a line.
- (ii) determine the equation of a straight line.
- (iii) use the graphing calculator to: draw lines, trace and zoom.

1. In the diagram, find the gradients of

(i) AB,

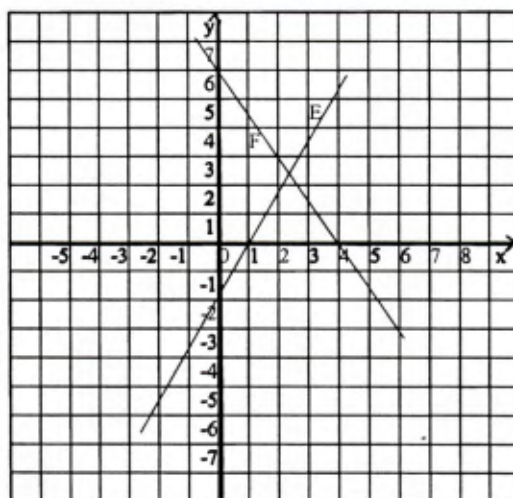
(ii) BC and

(iii) AC.



2. Find the gradient of the straight line passing through the points (1, 6) and (4, 21). Determine the equation of this line.

3. By looking at the gradients and the y-intercepts of the lettered lines on this diagram, write their equations. Using your graphing calculator, zoom in and trace the points where the line E and F intersect.



Appendix 11 Assessment 5

- 1 -

Aim:

To find out whether students

- (i) understand the relationship between the equations and graphs.
- (ii) make connections between the form of the equations and the shape of their graphs.

1. Match the sketch graphs given below with the following equations:

(a) $2y=x$

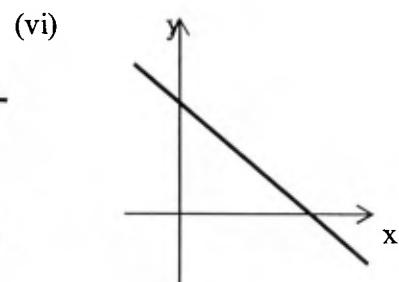
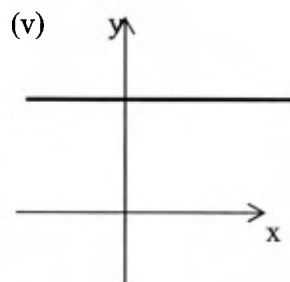
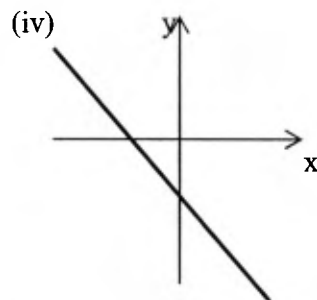
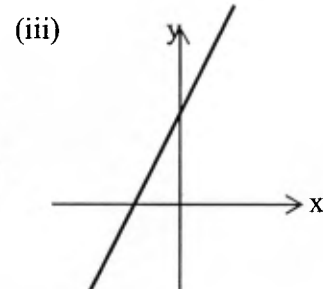
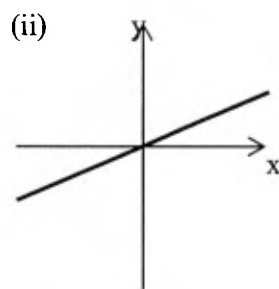
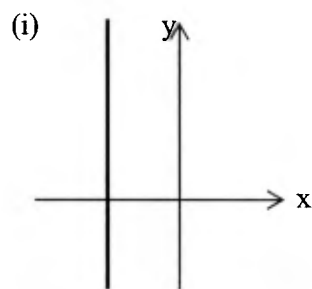
(b) $y=-3x+1$

(c) $y=2$

(d) $x=-1$

(e) $x+y=1$

(f) $3x=y+1$



- 2 -

2. Match the sketch graphs with the equations:

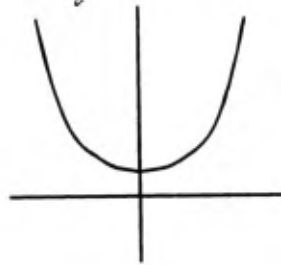
(a) $y = 2x^2 + 3$

(b) $y = \frac{5}{x}$

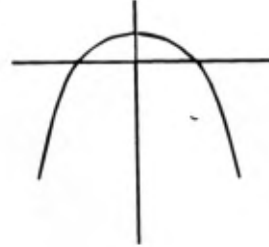
(c) $y = x^3 - x$

(d) $y = 5 - x^2$

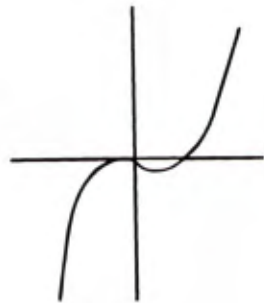
(i)



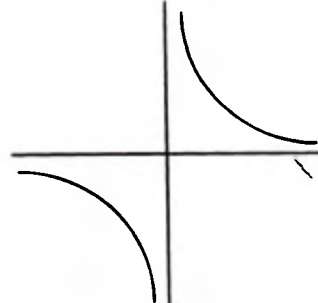
(ii)



(iii)



(iv)



Appendix 12 Pre-test

- 1 -

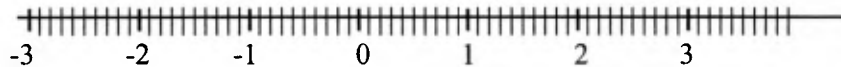
Name:.....

Form:.....

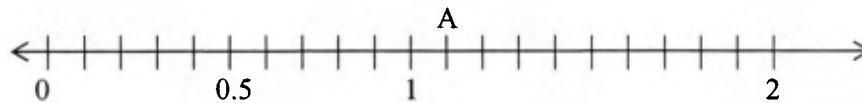
**Oops!**.....

This assessment is to identify which mathematical concepts you already know and to identify those that require assistance and attention to enable you to continue the activities found in the topic Equations and Graphs. As such, do not fear if you cannot answer some of the questions found in this paper.

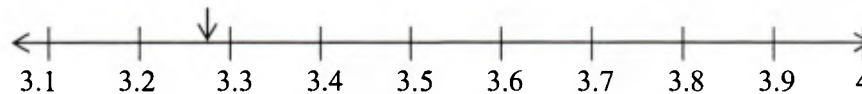
1. This is a number line. Mark 2.5 on it with a \times :



2. What is the number at point A?

Answer:

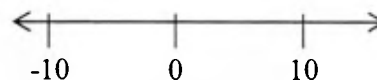
3. The arrow shows a point on the number line.
Write a number that could represent that point.

Answer:

4. Mark an X on the number line where $1\frac{3}{4}$ should be.



5. Which of the following sequences of numbers is in the order in which they occur from left to right on the number line?



- A. 0, $\frac{1}{2}$, -1
- B. 0, -1, $\frac{1}{2}$
- C. -1, $-\frac{1}{2}$, 0
- D. -1, 0, $-\frac{1}{2}$
- E. $\frac{1}{2}$, -1, 0

- 2 -

6. Write down the next three terms in these sequences:

(a) 0, 3, 6, 9, 12, 15,

(b) -10, -8, -6, -4,

(c) 0.6, 0.4, 0.2,

(d) $-1\frac{1}{2}$, -1, $-\frac{1}{2}$, 0,

7. Find the value of:

(a) $-8 + 3$

(b) $-3-8$

(c) $23-30$

(d) $2(9-3)$

(e) $-(4-7)$

8. Find y in the following equations. Leave your answer in terms of x.

(a) $y + x = 6$

(b) $2y = x$

(c) $y+2x-1=0$

- 3 -

(d) $y - 3 = x$

(e) $\frac{x}{y} = 4$

(f) $2y - 7 = x$

(g) $x + \frac{1}{4}y = 1$

(h) $y + 2x - 1 = 0$

9. Find the value of x if

(a) $x - 4 = 3$

(b) $4 - x = 7$

(c) $9 = 5x + 2$

(d) $30 = \frac{2}{5}x + 10$

- 4 -

10. If $x = 2$ and $y = 3$, find the value of xy .
11. If $m = 4$, $c = -2$, find the value of y in terms of x for $y = mx + c$
12. If $y = 3$, and $k = -1$, what is the value of x in the equation $y - k = 2x$
13. If $x = 9$, find the value of
- (a) x^2
 - (b) $2x^2$
 - (c) \sqrt{x}
 - (d) $-2x^2$
14. Write this number in the standard form: 0.00015

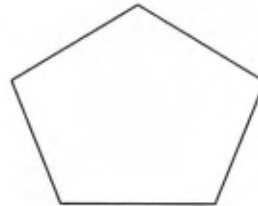
- 5 -

15. Expand the expression:

(a) $2(x+3)$

(b) $(x+2)^2$

16. See the figure opposite. Draw the line of symmetry for the figure:



17. Refer to the table opposite.

(a) What is the value of y when $x=3$?(b) What is the value of y when $x=n$?

x	y
1	8
3	
4	11
7	14
n	

18. From the table shown, a formula that could relate
- m
- and
- n
- is

A. $n=3m$

B. $n=-m^2 + 1$

C. $n=m^2 + 1$

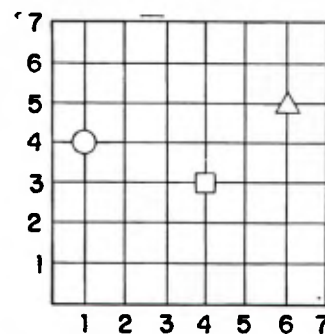
E. $n=2m+1$

m	-1	1	2	4
n	-1	3	5	9

19. The symbol
- Δ
- is on the point (6, 5) and the symbol
- \circ
- is on the point (1, 4)

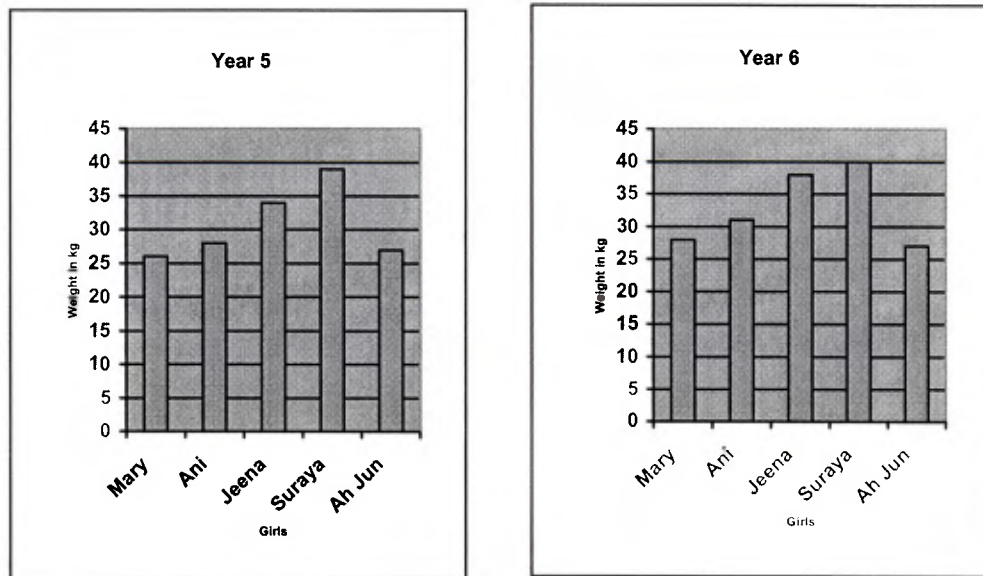
(a) What is the point of the symbol \square ?

(b) Mark X at the point (3, 5).



- 6 -

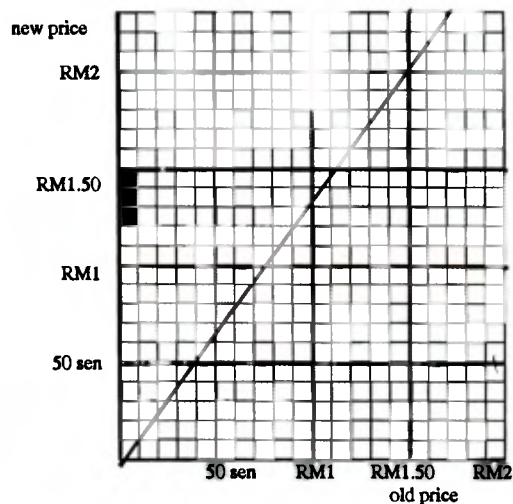
20. 5 girls made a chart to compare their weights when they were in Year 5 and again a year later when they were in Year 6.



- (a) Who was the lightest girl in Year 6?
- (b) Who was the heaviest girl in Year 5?
- (c) Who gained most weight during the year?

20. This graph shows new prices compared with old prices.

- (a) What is the new price of something with an old price of RM1.50?
- (b) What was the old price of something with a new price of RM1.20?
- (c) What is the old price of something with a new price of RM1?



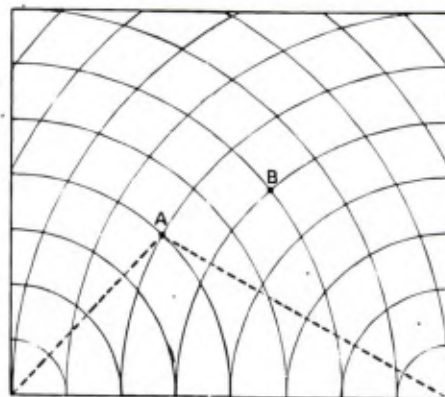
22. Mark the points (20, 15), (14, 3) and (5, -12) on the graph paper provided.

- 7 -

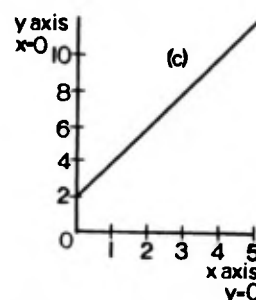
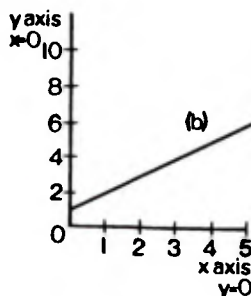
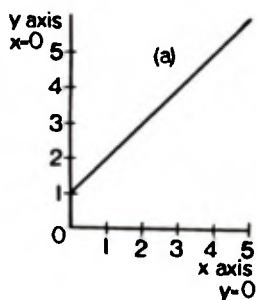
23. The coordinate at point A in the diagram opposite is (4, 6).

What is the coordinate at point B?

Mark the point (7, 6) in the diagram.



24. Here are three straight line graphs. Which are the two that showed the same information?



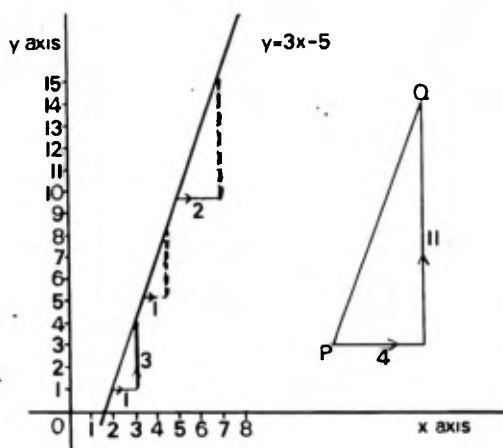
25. Here is the graph of $y=3x-5$.

- (i) How many units is the line of dots on the graph?

- (ii) How many units is the line of dashes? (upper section)

- (iii) Is the line joining PQ parallel to the first line?

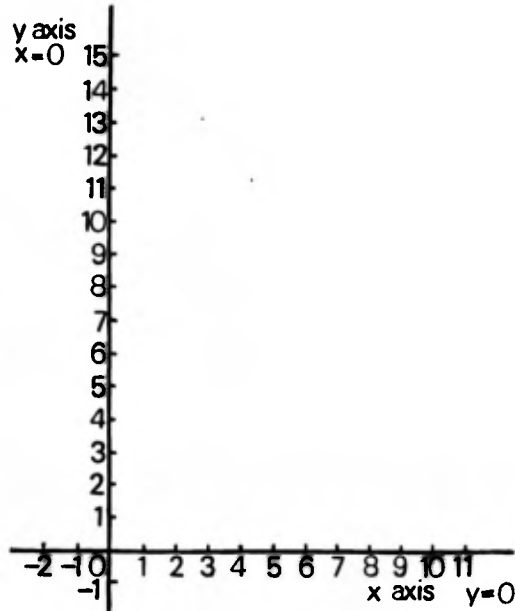
- (iv) Why do you think this?



- 8 -

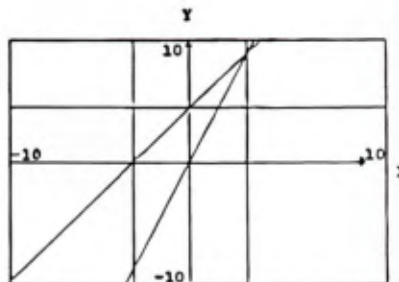
26. Refer to the diagram opposite.

- (i) Plot the points (2, 5), (3, 7), (5, 11).
- (ii) These points lie on a straight line.
Draw the line.
Find some other points on the line and write them down.
- (iii) The point (4.6, 10.2) also lies on the line.
Mark its position approximately.
- (iv) Plot the point $(1\frac{1}{2}, 4)$.
- (v) How many points do you think lie on the line altogether?
- (vi) Are there any points on the line between the points (2, 5) and (3, 7)?
If so, how many?



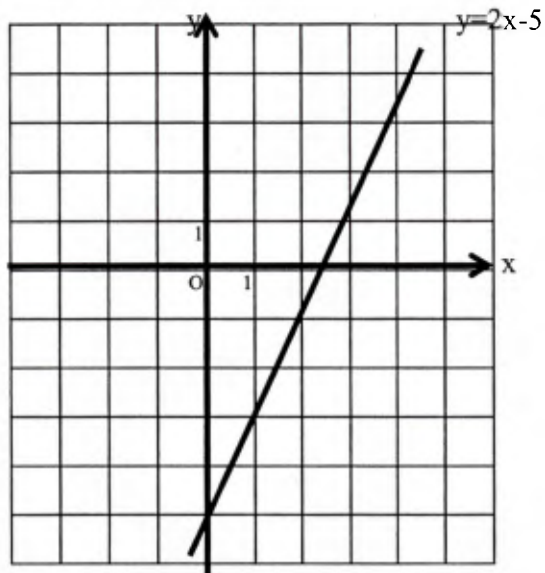
27. Match five out of the eight given equations with the line graphs. Explain your choice.

- (i) $x=4$
(ii) $y=4$
(iii) $y-2x=0$
(iv) $y=x+4$
(v) $y=3x$
(vi) $x=6$
(vii) $x=-4$
(viii) $y=-2x$



- 9 -

28. On the axes below, draw a line parallel to $y=2x-5$ that goes through the origin
0. Write an equation of the new line.



Thank you for trying
your best.



Appendix 13 Post-test

- 1 -

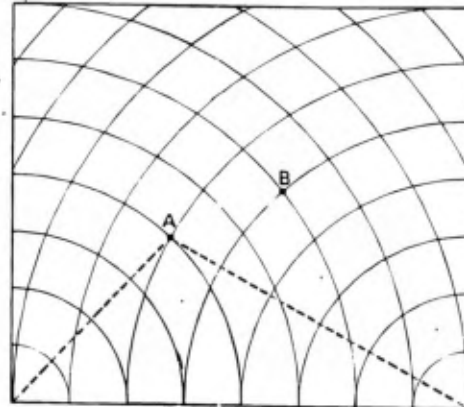
Name:

Please answer all questions.

1. The coordinate at point A in the diagram opposite is (4, 6).

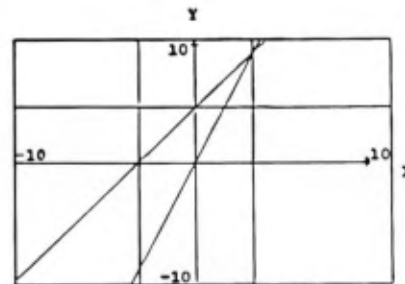
What is the coordinate at point B?

Mark the point (7, 6) in the diagram.

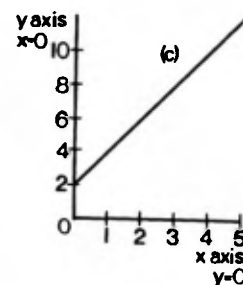
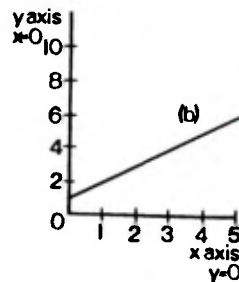
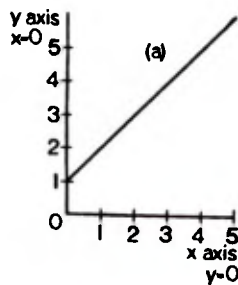


2. Match five out of the eight given equations with the line graphs. Explain your choice.

- | | |
|----------------|----------------|
| (i) $x=4$ | (ii) $y=4$ |
| (iii) $y-2x=0$ | (iv) $y=3x$ |
| (v) $y=x+4$ | (vi) $x=6$ |
| (vii) $x=-4$ | (viii) $y=-2x$ |



3. Here are three straight line graphs. Which are the two that showed the same information?



- 2 -

4. From the table shown, a formula that could relate a and b is

A. $b=3a$

B. $b=-a^2 + 1$

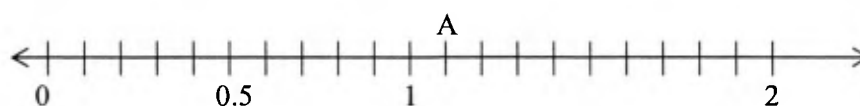
C. $b=a^2 + 1$

D. $b=2a+1$

a	-1	1	2	4
b	-1	3	5	9

5. Mark the points (20,15), (14,3) and (5,-12) on the graph paper provided. Then plot these points on your graphic calculator.

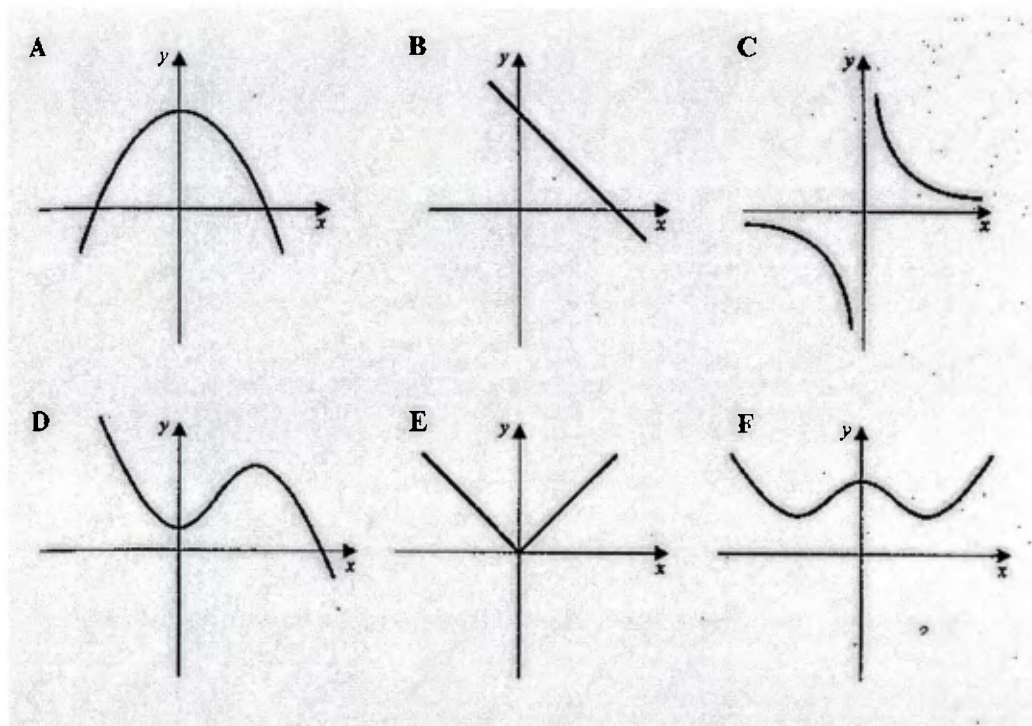
6. What is the number at point A?



Answer:

7. Here are six sketches of graphs. Write down the letter of the sketch which most closely fits these words:

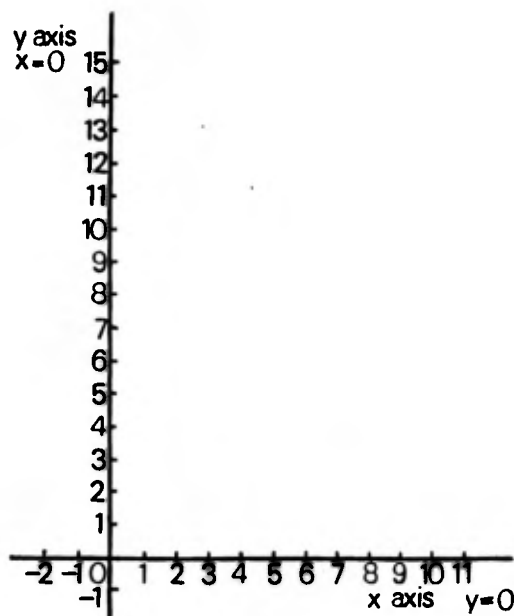
- (i) a cubic graph
 (ii) the graph of $y=1-x^2$
 (iii) a reciprocal graph



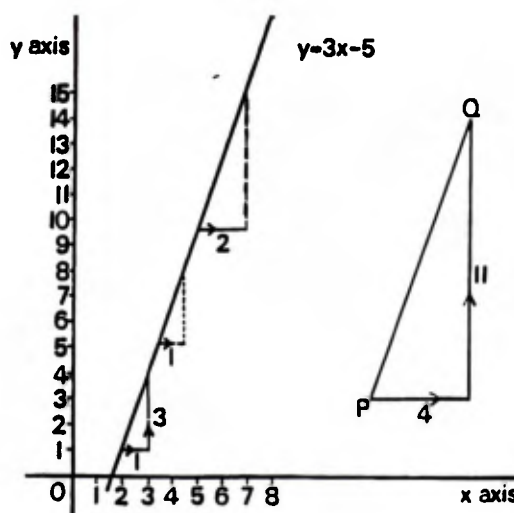
- 3 -

8. Refer to the diagram opposite.

- (i) Plot the points $(2, 5)$, $(3, 7)$, $(5, 11)$.
- (ii) These points lie on a straight line.
Draw the line.
Find some other points on the line and write them down.
- (iii) The point $(4.6, 10.2)$ also lies on the line.
Mark its position approximately.
- (iv) Plot the point $(1\frac{1}{2}, 4)$.
- (v) How many points do you think lie on the line altogether?
- (vi) Are there any points on the line between the points $(2, 5)$ and $(3, 7)$?
If so, how many?

9. Here is the graph of $y=3x-5$.

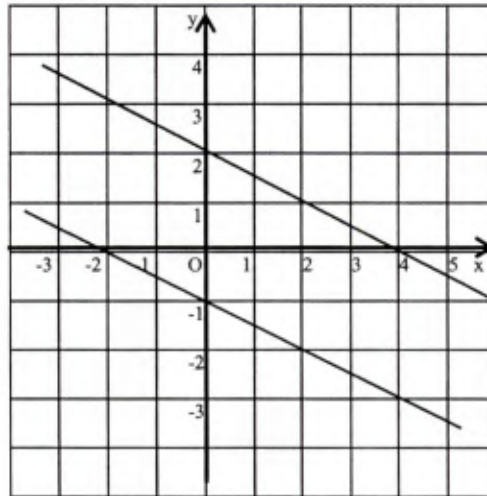
- (i) How many units is the line of dots on the graph?
- (ii) How many units is the line of dashes?
(upper section)
- (iii) Is the line joining PQ parallel to the first line?
- (iv) Why do you think this?



- 4 -

10. Write down:
- (a) the gradient of line (i)
 - (b) the equation of line (i)
 - (c) the equation of line (ii)

Explain your working.



11. Look at these four equations:

- (i) $2x+3y-5=0$
- (ii) $3x+2y+5=0$
- (iii) $4x+6y-5=0$
- (iv) $2x-3y+5=0$

Decide which pair of equations is

- (a) parallel
- (b) cuts the y-axis at the same point

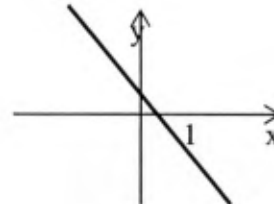
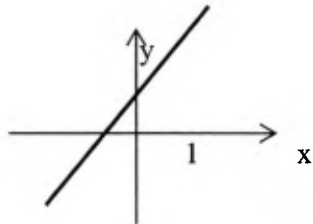
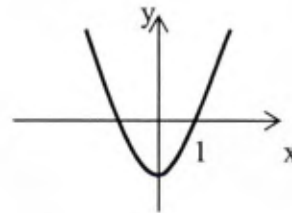
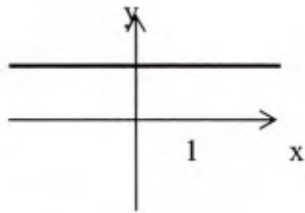
Explain your answer.

12. The equation of a straight line is given as $2y=5x-8$. Determine

- (i) the gradient of the line, and
- (ii) the y-intercept.

- 5 -

13. Look at these graphs carefully:

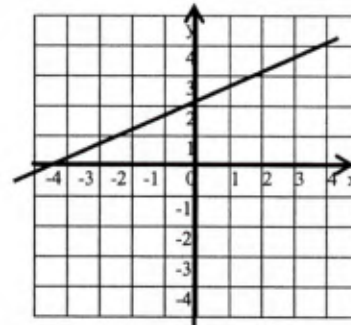


Which is the graph of $y = \frac{3}{2}x + 1$?

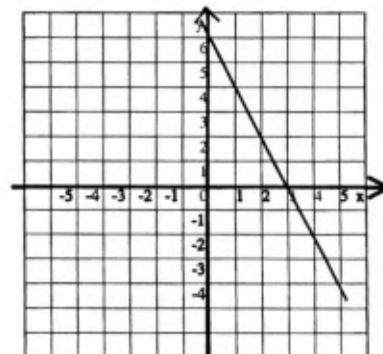
Why?

14. The equation of the line is

- A. $y = 2x - 4$
- B. $y = 2x + 2$
- C. $x - 2y + 2 = 0$
- D. $2y = x + 4$
- E. $y = -2x + 4$



15. Write the equation of the line opposite.



- 6 -

16. Match the following equations and the graphs:

$$y=5x$$

$$y=3x$$

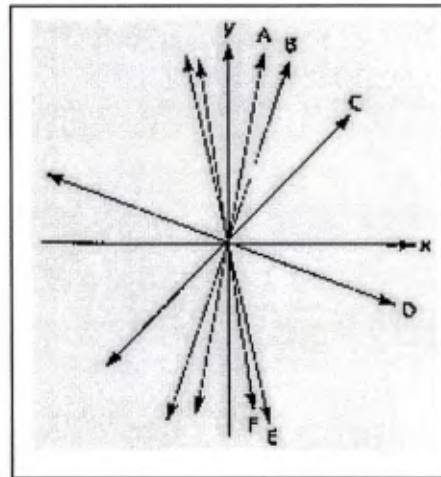
$$y= -4.6x$$

$$y = -\frac{1}{3}x$$

$$y=1.1x$$

$$y=-6x$$

Explain your answer.



17. On the axes below, draw a line parallel to $y=2x-5$ that goes through the origin
0. Write an equation of the new line.

